Solid State Physics
&
Electronics
for
M.Sc. Entrance

IIT-JAM | JNU | TIFR | BHU | ISM | JNCASR | DU

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PHYSICS-PH

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Chapter 1

CRYSTAL STRUCTURE

Solids: The solids are the forms of matter in which atoms and molecules are strongly bound and maintain definite volume and shape unless changed by applying external forces.

Solid state physics: The solid state physics is the study of the physical properties of the atomic/molecular matter in the solid state. These physical properties include crystal structure, bonding, thermal, electrical and magnetic properties.

Classification of solids:

Solids are classified into two categories:
(i) crystalline solids
(ii) non-crystalline solids (amorphous)

Crystalline solids:

This crystalline solids are defined as an ordered periodic arrangement of atoms, molecules or ions.

Properties of crystalline solids:

(1) The internal arrangement of atoms, ions or molecules of crystalline solid is in a definite order.
(2) The crystalline solids have a sharp melting point.
(3) Below the temperature of crystallization, the crystalline materials are in a stable state. As the stable state is the state of minimum energy, materials acquire a state of minimum energy on crystallization.
(4) The orderly arrangement of atoms, ions and molecules in a crystal is extended over a large volume of the crystal, indicating a long range order by the atoms, ions and molecules.
(5) Crystalline solids are anisotropic. This implies that physical properties such as refractive index, conductivity, thermal properties such as expansion etc. are different in different directions. This is due to orderly arrangement of the constituents.
(6) Crystalline solids have uniform chemical composition throughout.

Examples: Diamond, Fe, Ag, Au, Salt, Sugar and Mica etc.

Non-crystalline solids:

Solids in which constituents (atoms, ions or molecules) are arranged in irregular or random manner are called amorphous solids or non-crystalline solids.

Properties of non-crystalline solids:

(1) The constituents are arranged in irregular manner.
(2) They do not have sharp melting point because the bonds between the atoms are not equally strong.
(3) They are isotropic in nature.
(4) Amorphous solids are regarded as supercooled liquids or pseudo solids.

Single crystalline solids:

When the periodicity of the pattern extends throughout a certain piece of solid material, it is called single crystalline solids. Example: Diamond. In a single crystal, same symmetry exists everywhere throughout the crystal.

Polycrystalline solids:
When a large number of small crystal sections (called grains) of various shapes and sizes are packed to one another separated by the interfaces (called grain boundaries) so that the periodicity is interrupted at the grain boundaries, though the structure may be more stable, the solid is called polycrystalline.

Crystallography: The science which deals with the structure of crystalline solids is called crystallography.
Crystal structure: The atomic arrangement in a crystal is called crystal structure.

In nature, most of the solids exit in crystalline form because crystalline form is more stable as compared to amorphous form of solids. The energy released during the formation of crystalline solids is more than the energy released during the formation of amorphous solids. As more released during the formation, the crystal will have less energy, thus the crystal will be more stable.

In order to describe crystal structure in detail, first we will discuss the following terms:

Crystal translation vectors, symmetry operations, crystal lattices and basis.

Crystal translation vectors:

A crystal is regular arrangement of atoms. We can imagine points in space about which these atoms are located. These points are called as lattice points. Thus an ideal crystal is a body composed of atoms arranged on a lattice defined by three fundamental translation vectors $\vec{a}, \vec{b}, \vec{c}$ such that the atomic arrangement looks the same in every respect when viewed from any point $\vec{r}$ as when viewed from the point $\vec{r}'$.

Thus, $\vec{r}' = \vec{r} + \vec{T} \Rightarrow \vec{r}' = \vec{r} + n_1\vec{a} + n_2\vec{b} + n_3\vec{c}$ ... (i)

where n_1, n_2 and n_3 are arbitrary integers and $\vec{T}$ is called a lattice translation operation or crystal translation vector.

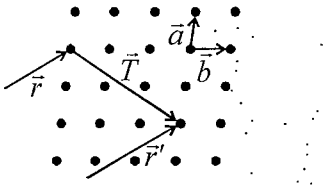
The crystal translation operation is defined as the displacement of a crystal parallel to itself by a crystal translation vector

$$\vec{T} = n_1\vec{a} + n_2\vec{b} + n_3\vec{c} \quad \text{... (ii)}$$

In two dimensional lattice, the vector $\vec{T}$ connects any two points, as shown in figure and given by the following relation

$$\vec{T} = n_1\vec{a} + n_2\vec{b} \quad \text{... (iii)}$$

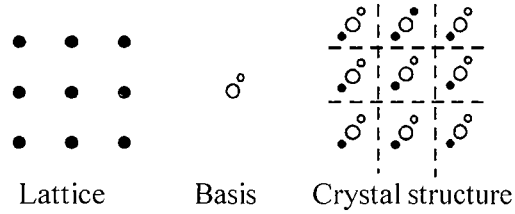
Lattice: A lattice is defined as parallel net like arrangement of points provided the environment about any point is identical with environment about any other point.
The figure shown below is an array of points in two dimensions, is called the plane lattice. If we extend this array of points in three dimensions then the array of points is called space lattice.



Basis: In order to get a crystal structure, an atom or group of atoms must be placed on each lattice point in regular manner. Such an atom or group of atoms is called the basis and acts as a building-block or unit for the crystal structure.

Mathematically, crystal structure can be expressed as

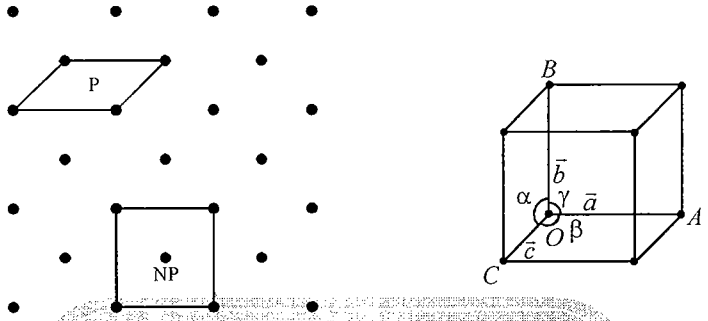
Space lattice + Basis = Crystal structure



Unit cell: The parallelograms formed by the translation vectors are regarded as builiding blocks for constructing the complete lattice and are called as unit cells of the lattice. In three dimensions, this unit cell will be a parallelopiped. Therefore, a unit cell is defined as the smallest unit of lattice points which on repetition produces the complete lattice. There are two types of unit cells:

***(i)* Primitive unit cell (P):** The lattice points (atoms) belonging to primitive unit cell lie at its corners. The effective number of lattice points in a primitive unit cell is one.

***(ii)* Non-primitive unit cell (NP):** The lattice points (atoms) belonging to non-primitive cell lie at corners as well as other locations. The effective number of lattice points in a non-primitive unit cell is greater than one.



Wigner-Seitz cell:

A method for the construction of unit cell from the lattice points was given by Wigner-Seitz. The procedure of the construction is as follows:

(1) connect a given lattice point to all the nearby lattice points

(2) draw normals at the midpoints of lines connecting the lattice points.

The smallest volume enclosed by the normals is the required primitive cell.

The cell constructed by this procedure is known as Wigner-Seitz cell.

In the figure, a two dimensional oblique lattice is constructed by this procedure and a hexagon is the result. Similarly, for a three dimensional BCC lattice, it is a truncated octahedron and for a FCC lattice, it is a rhombic dodecahedron, respectively.

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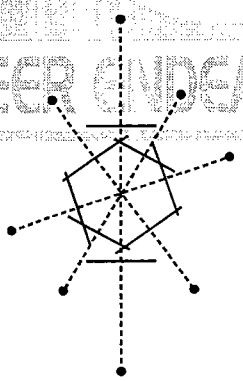


Construction of Wigner-Seitz primitive cell

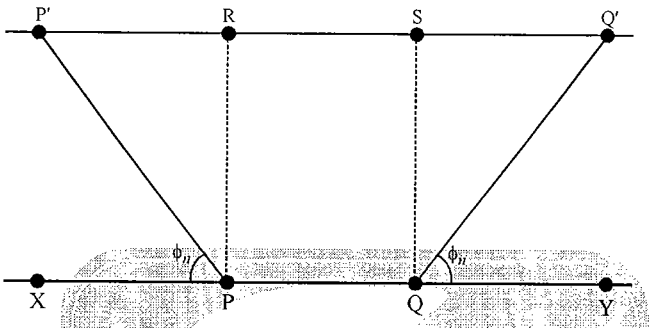
Crystal symmetry:

The operations that leaves the crystal and its environment invariant are called symmetry operations. There are four symmetry operations:

(i) Translational symmetry: A crystal translation is already defined in equations (ii) and (iii). Translation operations repeat the lattice infinite number of times along a given direction and leaves it unchanged.

(ii) Rotational symmetry: If after rotation of a lattice about an axis (or a point in two dimensional lattice) by some angle ϕ_n, the body appears as it did prior to rotation, the lattice is said to possess rotational symmetry. Axis of symmetry may thus be defined as a line such that the crystal assumes a congruent position (self-coincidence), for every rotation of $\left[\frac{360°}{n}\right]$. The value of n decide the fold of the axis.

Only one, two, three, four and six-fold rotation axes of symmetry alone are possible in single crystal. The fact that 5-fold rotation axis is not compatible with translation symmetry operation is proved as follows: Consider a lattice row XPQY.



Let the lattice translation be a and the lattice have n-fold rotation axis of symmetry.

Let us rotate the vectors PX and QY through an angle $\phi_n = \left(\frac{360°}{n}\right)$ in clockwise and anticlockwise directions respectively. The tips of the vectors P′ and Q′ in the new position must be lattice points if the lattice possess n-fold rotation axis of symmetry.

As per definition, rotation operation must be the lattice invariant. Clearly P′Q′ must be parallel to PQ and must be equal to an integral multiple of the lattice translation a,

$P'Q' = ma$ where m is an integer.

Thus, $\cos\phi_n = \frac{P'R}{PP'}$ and $P'R = PP'\cos\phi_n = a\cos\phi_n$ and $SQ' = a\cos\phi_n \Rightarrow P'Q' = P'R + RS + SQ'$

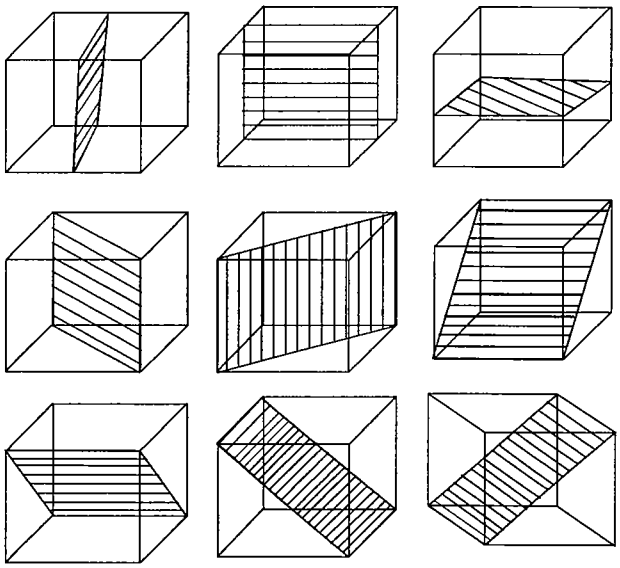
$\Rightarrow P'Q' = 2a\cos\phi_n + a = ma \Rightarrow 1 + 2\cos\phi_n = m \Rightarrow 2\cos\phi_n = m - 1 = N \Rightarrow \cos\phi_n = \left(\frac{m-1}{2}\right) = \frac{N}{2}$

N	$N/2$	$\cos\phi_n$	ϕ_n	N
−2	−1	−1	180°	2
−1	−1/2	−1/2	120°	3
0	0	0	90°	4
1	1/2	1/2	60°	6
2	1	1	360°	1

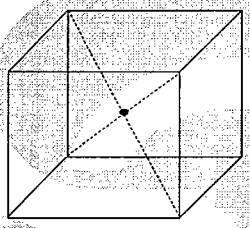
Hence, the permissible values of n are 2, 3, 4, 6, 1. Thus we conclude that 5-fold rotation is not permissible because it is not compatible with lattice translation symmetry.

Similarly, we can say that other rotations such as 7-fold rotation are also not permissible.

(iii) Reflection symmetry : A lattice possess reflection symmetry if there exist a plane (or a line into two dimensions) in the lattice which divides it into two identical halves which are mirror image of each other. Examples of this symmetry are shown in following figures.



(iv) Inversion symmetry : This is a point operation which is applicable to three-dimensional lattices only. This symmetry element implies that each point located at $\vec{r}$ relative to a lattice point has an identical point located at $-\vec{r}$ relative to the same lattice point.



Point group : Symmetry operations performed about a point or a line are called point group. There are total 10 point groups in two dimensional lattices and 32 point groups in three-dimensional lattices.

Space group : Symmetry operations performed by translation as well are called space group. There are total 17 space groups in two dimensional lattices and 230 space groups in three-dimensional lattices.

Types of lattices :

(i) Two-dimensional lattices : The four crystal systems of two dimensional lattices are given in following table.

Sr. No.	Crystal system	Bravais lattice	Conventional unit cell / Unit cell parameters
1.	Oblique	Oblique	Parallelogram $a \neq b, \gamma \neq 90°$
2.	Rectangular	(a) Rectangular primitive (b) Rectangular centered	Rectangle $a \neq b, \gamma = 90°$
3.	Square	Square	Square $a = b, \gamma = 90°$
4.	Hexagonal	Hexagonal	60° Rhombus $a = b, \gamma = 120°$

(ii) Three-dimensional lattices : There are 32 crystal structures based on geometrical considerations (i.e, symmetry and internal structure). But it is common practice to divide all crystal systems into seven groups or basic systems.

These seven basic crystal systems are shown in table with decreasing order of symmetry.

The crystallographic axes $\vec{a}, \vec{b}$ and $\vec{c}$ determines the size and shape of the unit cell. The angles α, β and γ

represent the angles between the vectors $\vec{b}$ and $\vec{c}$, $\vec{c}$ and $\vec{a}$ and $\vec{a}$ and $\vec{b}$ respectively. These α, β, γ and a, b, c are collectively known as lattice parameters.

Sr. No.	Crystal system	Lattice parameters	Bravais lattice	Lattice symbol	Examples
1.	Cubic	$a = b = c$ $\alpha = \beta = \gamma = 90°$	Simple Body-centered Face-centered	P I F	Cu, Ag, Fe, Na, NaCl, CsCl
2.	Tetragonal	$a = b \neq c$ $\alpha = \beta = \gamma = 90°$	Simple Body-centered	P I	β-Sn, TiO_2
3.	Orthorhombic	$a \neq b \neq c$ $\alpha = \beta = \gamma = 90°$	Simple Body-centered End-centered Face-centered	P I C F	Ga, Fe_3C (cementite)
4.	Rhombohedral or Trigonal	$a = b = c$ $\alpha = \beta = \gamma \neq 90°$	Simple	P	As, Sb, Bi
5.	Hexagonal	$a = b \neq c$ $\alpha = \beta = 90°$, $\gamma = 120°$	Simple	P	Mg, Zn, Cd, NiAs
6.	Monoclinic	$a \neq b \neq c$ $\alpha = \gamma = 90° \neq \beta$	Simple End-centered	P C	$CaSO_4.2H_2O$ (gypsum)
7.	Triclinic	$a \neq b \neq c$ $\alpha \neq \beta \neq \gamma \neq 90°$	Simple	P	$K_2Cr_2O_7$

Miller indices:

Crystal lattice may be regarded as made up of an aggregate of a set of parallel equidistant planes passing through the lattice points. These equidistant planes are called as lattice planes. The scheme to represent the orientation of lattice planes was given by Miller (a British crystallographer). He has given some indices to represent a set of planes. The indices of planes are known as Miller indices. For a general set of planes, Miller indices are written as $(h\,k\,l)$.

The steps involved in calculation of Miller indices is given below.

(i) Determine the coordinates of the intercepts made by the plane along the three crystallographic axes (x, y, z axes).

x	y	z	
pa	qb	rc	
$2a$	$3b$	c	(suppose)

$\Rightarrow p = 2, q = 3$ and $r = 1$

(ii) Express the intercepts as multiples of the unit cell dimensions or lattice parameter along the axes i.e.,

$\frac{2a}{a}$	$\frac{3b}{b}$	$\frac{c}{c}$
2	3	1

(iii) Determine the reciprocals of these numbers $\frac{1}{2}\quad\frac{1}{3}\quad\frac{1}{1}$.

(iv) Reduce these reciprocals to the smallest set of integral numbers and enclose them in bracket with comma.

$6\times\frac{1}{2}\qquad 6\times\frac{1}{3}\qquad 6\times 1$

$(3\qquad 2\qquad 6)$

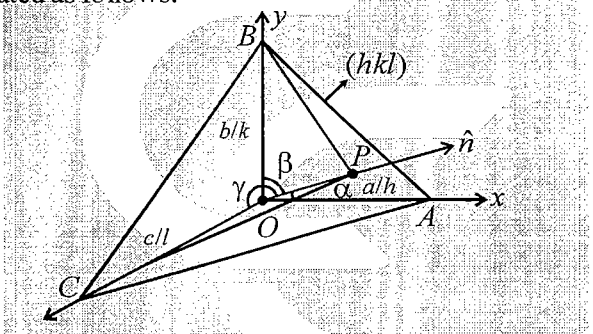
Thus the Miller indices of the planes is (3 2 6).

Thus, Miller indices may be defined as the reciprocals of the intercepts made by plane on the cyrstallographic axes when reduces to smallest numbers.

$$h:k:l=\frac{a}{\text{OA}}:\frac{b}{\text{OB}}:\frac{c}{\text{OC}}$$

where, a, b and c are lattice parameters along x-, y- and z-axes respectively. OA, OB and OC are intercepts on a-, b- and c-axis respectively.

Relation between interplanar spacing and lattice parameters : If the plane have $(h\,k\,l)$ miller indices then its intercepts can be calculated as follows:



$$h:k:l=\frac{a}{OA}:\frac{b}{OB}:\frac{c}{OC}$$

Therefore, intercepts $OA=\frac{a}{h}$, $OB=\frac{b}{k}$ and $OC=\frac{c}{l}$.

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Now, if we draw a perpendicular (normal) from origin (O) to the plane $(h\,k\,l)$ at P. Then OP will be the interplanar spacing (d) between two planes which are parallel to $(h\,k\,l)$ and also passes through O.

$$\therefore\quad d=OP=\frac{a}{h}\cos\alpha=\frac{b}{k}\cos\beta=\frac{c}{l}\cos\gamma$$

where α, β and γ represent the angles between the normal and the axes $\vec{a}$, $\vec{b}$ and $\vec{c}$ respectively and $\cos\alpha$, $\cos\beta$ and $\cos\gamma$ represent the direction cosines of the normal to the plane $(h\,k\,l)$.

Cosine rule: $\cos^2\alpha+\cos^2\beta+\cos^2\gamma=1$

$$\Rightarrow\quad\left(\frac{hd}{a}\right)^2+\left(\frac{kd}{b}\right)^2+\left(\frac{ld}{c}\right)^2=1$$

$$\Rightarrow\quad d^2\left(\frac{h^2}{a^2}+\frac{k^2}{b^2}+\frac{l^2}{c^2}\right)=1$$

$$\Rightarrow \quad d = \left(\frac{h^2}{a^2} + \frac{k^2}{b^2} + \frac{l^2}{c^2}\right)^{-1/2} \quad \text{or} \quad d = \frac{1}{\sqrt{\frac{h^2}{a^2} + \frac{k^2}{b^2} + \frac{l^2}{c^2}}}$$

There is another way to show this relation. Let $\hat{n}$ be the unit vector in the direction $\overrightarrow{OP}$.

$$\therefore \quad d = \hat{n} \cdot \frac{\vec{a}}{h} = \hat{n} \cdot \frac{\vec{b}}{k} = \hat{n} \cdot \frac{\vec{c}}{l}$$

The equation of plane, $f(x, y, z) = \frac{hx}{a} + \frac{ky}{b} + \frac{lz}{c}$

For a surface $f(x, y, z) =$ constant, ∇f represents the vector normal to it.

$$\therefore \quad \hat{n} = \frac{\nabla f}{|\nabla f|} = \frac{\left(\frac{h}{a}\right)\hat{i} + \left(\frac{k}{b}\right)\hat{j} + \left(\frac{l}{c}\right)\hat{k}}{\sqrt{\frac{h^2}{a^2} + \frac{k^2}{b^2} + \frac{l^2}{c^2}}}$$

$$\therefore \quad d = \hat{n} \cdot \frac{\vec{a}}{h} = \left\{\frac{\left(\frac{h}{a}\right)\hat{i} + \left(\frac{k}{b}\right)\hat{j} + \left(\frac{l}{c}\right)\hat{k}}{\sqrt{\left(\frac{h^2}{a^2} + \frac{k^2}{b^2} + \frac{l^2}{c^2}\right)}}\right\} \cdot \frac{\vec{a}}{h}$$

$$\Rightarrow \quad d = \frac{1}{\left(\frac{h^2}{a^2} + \frac{k^2}{b^2} + \frac{l^2}{c^2}\right)^{1/2}}$$

(where $a \neq b \neq c$, therefore this relation corresponds to orthorhombic structure)

For cubic structure, $a = b = c$, $d = \frac{a}{\sqrt{h^2 + k^2 + l^2}}$

For tetragonal structure, $a = b \neq c$, $d = \frac{1}{\sqrt{\left(\frac{h^2 + k^2}{a^2}\right) + \frac{l^2}{c^2}}}$.

Different crystal structures:

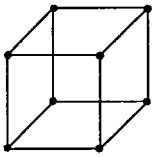
Cubic crystal structures:

(i) Simple cubic crystal structure (SC):

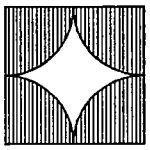
In a simple cubic structure there is one lattice point (atom) at each of the eight corners (shares 1/8 part of the atom) of the cubic unit cell. This is primitive unit cell structure of cubic crystal system. Properties of simple cubic strucutre are given below.

(a) Number of nearest neighbours:

If we take a lattice point at one corner as the origin, it is observed that this atom is surrounded by six equidistant nearest neighbours at coordinates (+a, 0, 0), (-a, 0 ,0), (0, +a, 0), (0, -a, 0), (0, 0, a) and (0, 0, -a). The coordination number is defined as the number of nearest neighbours to a given atom in the crystal lattice. Thus, for simple cubic structure, the coordination number is six. The distance of these nearest neighbours is a.



Unit cell of SC structure

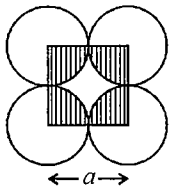


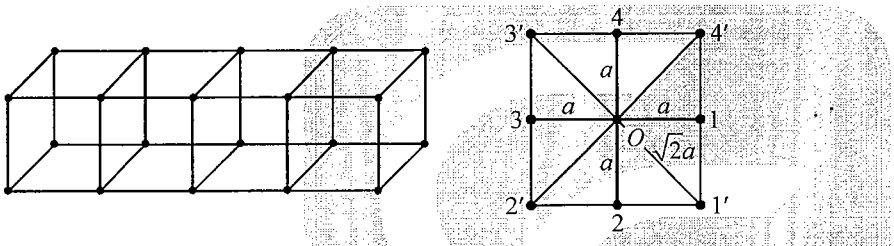
Plane view of SC structure

In this structure, each corner atom is shared by eight unit cell.

(b) Number of second nearest neighbours: There are 12 second nearest neighbours at distance $\sqrt{2}a$.

(c) Atomic radius: From this figure, $a = 2r \Rightarrow r = \frac{a}{2}$.





(d) Number of atoms per unit cell: The number of effective lattice points (atoms) per unit cell is given by

$$n_{eff} = n_i + \frac{n_f}{2} + \frac{n_c}{8}$$

where, n_i is the number of lattice points (atoms) completely inside the cell and n_f and n_c represent the lattice points occupying face center and corner positions of the cell, respectively.

For SC structure, $n_i = 0, n_f = 0$ and $n_c = 8, n_{eff} = 1 \times 0 + \frac{1}{2} \times 0 + \frac{1}{8} \times 8 = 1$.

(e) Atomic packing fraction: The packing of atoms in a unit cell of the crystal structure of a material is represented by atomic packing fraction (APF). The APF is defined as the ratio of volume occupied by the atoms in a unit cell to that of the volume of the unit cell.

$$\text{APF} = \left(\frac{\text{volume of the atoms per unit cell}}{\text{volume of the unit cell}} \right) = \left(\frac{n_{eff} \times \text{volume of each atom}}{\text{volume of unit cell}} \right)$$

For simple cubic structure, $n_{eff} = 1$

$$\therefore \quad \text{APF} = \frac{1 \times \frac{4}{3}\pi r^3}{a^3} = \frac{1 \times \frac{4}{3}\pi \left(\frac{a}{2}\right)^3}{a^3} = \frac{4}{3}\pi \times \frac{1}{8} = \frac{\pi}{6} = 0.52.$$

$\therefore$ APF % = 52%.

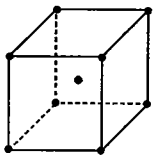
(f) Type of packing: Simple cubic structure is a loose-packed structure.

(g) Example : Only Polonium exhibits this type of structure at room temperature.

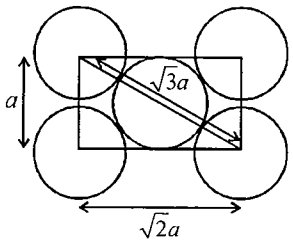
(ii) Body centered cubic structure (BCC):

In the BCC structure, there is one lattice point (atom) at each corners (shares 1/8th part of the atom) of the cubic unit cell and one lattice point (atom) exit at the body center. This is non-primitive unit cell structure of cubic crystal system.

The corner atoms do not touch each other, but each corner atom touches the body centered atom along the body diagonal.



Conventional unit cell of BCC structure



Plane view of BCC structure

(a) Number of nearest neighbours: If we take, the body centered lattice point (atom) of cell as origin then all the corner atoms at coordinates $\left(+\frac{a}{2},+\frac{a}{2},+\frac{a}{2}\right),\left(+\frac{a}{2},+\frac{a}{2},-\frac{a}{2}\right),\left(+\frac{a}{2},-\frac{a}{2},+\frac{a}{2}\right),\left(+\frac{a}{2},-\frac{a}{2},-\frac{a}{2}\right),$ $\left(-\frac{a}{2},+\frac{a}{2},+\frac{a}{2}\right),\left(-\frac{a}{2},+\frac{a}{2},-\frac{a}{2}\right),\left(-\frac{a}{2},-\frac{a}{2},+\frac{a}{2}\right)$ and $\left(-\frac{a}{2},-\frac{a}{2},-\frac{a}{2}\right)$ will be nearest neighbours to the body centered atom. Since, there are 8 corners therefore, number of nearest neighbours (i.e. coordination number) will be 8. The distance of these nearest neighbours from reference lattice point is $\frac{\sqrt{3}a}{2}$.

(b) Number of second nearest neighbours: There are 6 second nearest neighbours at distance a in BCC structures.

(c) Atomic radius: From above figure, $4r=\sqrt{3}a \Rightarrow r=\frac{\sqrt{3}a}{4}$.

(d) Number of atoms per unit cell: The number of effective lattice points (atoms) per unit cell is given by

$$n_{eff}=n_i+\frac{n_f}{2}+\frac{n_c}{8}$$

where, n_i is the number of lattice points (atoms) completely inside the cell and n_f and n_c represent the lattice points occupying face center and corner positions of the cell, respectively.

For BCC structure, $n_i=1$, $n_f=0$, and $n_c=8$, $n_{eff}=1\times1+\frac{1}{2}\times0+\frac{1}{8}\times8=2$.

(e) Atomic packing fraction: $\text{APF}=\left[\frac{n_{eff}\times\text{volume of each atom}}{\text{volume of unit cell}}\right]$

For BCC structure, $n_{eff}=2$

$$\therefore\ \text{APF}=\frac{2\times\frac{4}{3}\pi r^3}{a^3}=\frac{2\times\frac{4}{3}\pi\left(\frac{a\sqrt{3}}{4}\right)^3}{a^3}=\frac{\sqrt{3}\pi}{8}=0.68$$

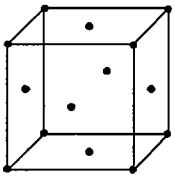
$\therefore$ APF% = 68%

(f) Type of packing: Body centered cubic structure is a loose-packed structure.

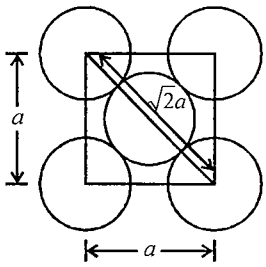
(g) Examples: Li, Na, K, Mo etc.

(iii) Face-centered cubic structure (FCC):

In FCC structure, one atom lies at each corner of the cube in addition to one atom at the center of each face. This is also a non-primitive unit cell structure of cubic crystal system.



Conventional unit cell of FCC structure



Plane view of FCC structure

(a) Number of nearest neighbours : If we take, one of the face centered lattice point (atom) of unit cell as origin then all the 12 corner atoms at coordinates $\left(+\frac{a}{2},+\frac{a}{2},0\right),\left(+\frac{a}{2},-\frac{a}{2},0\right),\left(-\frac{a}{2},-\frac{a}{2},0\right),\left(-\frac{a}{2},+\frac{a}{2},0\right),$ $\left(+\frac{a}{2},0,+\frac{a}{2}\right),\left(+\frac{a}{2},0,-\frac{a}{2}\right),\left(-\frac{a}{2},0,-\frac{a}{2}\right),\left(-\frac{a}{2},0,+\frac{a}{2}\right),\left(0,+\frac{a}{2},+\frac{a}{2}\right),\left(0,+\frac{a}{2},-\frac{a}{2}\right),\left(0,-\frac{a}{2},-\frac{a}{2}\right)$ and $\left(0,-\frac{a}{2},+\frac{a}{2}\right)$ will be nearest neighbours to this face centered atom. These atoms will lie at corners of the three faces in x, y and z-axes passing through the reference atom. Hence, total number of nearest neighbours (i.e. coordination number) will be 12. The distance of these nearest neighbours from reference lattice point is $\frac{\sqrt{2}a}{2}$ or $\frac{a}{\sqrt{2}}$.

(b) Number of second nearest neighbours : There are 6 second nearest neighbours at distance a in FCC structures.

(c) Atomic radius: From above figure, $4r=\sqrt{2}a \Rightarrow r=\frac{\sqrt{2}a}{4}=\frac{a}{2\sqrt{2}}$.

(d) Number of atoms per unit cell: The number of effective lattice points (atoms) per unit cell is given by

$$\therefore \quad n_{eff}=n_i+\frac{n_f}{2}+\frac{n_c}{8}$$

where, n_i is the number of lattice points (atoms) completely inside the cell and n_f and n_c represent the lattice points occupying face center and corner positions of the cell, respectively.

For FCC structure, $n_i=0$, $n_f=6$ and $n_c=8$, $n_{eff}=1\times 0+\frac{1}{2}\times 6+\frac{1}{8}\times 8=3+1=4$.

(e) Atomic packing fraction : $\text{APF}=\left[\frac{n_{eff}\times \text{volume of each atom}}{\text{volume of unit cell}}\right]$

For FCC structure, $n_{eff}=4$

$$\therefore \quad \text{APF} = \frac{4 \times \frac{4}{3}\pi r^3}{a^3} = \frac{4 \times \frac{4}{3}\pi \left(\frac{\sqrt{2}a}{4}\right)^3}{a^3} = \frac{\sqrt{2}\pi}{6} = 0.74$$

$\therefore$ APF% = 74%

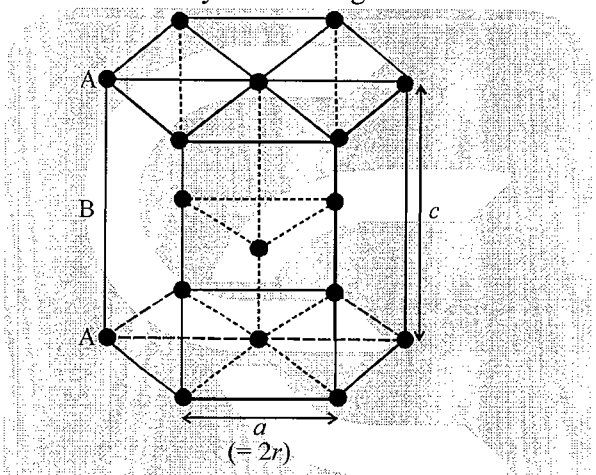
(f) Type of packing: Face-centered cubic strucutre is a close-packed structure.

(g) Examples: Cu, Au, Ag, Al etc.

Hexagonal close packed structure (HCP):

The specific hexagonal structure formed by Magnesium is shown in figure. The unit cell contains one atom at each corner, one atom at the centre of each hexagonal faces and three more atoms within the body of the cell.

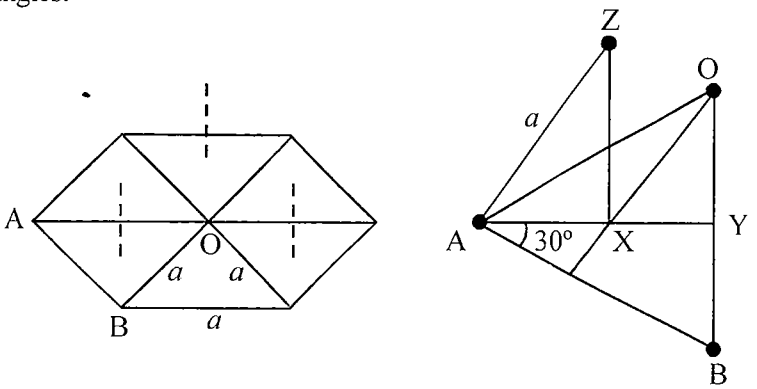
(a) Number of nearest neighbours: Each atom touches six atoms in its plane, three atoms in the layer below its plane, and three atoms in the layer above. Hence the coordination number of this structure is 12. Further the atoms touch each other along the edge of the hexagon. Thus, $a = 2r$. The top layer contains seven atoms. Each corner atom is shared by 6 surrounding hexagon cells and the centre atom is shared by two adjacent cells. The three atoms within the body of the cell are fully contributing to the cell.



Conventional unit cell of HCP structure

(b) Number of atoms per unit cell: In this structure, there are 12 atoms at 12 corners of the hexagonal unit cell, two atoms at the center of top and bottom hexagons and three atoms inside the unit cell. Thus, the effective number of atoms in a unit cell is $\frac{1}{6} \times 12 + \frac{1}{2} \times 2 + 1 \times 3 = 6$.

(c) Calculation of (c/a) ratio for an ideal HCP structure : Let c be the height of the unit cell and a equilateral triangles at the top or base of the hexagonal cell. These three atoms just rest on the three atoms at the corners of the triangles.



Plane view of HCP structure

In the triangle ABY, $\cos 30° = \frac{AY}{AB}$.

$AY = a\cos 30° = \frac{a\sqrt{3}}{2}$ and $(AZ)^2 = (AX)^2 + (ZX)^2$ ... (i)

In triangle AXZ, $AX = \frac{2}{3}AY = \frac{2}{3}\left(\frac{a\sqrt{3}}{2}\right) = \frac{a}{\sqrt{3}}$

Substituting AZ, AX and ZX in equation (i), we get

$$a^2 = \frac{a^2}{3} + \frac{c^2}{4} \Rightarrow \frac{c^2}{4} = a^2 - \frac{a^2}{3} = \frac{2a^2}{3} \Rightarrow \frac{c^2}{a^2} = \frac{8}{3} \Rightarrow \frac{c}{a} = \sqrt{\left(\frac{8}{3}\right)}$$

(d) Volume of the unit cell :

Area of the base = six times the area of the triangle AOB

Area of the triangle, $AOB = \frac{1}{2} \times BO \times AY = \frac{1}{2} \times a \times \frac{\sqrt{3}a}{2} = \frac{\sqrt{3}a^2}{4}$

Thus, area of the base $= 6 \times \frac{\sqrt{3}a^2}{4} = \frac{3}{2}\sqrt{3}a^2$

Volume of the cell $= \frac{3}{2}\sqrt{3}a^2c$.

(e) Calculation of APF for HCP structure :

Coordination number, $N = 12$; Lattice constant, $a = 2r$

Nearest neighbour distance, $2r = a$; Number of atoms per unit cell, $n = 6$

Volume of one atom in a unit cell $= \frac{4}{3}\pi r^3 = \frac{4}{3}\pi\left(\frac{a}{2}\right)^3 = \frac{\pi a^3}{6}$

Volume of unit cell $= \frac{3}{2}\sqrt{3}a^2c$ with $c/a = \left(\frac{8}{3}\right)^{1/2}$

Atomic packing factor, $APF = \left[\frac{n_{eff} \times \text{volume of each atom}}{\text{volume of unit cell}}\right]$

$$\therefore \quad APF = \frac{6 \times (\pi a^3/6)}{\frac{3}{2}\sqrt{3}\,a^2c} = \frac{2\pi}{3\sqrt{3}}\left(\frac{a}{c}\right) = \frac{2\pi}{3\sqrt{3}}\left(\frac{3}{8}\right)^{1/2} = \frac{\sqrt{2}\pi}{6} = \frac{\pi}{3\sqrt{2}} = 0.74$$

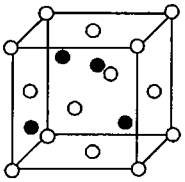
$\therefore$ APF% = 74%

(f) Type of packing: The packing fraction of FCC and HCP are 74% and hence they are known as close-packed structures.

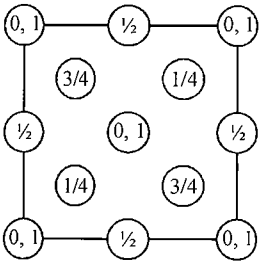
(g) Examples: Magnesium, zinc, cadmium etc.

Diamond Cubic Structure:

The diamond lattice can be considered to be formed by two interpenetrating FCC lattices along the body diagonal by one-fourth of cube edge. One sublattice has its origin at the point (0, 0, 0) and the other at a point quarter of the way along the body diagonal [i.e. at the point (a/4, a/4, a/4)]. The basic diamond lattice and the atomic positions in the cubic cell of diamond lattice are shown in figure. The fractions denote height about the base in units of cube edge. The point at 0 and 1/2 are on the FCC lattice, those at 1/4 and 3/4 are on a similar lattice displaced along the body diagonal by 1/4th of the cube edge.



○ : Carbon atom occupying DC structure
● : Carbon atom occupying tetrahedral sites
Unit cell of DC structure



Plane view of atomic positions in DC unit cell. Numbers in the circles indicate fractional heights of the carbon atoms.

This is not a Bravais lattice.

(a) Number of nearest neighbours (coordination numbers): Each carbon atom has four nearest neighbours at distance $\sqrt{3}a/4$. Therefore, the co-ordination number is 4.

(b) Number of second nearest neighbours: There are 12 second nearest neighbours at distance $a/\sqrt{2}$.

(c) Atomic radius: From the plane view of DC structure it is clear that $8r = \sqrt{3}a \Rightarrow r = \frac{\sqrt{3}a}{8}$.

(d) Number of atoms per unit cell: $n_{eff} = \frac{1}{8} \times 8 + \frac{1}{2} \times 6 + 4 = 8$

(e) Atomic packing fraction : $\text{APF} = \left[\frac{n_{eff} \times \text{volume of each atom}}{\text{volume of unit cell}}\right]$

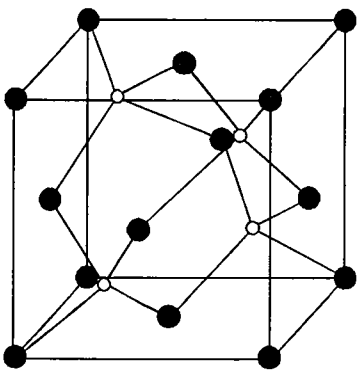
$$\therefore \quad \text{APF} = \frac{8 \times \frac{4}{3}\pi r^3}{a^3} = \frac{8 \times \frac{4}{3}\pi\left(\frac{\sqrt{3}a}{8}\right)^3}{a^3} = 8 \times \frac{4}{3}\pi \times \frac{3\sqrt{3}}{8 \times 8 \times 8} = \frac{\sqrt{3}\pi}{16} = 0.34$$

$\therefore$ APF % = 34 %.

(f) Type of packing: Diamond cubic is a loose-packed structure.

(g) Example : Carbon, silicon, germanium etc.

Zinc Blende structure (ZnS) : Zinc blende structure is almost identical to the diamond structure except that the two interpenetrating FCC sublattices are of different atoms and displaced from each other by one-quarter of the body diagonal.



○ Zn-atom occupying tetrahedral site
● S-atoms occupying FCC position
The unit cell of Zinc blende (ZnS) structure

The cubic zinc sulphide structure results when Zn-atoms are placed on one FCC lattice and S-atoms on the other FCC lattice as shown in figure. The conventional cell of this structure is a cube. There are four molecules per conventional cell. For each atom, there are four equally distant atoms of opposite kind arranged at a regular tetrahedron.

The ZnS is not an atomic structure. It is a molecular structure. For this structure, nearest neighbour distance $= r_{Zn} + r_S$. In a unit cell of ZnS, there will be 4 Zn-atoms and 4 S-atoms.

(a) Atomic radius: $a = \frac{4}{\sqrt{3}}(r_{Zn} + r_S)$

(b) Atomic packing fraction: $\text{APF} = \left[\frac{n_{eff} \times \text{volume of each atom}}{\text{volume of unit cell}}\right]$

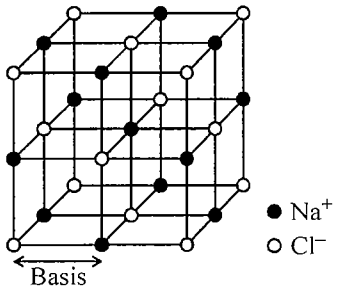
$$\therefore \quad \text{APF} = \frac{4 \times \left(\frac{4}{3}\pi r_{Zn}^3 + \frac{4}{3}\pi r_S^3\right)}{a^3} = \frac{4 \times \frac{4}{3}\pi r_{Zn}^3 + 4 \times \frac{4}{3}\pi r_S^3}{\left(\frac{4}{\sqrt{3}}\right)^3 (r_{Zn} + r_S)^3} = \left\{\frac{r_S^3\left(1 + \frac{r_{Zn}^3}{r_S^3}\right)}{r_S^3\left(1 + \frac{r_{Zn}}{r_S}\right)^3}\right\} \times \left(\frac{\sqrt{3}\pi}{4}\right)$$

$$= \frac{\sqrt{3}}{4}\pi \left\{\frac{1 + \left(\frac{r_{Zn}}{r_S}\right)^3}{\left(1 + \frac{r_{Zn}}{r_S}\right)^3}\right\}$$

(c) Examples: The compounds which have cubic zinc sulphide structure are CuCl, ZnS, InSb and CdS.

Sodium chloride structure:

In sodium chloride, sodium loses its outer electron and acquires an excess of positive charge while the chlorine atom accepts one electron from sodium and becomes a negative ion. Now, due to the electrostatic forces between the excess charges, the two ions attract each other. Further, due to the strong forces of repulsion as their outer electron shells come into close proximity, the two ions cannot approach each other to within less than certain distance. When attraction and repulsion balance, equilibrium is obtained. The Bravais lattice is a face centred cube, with a basis of one Na^+ ion and one Cl^- ion separated by one-half the body diagonal of the unit cell. There are four molecules of NaCl in a unit cell, with ions in the positions.



Unit cell of sodium chloride structure

$$Na^+ : \left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right), \left(0, 0, \frac{1}{2}\right), \left(0, \frac{1}{2}, 0\right), \left(\frac{1}{2}, 0, 0\right)$$

$$Cl^- : (0, 0, 0), \left(\frac{1}{2}, \frac{1}{2}, 0\right), \left(\frac{1}{2}, 0, \frac{1}{2}\right), \left(0, \frac{1}{2}, \frac{1}{2}\right)$$

(a) Number of nearest neighbours (coordination number): Nearest neighbours of Na^+ ions are 6 Cl^- ions. Similarly nearest neighbours of Cl^- ions ions are 6 Na^+ ions. Therefore, the co-ordination number of NaCl structure is 6. The distance of nearest neighbour is $r_{Na} + r_{Cl} = \frac{a}{2}$.

(b) Second nearest neighbours: There are 12 second nearest neighbours for each Na^+ and Cl^- ions at $\frac{a}{\sqrt{2}}$ distance.

(c) Number of atoms per unit cell: Total number of Cl^- ions $= 8 \times \frac{1}{8} + 6 \times \frac{1}{2} = 4$

Total number of Na^+ ions $= \frac{1}{4} \times 12 + 1 = 3 + 1 = 4$

This is also a molecular crystal structure.

(d) Atomic packing fraction: $\text{APF} = \left[\frac{n_{eff} \times \text{volume of each atom}}{\text{volume of unit cell}}\right]$

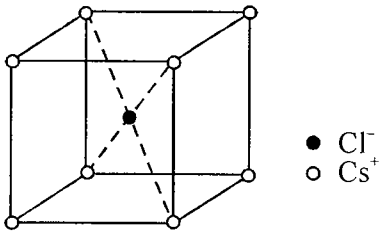
$$\therefore \quad \text{APF} = \frac{4 \times \left(\frac{4}{3}\pi r_{Na}^3 + \frac{4}{3}\pi r_{Cl}^3\right)}{a^3} = \frac{4 \times \frac{4}{3}\pi r_{Na}^3 + 4 \times \frac{4}{3}\pi r_{Cl}^3}{8\left(r_{Na} + r_{Cl}\right)^3} = \frac{2\pi}{3}\frac{\left(r_{Na}^3 + r_{Cl}^3\right)}{\left(r_{Na} + r_{Cl}\right)^3} = \frac{2}{3}\pi\left\{\frac{1 + \left(\frac{r_{Cl}}{r_{Na}}\right)^3}{\left(1 + \frac{r_{Cl}}{r_{Na}}\right)^3}\right\}$$

(e) Examples : Representative crystals having NaCl structure are KCl, KBr, MgO, AgBr, etc.

Cesium chloride structure:

The space lattice of cesium chloride (CsCl) structure is simple cubic. The basis has one Cs^+ ion at (0, 0, 0) and one Cl^- ion at $\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)$. Each ion is at the centre of a cube of ions of the opposite kind, the coordination number is thus eight. Obviously, the lattice points of CsCl are two interpenetrating simple cubic lattices, the corner of one sublattice is the body centre of the other. One sublattice is occupied by Cs^+ ions and the other by Cl^- ions.

The unit cell of CsCl structure is shown in the figure shown below.



The unit cell of Cesium chloride structure

(a) Number of nearest neighbours: There are 8 nearest neighbour lattice points (atoms) at distance $\left(\frac{\sqrt{3}a}{2} = r_{Cs} + r_{Cl}\right)$ in CsCl structure. For Cs^+ ion nearest neighbours are 8 Cl^- ions. Similarly, for Cl^- ion nearest neighbours are 8 Cs^+ ions.

(b) Second nearest neighbours: There are 6 second nearest neighbours at distance a.

(c) Number of atoms per unit cell : There are two ions (one Cs^+ and another Cl^-) per unit cell i.e. one molecule of CsCl.

(d) Atomic packing fraction:

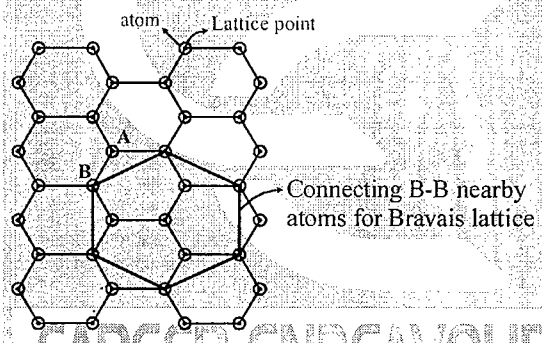
$$APF = \left[\frac{n_{eff} \times \text{volume of each atom}}{\text{volume of unit cell}}\right] = \frac{1 \cdot \frac{4}{3}\pi\left(r_{Cs}^3 + r_{Cl}^3\right)}{a^3} = \frac{\frac{4}{3}\pi\left(r_{Cs}^3 + r_{Cl}^3\right)}{\left(\frac{2}{\sqrt{3}}\left(r_{Cs} + r_{Cl}\right)\right)^3}$$

$$= \frac{\sqrt{3}}{2}\pi\frac{\left(r_{Cs}^3 + r_{Cl}^3\right)}{\left(r_{Cs} + r_{Cl}\right)^3} = \frac{\sqrt{3}}{2}\pi\frac{\left(1 + \frac{r_{Cl}^3}{r_{Cs}^3}\right)}{\left(1 + \frac{r_{Cl}}{r_{Cs}}\right)^3}$$

(e) Examples: CsCl, RbCl, LiHg etc.

Graphene:

Graphite structure is made up of graphene layers graphene has honeycomb structure i.e., carbon atoms are arranged in the honeycomb structure. In 2-dimensions one C-atom is shared by 3 nearby C-atoms. This is the graphite structure in 2-dimensional. In 3-dimensions, one C-atom is shared by 6 nearby C-atoms.



Graphite structure is layered structure but graphene is a single layered structure (single layer of carbon atom).
Graphite is conductor because there is one electron free per carbon atom.
If consider lattice point on each atom position then it will not form a basis because surroundings of A and B are different.
Only A-A atoms or B-B atoms can form basis as they have same surrounding.
So either connect nearby A-A atoms or B-B atoms. This hexagon (red) is the unit cell of graphene.

(1) This unit cell is non-primitive.

(2) **Number of atoms per unit cell:**

$$\text{Corners atoms} = \frac{1}{3} \times 6 = 2$$

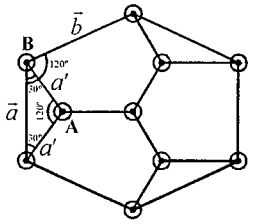
$$\text{Inside atoms} = 1 \times 4 = 4$$

Therefore, total numbers of atoms/unit cell = 6

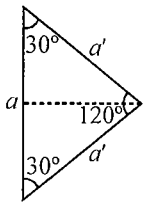
The side of unit cell is lattice parameter a. All the sides of this structure will be same.

Let lattice parameters is a. Thus, $|\vec{a}| = |\vec{b}|, \gamma = 120°$

(3) **Nearest neighbour distance:** $2r = a' = \frac{a}{\sqrt{3}}$.



Primitive unit cell



From figure, $\cos 30° = \frac{a/2}{a'}$

$\Rightarrow \frac{\sqrt{3}}{2} = \frac{a}{2a'}$

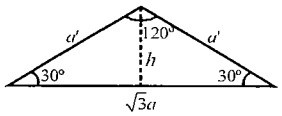
$\Rightarrow a' = \frac{a}{\sqrt{3}}$

(4) Number of nearest neighbours = 3.
In 2-dimensions, there will be close packing if number of nearest neighbours are 6 but here 3 so packing will be loose.

(5) Second nearest neighbour distance = a

(6) Number of second nearest neighbours = 6

(7) Area of triangle $= \frac{1}{2} \times \sqrt{3}a \times \frac{a}{2} = \frac{\sqrt{3}a^2}{4}$



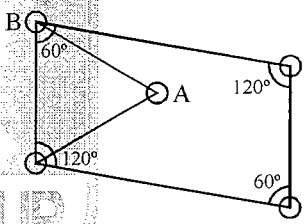
$h^2 = a^2 - \frac{3}{4}a^2 = \frac{a^2}{4} \Rightarrow h = \frac{a}{2}$

Area of 2 triangle $= \frac{\sqrt{3}a^2}{2}$

Area of unit cell $= \sqrt{3}a^2 + \frac{\sqrt{3}a^2}{2} = \sqrt{3}\left(1 + \frac{1}{2}\right)a^2 = \frac{3\sqrt{3}a^2}{2}$

(8) Area of primitive unit cell $= \frac{1}{3} \times$ area of unit cell $= \frac{1}{3}\left(\frac{3\sqrt{3}}{2}\right)a^2 = \frac{\sqrt{3}a^2}{2}$

(9) Number of atoms per primitive unit cell = 1 + 1 = 2
Therefore, 2 atoms of graphene inside the primitive cell.
Thus, graphene has 2 atom basis (i.e., one atom of A type and one of B-type)



(10) **Number of atoms per unit area:**

Number of atoms per unit area $= \frac{6}{\left(\frac{3\sqrt{3}a^2}{2}\right)} = \frac{4}{\sqrt{3}a^2}$ Number of atoms $= \left(\frac{1}{6} + \frac{1}{3} + \frac{1}{3} + \frac{1}{6}\right) = 1$

(11) **Atomic radius :** $r = \frac{a}{2\sqrt{3}}$

(12) **Type of packing:** loose

(13) **Example :** Graphene

(14) **Atomic Packing Fraction (APF):**

$$\text{APF} = \frac{\text{area of the atoms in unit cell}}{\text{area of unit cell}} = \frac{6 \times \pi r^2}{\frac{3\sqrt{3}a^2}{2}} = \frac{12\pi r^2}{3\sqrt{3} \times 4 \times 3 \times r^2} = \frac{\pi}{3\sqrt{3}} = 0.604.$$

$\{a = 2\sqrt{3}r$ and area of the atoms $= \pi r^2\}$

Therefore, APF = 60 %

Graphite : Graphite is a form of carbon and crystalises in simple hexagonal structure. It contains 4 atoms per primitive unit cell.

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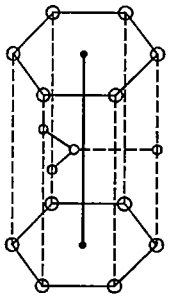
Number of atom per unit cell: Number of atoms per unit cell $= \frac{1}{6} \times 12 + \frac{1}{3} \times 3 = 3 + 1 = 4$



Height of the unit cell = 4.8 times the nearest neighbours distance

Nearest neighbour distance implies $2r = a$, a is lattice parameter

Number of nearest neighbour = 3.

One C-atom can make covalent bond with 3 nearby atoms. C-atoms of one layer can not make bond with C-atom of second layer. That's why graphite sheets slides on each other. Therefore, graphite is soft because layer do not have bonding.

Since, semiconductor are not conducting at $T = 0$ K and metals are conducting at $T = 0$ K

Graphite is a semi-metal.

Relation between lattice constant and density:

Let the lattice parameter of the unit cell is a. Therefore, the volume of the unit cell is a^3.

If ρ be the density of the unit cell then the mass of each unit cell $= a^3 \rho$ ...(1)

If M is the molecular weight and N is the Avogadro number.

$\therefore$ Mass of each molecule $= \frac{M}{N}$

As if n is the number of molecules per unit cell, then the mass in each unit cell $= n \times \frac{M}{N}$ ... (2)

From equations (1) and (2), we get

$$a^3 \rho = n\frac{M}{N} \quad \Rightarrow \quad a = \left(\frac{nM}{N\rho}\right)^{1/3}$$

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Atomic density: The number of atoms per unit area of a lattice plane is called as atomic density.

Atomic density $(\rho) = \frac{nd}{V}$, where V = volume of unit cell and n is the number of atoms in it and d is the interplanar spacing.

$\therefore \quad \rho = \frac{nd}{abc}$

If $n = 1 \Rightarrow \rho = \frac{d}{abc}$

Planes in different crystals:

1. **Simple cubic structure :**

   Plane analysis in semiconductor

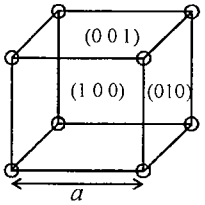
   First consider (1 0 0) plane

   In planes (0 1 0), (0 0 1), (1 0 0), the properties are same.

Number of atoms on (1 0 0) type plane $= \frac{1}{4} \times 4 = 1$

Area of plane $= a^2$

Therefore, atomic density = number of atoms per unit area $= \frac{1}{a^2}$

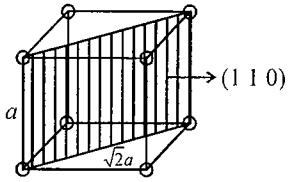


For a plane, consider atoms as disc (in two dimension)

(1 1 0) plane :

Number of atoms $= \frac{1}{4} \times 4 = 1$.

Area $= \sqrt{2}a^2$ (Since, shape is rectangular)

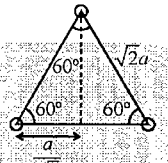


Therefore, atomic density $= \frac{1}{\sqrt{2}a^2}$

(1 1 1) plane :

Number of atoms $= \frac{1}{6} \times 3 = \frac{1}{2}$

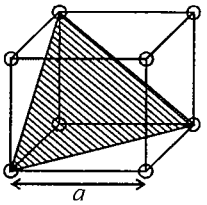
Area $= \frac{1}{2} \times$ base $\times$ height (Since, shape is triangular)



$= \frac{1}{2}\left(\sqrt{2}a\right)\sqrt{\frac{3}{2}}a = \frac{\sqrt{3}a^2}{2}$

$h^2 = 2a^2 - \frac{a^2}{2}$

$h^2 = \frac{3a^2}{2} \Rightarrow h = \sqrt{\frac{3}{2}}a$



Therefore, atomic density $= \frac{1}{\sqrt{3}a^2}$

Example: Which plane in simple cubic is closed pack plane ?

In. Plane having (1 0 0) largest atomic density will be closed packed plane.

Example: Which direction is closest pack in simple cubic lattice ?

In. In one dimensions, atom is considered as a line
In 2 dimensions, atom is considered as a disc
In three dimensions, atom is considered as a sphere
Directions having largest line density will be closest pack

Simple cubic structure:

Among planes (1 0 0), (1 1 0), (1 1 1), (1 $\bar{1}$ 0), line density of (1 0 0) is largest.
Therefore, (1 0 0) is the closest pack direction.

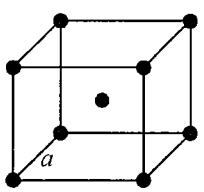
[In distance a there are 1 atom $\left(\frac{1}{2} + \frac{1}{2} = 1\right)$]. (i.e., line density = $1/a$)

Body Centred Cubic Structure (BCC):

(1 0 0) plane :

Number of atoms = 1 (no new atom on faces)
Area = a^2

Plannar density $= \frac{1}{a^2}$

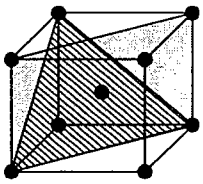


Line density $= \frac{1}{a}$

(1 1 0) plane :

Number of atoms = 2

Area $= \sqrt{2}a^2$



Planner density $= \frac{\sqrt{2}}{a^2}$

Line density $= \frac{1}{\sqrt{2}}a$

(1 1 1) plane :

Number of atoms $= \frac{1}{2}$

(Body centered atom will not contribute to (1 1 1) plane)

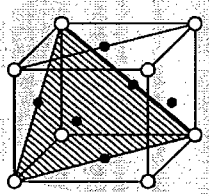
Area $= \frac{\sqrt{3}a^2}{2}$; Planner density $= \frac{1}{\sqrt{3}a^2}$; Line density $= \frac{2}{\sqrt{3}a}$

For BCC closest pack plane is (1 1 0).

Face Centred Cubic Structure (FCC):

(1 0 0) plane :

Number of atoms $= \frac{1}{4} \times \overset{\text{corners}}{4} + \overset{\text{face}}{1} = 2$



Planner density $= \frac{2}{a^2}$

Line density $= \frac{1}{a}$

(1 1 0) plane :

Number of atoms $= \frac{1}{4} \times 4 + \frac{1}{2} \times 2 = 2$

Planner density $= \frac{\sqrt{2}}{a^2}$

Line density $= \frac{\sqrt{2}}{a}$

(1 1 1) plane :

Number of atoms $= \frac{1}{6} \times 3 + \frac{1}{2} \times 3 = 2$

Planner density $= \frac{4}{\sqrt{3}a^2}$

Line density $= \frac{1}{\sqrt{3}a}$

For FCC closest pack plane is (1 1 1).

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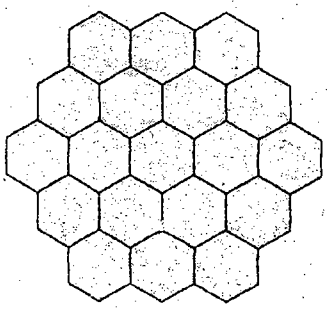
SOLVED PROBLEMS

In a single layer of graphite, called graphene, the carbon atoms from a hexagonal lattice (see figure). How many carbon atoms are there in a unit cell of the lattice? **[JNU 2010]**



(a) one (b) two (c) three (d) six

In. Each atom is shared by three unit cells.

Therefore, number of atoms/unit cell $= \frac{1}{3} \times 6 = 2$

Correct option is (b)

Helium atoms at low temperatures make a perfect closed pack structure of hexagonal lattice with parameters $a = 0.36$ nm and $c = 0.59$ nm. The density of the crystal is approximately **[JNU 2012]**

(a) 2000 kg/m^3 (b) 100 kg/m^3 (c) 123 kg/m^3 (d) 200 kg/m^3

In. In a conventional cell of hexagonal structure number of atoms present (N) = 6.

Volume of conventional unit cell (V) $= 6\frac{\sqrt{3}}{4} a^2 \cdot c$

Number density $(n) = \frac{N}{V} = \frac{6}{V} = \frac{6}{6 \times \frac{\sqrt{3}}{4} a^2 c} = \frac{4}{\sqrt{3} a^2 c}$

Mass density = Mass of one helium atom × number density

$$= 4 \times 1.67 \times 10^{-27} \times \frac{4 \times 10^{27}}{\sqrt{3} \times (0.36)^2 \times 0.59} = 200.88 \text{ kg/m}^3$$

Correct option is (d)

Consider a regular arrangement of identical spheres in a face-centred cubic (fcc) structure in which the centres of the respective spheres are located at each of the eight corners and the centres of the six surfaces of a unit cube. The fraction of each cubic unit cell occupied by the spheres in the close-pack configuration is **[JNU 2013]**

(a) 0.50 (b) 0.62 (c) 0.74 (d) 0.88

In. $\text{APF} = \frac{\text{Number of atom} \times \text{volume of sphere}}{\text{volume}} = \frac{4 \times \frac{4}{3} \pi r^3}{a^3}$

$$= \frac{4 \times \frac{4}{3} \pi \left(\frac{a}{2\sqrt{2}} \right)^3}{a} = 0.74$$

Correct answer is (c)

4. The density of copper, which has a face-centred cubic (fcc) lattice structure, is 8.9 gm/cm^3. Given that the molar mass of copper is 63.55, the length of a side of the cubic cell is **[JNU 2015]**

(a) 0.45 nm (b) 0.57 nm (c) 0.29 nm (d) 0.23 nm (e) 0.36 nm

Soln. We know that,

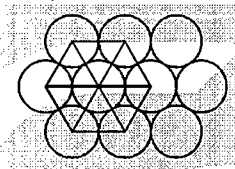
$$\rho = \frac{n \times M}{Na^3} \Rightarrow a = \left(\frac{nM}{N \times \rho}\right)^{1/3} = \left(\frac{4 \times 63.55}{6.023 \times 10^{-26} \times 8900}\right)^{1/3} = 0.36\ nm \quad \left[\because \rho = \frac{8.9 \times 10^{-3}}{\left(10^{-2}\right)^3} = 8900\ kg/m^3\right]$$

Correct answer is (e)

5. A flat surface is covered with non-overlapping disks of same size. What is the largest fraction of the area that can be covered ?

(a) $\frac{3}{\pi}$ (b) $\frac{5\pi}{6}$ (c) $\frac{6}{7}$ (d) $\frac{\pi}{2\sqrt{3}}$ **[JEST-2013]**

Soln. Let the radius of the circular disk is a. Then, area of the circular disk = πr^2



Here, centers of three disks make an equilateral triangle with area = $\frac{\sqrt{3}a^2}{4} = \frac{\sqrt{3}(2r)^2}{4}$.

Therefore, packing fraction of the triangular lattice = $\left(\frac{\frac{3}{6} \times \pi r^2}{\frac{\sqrt{3}a^2}{4}}\right) = \left(\frac{\frac{3}{6} \times \pi r^2}{\frac{\sqrt{3} \times (2r)^2}{4}}\right) = \frac{\pi}{2\sqrt{3}}$.

Therefore, the fractional area of the flat surface that can be covered by these circular discs = $\frac{\pi}{2\sqrt{3}}$.

Hence, correct option is (d).

6. A metal suffers a structural phase transition from face-centered cubic (FCC) to the simple cubic (SC) structure. It is observed that this phase transition does not involve any change of volume. The nearest neighbour distances d_{fcc} and d_{sc} for the FCC and the SC structures respectively are in the ratio (d_{fcc}/d_{sc}) [Given: $2^{1/3} = 1.26$]

(a) 1.029 (b) 1.122 (c) 1.374 (d) 1.130 **[JEST-2013]**

Soln. For FCC crystal strucutre, the nearest neighbour distance, $d_{fcc} = \frac{\sqrt{2}}{2} a_{fcc}$ ... (i)

For SC crystal strucutre, the nearest neighbour distance, $d_{sc} = a_{sc}$ ... (ii)

Since, volume does not change in this phase transformation from FCC to SC structure.

Therefore, density of FCC structure = density of SC structure

i.e., $\frac{4}{a_{fcc}^3} = \frac{1}{a_{sc}^3}$

$\Rightarrow \frac{a_{fcc}^3}{a_{sc}^3} = 4 \Rightarrow \frac{a_{fcc}}{a_{sc}} = (4)^{1/3} = (2)^{2/3}$ ... (iii)

Now, from equations (i) and (ii), we get

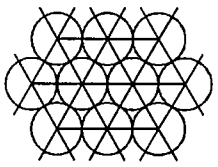
$$\frac{d_{fcc}}{d_{sc}}=\frac{\left(\frac{\sqrt{2}}{2}a_{fcc}\right)}{a_{sc}}=\frac{1}{\sqrt{2}}\times\frac{a_{fcc}}{a_{sc}}=\frac{1}{\sqrt{2}}\times(2)^{2/3}=\frac{(2)^{2/3}}{(2)^{1/2}}=(2)^{\frac{2}{3}-\frac{1}{2}}=(2)^{\frac{1}{6}}=(2^{\frac{1}{3}})^{\frac{1}{2}}=(1.26)^{\frac{1}{2}}=1.122.$$

Hence, correct option is (b).

Circular discs of radius 1 m each are placed on a plane so as to form a closely packed triangular lattice. The number of discs per unit area is approximately equal to

(a) 0.86 m^{-2} (b) 0.43 m^{-2} (c) 0.29 m^{-2} (d) 0.14 m^{-2} **[JEST-2014]**

Sln. Given, radius of circular disc $r = 1$ m



The circular discs will form a equilateral triangular lattice of side $a = 2r = 2\times1$ m $= 2$ m, where r is the radius of the circular discs.

Now, area of the triangular lattices $=\frac{\sqrt{3}}{4}a^2=\frac{\sqrt{3}}{4}\times(2r)^2=\sqrt{3}r^2=\sqrt{3}(1)^2=\sqrt{3}\,m^2$.

Number of circular discs inside the triangular lattice $=\frac{1}{6}\times3=0.5$.

Therefore, number of discs/area $=\frac{0.5}{\sqrt{3}}=\frac{0.5}{\sqrt{3}}m^{-2}=0.28867=0.29\,m^{-2}$

Hence, correct option is (c).

The total number of Na^+ and Cl^- ions per unit cell of NaCl is, **[JEST 2015]**

(a) 2 (b) 4 (c) 6 (d) 8

Sln. The total number of Na^+ and Cl^- ions per unit cell of NaCl is 8.

Correct option is (d)

The number of different Bravais lattices possible in two dimension is

(a) 2 (b) 3 (c) 5 (d) 6 **[JEST-2016]**

Sln. The number of different Bravais lattices possible in two dimension is 5. The lattices are oblique ($a \neq b, \gamma \neq 90°$), square ($a = b, \gamma = 90°$), primitive rectangular ($a \neq b, \gamma = 90°$), non-primitive rectangular ($a \neq b, \gamma = 90°$) and hexagonal ($a = b, \gamma = 120°$).

Hence, correct option is (c).

In a simple cubic lattice of lattice constant 0.287 nm, the number of atoms per mm^2 along the 111 plane is

(a) 2.11×10^{13} (b) 1.73×10^{13} (c) 1.29×10^{13} (d) 1.21×10^{13} **[TIFR 2016]**

Sln. **Given:** Lattice parameter, a = 0.287 nm = 2.87×10^{-10} m

Number of atoms on (111) plane, $n=3\times\frac{1}{6}=0.5$

Area of the (111) place in simple cubic lattice

$$=\frac{\sqrt{3}\left(\sqrt{2}a\right)^2}{4}=\frac{\sqrt{3}\times2a^2}{4}=\frac{2\times1.732\times\left(2.87\times10^{-10}\right)^2}{4}=7.133\times10^{-20}m^2=7.133\times10^{-14}\,mm^2$$

Therefore, number of atoms per mm^2

$$= \frac{\text{number of atoms on } (111) \text{ plane}}{\text{area of the } (111) \text{ plane}} = \frac{0.5}{7.133\times10^{-14}} = \frac{0.5}{7.133}\times10^{14} = 0.7\times10^{13} \text{ atoms/mm}^2$$

No option is matched.

11. If the molecular weight and density of rock salt (NaCl) with fcc structure are 60 and 2160 kg/m^3, respectively, the lattice constant in Å is about: **[DU 2014]**

(a) 6.78 (b) 5.69 (c) 4.51 (d) 3.10

Soln. Molecular weight $(M) = 60$, density $(\rho) = 2160$ kg/m^3

Now, $a^3 = \frac{nM}{N\rho} = \frac{4\times60}{6.023\times10^{26}\times2160} = 0.01845\times10^{-26} \text{ m}^3 = 184.5\times10^{-30} \text{ m}^3$

$\Rightarrow a = 5.69\times10^{-10}$ m

$\Rightarrow a = 5.69$ Å

Hence, correct option is (b).

12. Packing fraction of an fcc lattice is **[BHU 2012]**

(a) 0.52 (b) 0.32 (c) 0.74 (d) 0.62

Soln. Packing fraction of an fcc lattice is 0.74

Hence, correct option is (c).

13. In a simple cubic lattice $a = 2$ Å, then planar atomic density of (1, 0, 0) plane is **[BHU 2012]**

(a) 10^{15} atoms/cm^2 (b) 2.5×10^{15} atoms/cm^2

(c) 5×10^{15} atoms/cm^2 (d) 7.5×10^{15} atoms/cm^2

Soln. $a = 2$ Å, the planar atomic density of (100) plane is d_{100}/a^3

$$= \frac{a}{a^3} = \frac{1}{a^2} = \frac{1}{(2\times10^{-10})^2} \text{ atoms/m}^2$$

$$= \frac{1}{4}\times10^{20} \text{ atoms/m}^2 = \frac{0.25\times10^{20}}{10^4} \text{ atoms/cm}^2 = 2.5\times10^{15} \text{ atoms/cm}^2.$$

Hence, correct option is (b).

14. The crystal structure of diamond is **[BHU 2012]**

(a) fcc with two atom basis of (0 0 0) and $\frac{a}{4}\left(\hat{i}+\hat{j}+\hat{k}\right)$

(b) simple cubic with two atom basis of (0 0 0) and $\frac{a}{2}\left(\hat{i}+\hat{j}+\hat{k}\right)$

(c) fcc with two atom basis of (0 0 0) and $\frac{a}{2}\left(\hat{i}+\hat{j}+\hat{k}\right)$

(d) bcc with one atom basis

Soln. The crystal structure diamond is fcc with two atom basis of (0 0 0) and $\frac{a}{4}\left(\hat{i}+\hat{j}+\hat{k}\right)$

Hence, correct option is (a).

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. For an FCC lattice the ratio of $d_{200} : d_{220} : d_{222}$ is **[BHU 2013]**

(a) $\sqrt{3}:\sqrt{6}:\sqrt{2}$ (b) $1:\sqrt{2}:\sqrt{6}$ (c) $1:2:3$ (d) $\sqrt{6}:\sqrt{3}:\sqrt{2}$

ln. $d_{200} : d_{220} : d_{222} = \frac{a}{\sqrt{2^2+0^2+0^2}} : \frac{a}{\sqrt{2^2+2^2+0^2}} : \frac{a}{\sqrt{2^2+2^2+2^2}}$

$= \frac{a}{\sqrt{4}} : \frac{a}{\sqrt{8}} : \frac{a}{\sqrt{12}} = \frac{a}{2} : \frac{a}{2\sqrt{2}} : \frac{a}{2\sqrt{3}} = \sqrt{6}:\sqrt{3}:\sqrt{2}.$

Hence, correct option is (d).

. Magnesium has h.c.p. structure. The radius of magnesium atom is 0.1605 nm. The volume of unit cell of magnesium is **[BHU 2013]**

(a) $0.7\times10^{-28}\ m^3$ (b) $2.8\times10^{-28}\ m^3$ (c) $1.4\times10^{-28}\ m^3$ (d) $0.35\times10^{-28}\ m^3$

ln. $r = 0.1605$ m , For fcc, $a = 2r = 2\times0.1605\,m = 0.321$ nm and $\frac{c}{a} = \sqrt{\frac{8}{3}} \Rightarrow c = a\times\sqrt{\frac{8}{3}} = 0.5242$ nm .

Thus, volume of the unit cell $V = \frac{6\times\sqrt{3}a^2\times c}{4} = \frac{6\times\sqrt{3}\times(0.321)^2\times(0.5242)}{4}\times10^{-27}\ m^3$

$= 0.14\times10^{-27}\ m^3 = 1.4\times10^{-28}\ m^3.$

Hence, correct option is (c).

. The spacing d_{hkl} of the planes (hkl) in a tetragonal crystal is **[BHU 2013]**

(a) $\left[\frac{h^2}{a^2}+\frac{k^2}{b^2}+\frac{l^2}{c^2}\right]^{-1/2}$ (b) $\left[\frac{h^2+k^2}{a^2}+\frac{l^2}{c^2}\right]^{-1/2}$

(c) $\left[\frac{4}{3}\left(\frac{h^2+hk+k^2}{a^2}\right)+\frac{l^2}{c^2}\right]^{-1/2}$ (d) $a\left[h^2+k^2+l^2\right]^{-1/2}$

ln. For tetragonal crystal, $a = b \neq c$, $\alpha = \beta = \gamma = 90°$

The spacing d_{hkl} of the planes (hkl) in a tetragonal crystal is given by

$$d_{hkl} = \left[\frac{h^2+k^2}{a^2}+\frac{l^2}{c^2}\right]^{-1/2}$$

Hence, correct option is (b).

3. Diamond has the following crystal structure **[BHU 2013]**

(a) hexagonal (b) simple cubic

(c) face-centered cubic (d) body-cented cubic

ln. Diamond has fcc crystal structure.

Hence, correct option is (c).

). A crystal system whose unit cell is specified by $a \neq b \neq c$, $\alpha = \gamma = 90 \neq \beta$ is known as **[BHU 2013]**

(a) monoclinic (b) rhombohedral (c) tetragonal (d) orthorhombic

ln. The crystal system whose unit cell is specified by $a \neq b \neq c$, $\alpha = \gamma = 90° \neq \beta$ is known as monoclinic crystal.

Hence, correct option is (a).

20. For hexagonal close-packed structure, the ratio of lattice parameters a and c, i.e., c/a is given by

(a) $\frac{1}{2}\left(\frac{8}{3}\right)^{1/2}$ (b) $\left(\frac{3}{8}\right)^{1/2}$ (c) $\left(\frac{8}{3}\right)^{1/2}$ (d) $\frac{1}{2}\left(\frac{3}{8}\right)^{1/2}$ **[BHU 2013]**

Soln. For HCP structure, the ratio of a and c is c/a is given by $\frac{c}{a}=\sqrt{\frac{8}{3}}$.

Hence, correct option is (c).

21. The number of ions in the unit cell of CsCl crystal is **[BHU 2013]**

(a) 1 (b) 2 (c) 3 (d) 4

Soln. The total number of ions in the unit cell of CsCl crystal is 2.
Hence, correct option is (b).

22. The Miller indices of the plane parallel to the x and y axes are **[BHU 2013]**

(a) (100) (b) (111) (c) (001) (d) (010)

Soln. Let the miller indices are (hkl). Thus, $h:k:l=\frac{a}{OA}:\frac{b}{OB}:\frac{c}{OC}=\frac{a}{\infty}:\frac{b}{\infty}:\frac{c}{c}$, $h:k:l=0:0:1$

Therefore, miller indices are (001).
Hence, correct option is (c).

23. The number of lattice points in a primitive cell is **[BHU 2013]**

(a) 2 (b) 3 (c) 1 (d) 4

Soln. The number of lattice points in a primitive cell is 1.
Hence, correct option is (c).

24. If n is the number of atoms in the unit cell of the cubic system, N and M are the Avogadro's number and atomic weight respectively and ρ is the density of the element, then the lattice constant a is given by

(a) $\left[\frac{M\rho}{nN}\right]^{1/3}$ (b) $\left[\frac{nM}{N\rho}\right]^{1/3}$ (c) $\left[\frac{nN}{M\rho}\right]^{1/3}$ (d) $\left[\frac{\rho N}{Mn}\right]^{1/3}$ **[BHU 2013]**

CAREER ENDEAVOUR

Soln. For unit cell of cubic system, the density is given by $\rho=\frac{nM}{Na^3} \Rightarrow a^3=\frac{nM}{N\rho} \Rightarrow a=\left[\frac{nM}{N\rho}\right]^{1/3}$.

Hence, correct option is (b).

25. ABABA..... represents an arrangement of layers called : **[BHU 2014]**

(a) hexagonal closed packing (b) cubic closed packing
(c) body centered cubic packing (d) flourite close packing

Soln. ABABA..... represents an arrangement of layers called hexagonal closed packing.
Hence, correct option is (a).

26. In diamond the coordination number of carbon is : **[BHU 2014]**

(a) 4 and its unit cell has 8 carbon atoms (b) 4 and its unit cell has 6 carbon atoms
(c) 6 and its unit cell has 4 carbon atoms (d) 4 and its unit cell has 4 carbon atoms

Soln. Coordination number of carbon in diamond cubic lattice is 4 and its unit cell has 8 carbon atoms.
Hence, correct option is (a).

A compound formed by elements A and B crystallizes in cubic structure, in which atoms of A are at the corners while that of B are at the face center. The formula of the compound is **[BHU 2014]**

(a) AB_3 (b) AB (c) AB_6 (d) A_2B

In. If atoms of element A lies at the corners and atoms of elements B are at the corners, then there will be total one atom of element A and three atoms of element B per unit cell. Thus, the formula of compound formed by elements A and B will be AB_3.

Hence, correct option is (a).

For a given cubic crystal lattice parameter a is 3.18 Å. The d spacing for a (III) plane is :

(a) 2.25 Å (b) 1.84 Å (c) 3.18 Å (d) 3.90 Å **[BHU 2014]**

In. Since, $d_{hkl} = \frac{a}{\sqrt{(h^2+k^2+l^2)}} \Rightarrow d_{111} = \frac{3.18\text{ Å}}{\sqrt{1^2+1^2+1^2}} = \frac{3.18}{\sqrt{3}}\text{Å} = 1.84\text{ Å}$

Hence, correct option is (b).

For a simple cubic lattice, the ratio of $d_{100} : d_{110} : d_{111}$ is **[BHU 2015]**

(a) $\sqrt{2}:\sqrt{3}:\sqrt{6}$ (b) $\sqrt{6}:\sqrt{3}:\sqrt{2}$ (c) $\sqrt{2}:\sqrt{6}:\sqrt{3}$ (d) $\sqrt{3}:\sqrt{2}:\sqrt{6}$

In. $d_{100} : d_{110} : d_{111} = \frac{a}{\sqrt{1^2+0^2+0^2}} : \frac{a}{\sqrt{1^2+1^2+0^2}} : \frac{a}{\sqrt{1^2+1^2+1^2}}$

$= \frac{a}{\sqrt{1}} : \frac{a}{\sqrt{2}} : \frac{a}{\sqrt{3}} = \sqrt{6}:\sqrt{3}:\sqrt{2}.$

Hence, correct option is (b).

Given that the cube edge of diamond is 0.356 nm. The number of atoms per metre cube is

(a) 0.88×10^{29} (b) 5.31×10^{29} (c) 2.77×10^{29} (d) 1.77×10^{29} **[BHU 2015]**

In. Cube edge of diamond (a) = 0.356 nm.

The number atoms per metre cube $= \frac{n}{a^3}$, where n is number of atoms per unit cell.

$= \frac{8}{(0.356\times10^{-9})^3} = 177.3\times10^{27} = 1.77\times10^{29}$ atoms/m^3.

Hence, correct option is (d).

Sodium chloride has the following crystal structure **[BHU 2015]**

(a) hexagonal (b) face-centered cubic

(c) simple cubic (d) body-centered cubic

In. Sodium chloride (NaCl) has the face-centered cubic crystal structure.

Hence, correct option is (b).

The spacing d_{hkl} of the planes (hkl) in a hexagonal crystal is **[BHU 2015]**

(a) $\left[\frac{4}{3}\left(\frac{h^2+hk+k^2}{a^2}\right)+\frac{l^2}{c^2}\right]^{-1/2}$ (b) $a\left[h^2+k^2+l^2\right]^{-1/2}$

(c) $\left[\frac{h^2}{a^2}+\frac{k^2}{b^2}+\frac{l^2}{c^2}\right]^{-1/2}$ (d) $\left[\frac{h^2+k^2}{a^2}+\frac{l^2}{c^2}\right]^{-1/2}$

In. For (hkl) planes, the lattice spacing (d_{hkl}) is given by

$$d_{hkl}=\left[\frac{4}{3}\left(\frac{h^2+hk+k^2}{a^2}\right)+\frac{l^2}{c^2}\right]^{-1/2}$$

Hence, correct option is (a).

33. A crystal system whose unit cell is specified by $a=b\neq c,\ \alpha=\beta=\gamma=90$ is known as **[BHU 2015]**

(a) simple cubic (b) hexagonal (c) monoclinic (d) tetragonal

Soln. The lattice parameters a, b, c and α, β, γ given by $a=b\neq c,\ \alpha=\beta=\gamma=90°$ are characterizing the tetragonal lattice.

Hence, correct option is (d).

34. The Miller indices of a set of parallel planes which makes intercepts in the ratio of $1:\frac{1}{2}:\frac{1}{4}$ is given by

(a) (421) (b) (124) (c) (122) (d) (142) **[BHU 2015]**

Soln. Intercepts $OA : OB : OC = 1:\frac{1}{2}:\frac{1}{4}$

$$\therefore\ h:k:l=\frac{1}{OA}:\frac{1}{OB}:\frac{1}{OC}=\frac{1}{1}:\frac{1}{(1/2)}:\frac{1}{(1/4)}=1:2:4$$

$\therefore\ (hkl)=(124)$.

Hence, correct option is (b).

35. The number of ions in the unit cell of NaCl crystal is **[BHU 2015]**

(a) 1 (b) 2 (c) 4 (d) 8

Soln. The total number of ions in the unit cell of NaCl crystal is 8.

Hence, correct option is (d).

36. The number of lattice points in a BCC unit cell is **[BHU 2015]**

(a) 1 (b) 2 (c) 3 (d) 4

Soln. The number of lattice points in a BCC unit cell is 2.

Hence, correct option is (b).

CAREER ENDEAVOUR

PRACTICE SET

(Multiple Choice Questions (MCQ)):

The coordination number of a simple cubic (sc) structure is
(a) 4 (b) 6 (c) 8 (d) 12

The coordination number of a bcc structure is
(a) 4 (b) 6 (c) 8 (d) 12

The coordination number of a fcc structure is
(a) 4 (b) 6 (c) 8 (d) 12

The coordination number of a hcp structure is
(a) 3 (b) 6 (c) 9 (d) 12

The number of two dimensional lattices are
(a) 3 (b) 5 (c) 7 (d) 9

The number of macroscopic symmetry elements in a cube is
(a) 13 (b) 31 (c) 23 (d) 32

The number of identical sphere arranged in simple cubic form of side a is
(a) $\frac{a}{2}$ (b) $\frac{a}{2\sqrt{2}}$ (c) $\frac{\sqrt{3}a}{2}$ (d) $\frac{\sqrt{3}a}{4}$

The number of molecules per unit cell of NaCl structure is
(a) 1 (b) 2 (c) 4 (d) 8

The radius of identical spheres arranged in bcc form of side a is
(a) $\frac{a}{2}$ (b) $\frac{a}{2\sqrt{2}}$ (c) $\frac{\sqrt{3}a}{2}$ (d) $\frac{\sqrt{3}a}{4}$

The radius of identical spheres arranged in fcc form of side a is
(a) $\frac{a}{2}$ (b) $\frac{a}{2\sqrt{2}}$ (c) $\frac{\sqrt{3}a}{2}$ (d) $\frac{\sqrt{3}a}{4}$

CAREER ENDEAVOUR

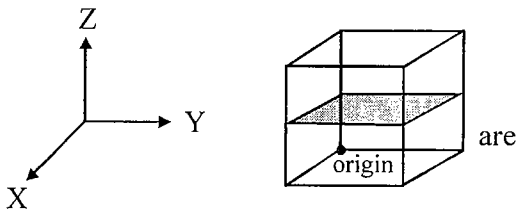
The pure rotational symmetry which is not possible in crystalline solid is
(a) 2-fold (b) 3-fold (c) 4-fold (d) 5-fold

If 0.28 nm is the interionic distance in NaCl crystal, the lattice parameter is
(a) 0.14 nm (b) 0.56 nm (c) 0.08 nm (d) 0.28 nm

The maximum radius of the interstitial sphere that can just fit into the void between the body centred atom of bcc structure is
(a) $r\left[\left(2/\sqrt{3}\right)-1\right]$ (b) $r\left[\left(\sqrt{3}/2\right)-1\right]$ (c) $r\left[\sqrt{3}-1\right]$ (d) $r\left[\sqrt{2}-1\right]$

Magnesium crystallizes in hcp structure. If the lattice constant is 0.32 nm, the nearest neighbour distance in magnesium is
(a) 0.32 nm (b) 0.16 nm (c) 0.64 nm (d) none of these

15. The miller indices of the shaded plane



are

(a) (001) (b) (002) (c) $\left(00\frac{1}{2}\right)$ (d) none

(Multiple Select Questions (MSQ)):

1. Indicate the ***INCORRECT*** statement(s) about the simple cubic structure
   (a) Atoms/lattice points per unit cell is one.
   (b) If r is the radius of atom, the volume of unit cell is equal to $8r^3$.
   (c) The packing fraction of simple cubic structure is 68%.
   (d) In a simple cubic lattice there is one atom at each centre and one at body centre.
2. For closed-packed structure, the correct statement(s) is/are
   (a) the packing fraction should be less than 74%.
   (b) the packing fraction should be larger than or equal to 74%.
   (c) the coordination number should be 12.
   (d) the coordination number should be 8.
3. For NaCl structure, the correct statement(s) is/are
   (a) each ion is surrounded by six nearest neighbours of the opposite kind.
   (b) the coordination number is six.
   (c) representative crystals having NaCl structure are KCl, KBr, MgO, AgBr.
   (d) There are four molecules of NaCl in one unit cell.
4. Indicate the ***CORRECT*** statement(s) about the diamond cubic structure
   (a) the space lattice of dimaond is fcc with a basis of two carbon atoms assocaited with each lattice point.
   (b) The number of atoms per unit cell is 4.
   (c) the packing fraction is 34 %
   (d) the lattice parameter (a) and radius (r) are related as $a = \frac{8r}{\sqrt{3}}$
5. Indicate the ***INCORRECT*** statement(s) about the hexagonal close packed structure
   (a) the ratio of height(c) to lattice parameter (a) is $\frac{c}{a} = \sqrt{\frac{8}{3}}$
   (b) the total number of atoms in a unit cell is 3.
   (c) the volume of unit cell is equal to $\frac{3 \times \sqrt{3}\, a^2 c}{6}$
   (d) The coordination number is 8.

CAREER ENDEAVOUR

(Numerical Answer Type (NAT)):

Lead is a fcc centred cubic with an atomic radius of 0.1746 nm. The spacing of (2 2 0) planes (in units of nm) is _____

If the lattice constant of bcc lattice structure is 0.32 nm. The nearest neighbour distance (in nm) in the structure is _____

A metal crystallizes into two cubic phases fcc and bcc, whose unit cell parameters are 3.5Å and 3.0Å, respectively. The ratio of their densities is _____

Sodium has bcc structure whose atomic weight is 23. Its density is 0.96×10^3 kg/m^3. The lattice parameter (in Å) is _____

Silver crystallizes into fcc structure, where the distance between the nearest neighbour silver atoms is 2.87Å. The atomic weight of silver is 107.88. The density (in units of kg/m^3) is _____ $\times10^3$.

ANSWER KEY						
Multiple Choice Questions (MCQ)						
Questions	1	2	3	4	5	6
Answers	b	c	d	d	b	c
Questions	7	8	9	10	11	12
Answers	a	c	d	b	d	b
Questions	13	14	15			
Answers	a	a	b			
Multiple Select Questions (MSQ)						
Questions	1	2	3	4	5	
Answers	c,d	b,c	a,b,c,d	a,c,d	b,c,d	
Numerical Answer Type (NAT)						
Questions	1	2	3	4	5	
Answers	0.0873	0.277	1.259	4.3	10.71	

Chapter 2

RECIPROCAL LATTICE & X-RAY DIFFRACTION

Reciprocal lattice:

Every crystal has two lattices associated with it, the crystal lattice (or direct space lattice) and the reciprocal lattice.

The concept of reciprocal lattice was introduced for the purpose of tabulating two important properties of crystal planes : their slope and their interplanar spacing.

Each set of parallel planes in a direct lattice can be represented by a normal to these planes having length equal to the reciprocal of the interplanar spacing. The normals are drawn with reference to any arbitrary origin and points are marked at their ends. These points form a regular arrangement which is called reciprocal lattice. Thus each point in a reciprocal lattice is a representative point of a particular set of parallel planes. It is easier to deal with such points than to deal with set of planes. That's why we use reciprocal lattice.

Thus reciprocal lattice vector is a vector whose magnitude is equal to the reciprocal of the interplanar spacing i.e., $\frac{1}{d_{hkl}}$ and direction is parallel to the normal to (hkl) plane.

Why reciprocal lattice is so named? The reciprocal lattice is so named because the length assigned to each normal representing the reciprocal lattice is proportional to the reciprocal of the interplanar spacing of that plane in ordinary space.

CAREER ENDEAVOUR

Primitive translation vectors of reciprocal lattice:

Let $\vec{a}, \vec{b}, \vec{c}$ be the primitive translation vectors of a direct space lattice for a crystal forming a primitive unit cell. The volume of this unit cell is given by

$$V = \vec{a}.\left(\vec{b}\times\vec{c}\right) = \vec{b}.\left(\vec{c}\times\vec{a}\right) = \vec{c}.\left(\vec{a}\times\vec{b}\right)$$

The vectors given by $\vec{A} = 2\pi\frac{\vec{b}\times\vec{c}}{\vec{a}.\left(\vec{b}\times\vec{c}\right)}$; $\vec{B} = 2\pi\frac{\vec{c}\times\vec{a}}{\vec{b}.\left(\vec{c}\times\vec{a}\right)}$; $\vec{C} = 2\pi\frac{\vec{a}\times\vec{b}}{\vec{c}.\left(\vec{a}\times\vec{b}\right)}$

are known as reciprocal lattice vectors or primitive translation vectors of reciprocal lattice.

$$\vec{A}.\vec{a} = 2\pi,\ \vec{A}.\vec{b} = 0 \text{ and } \vec{A}.\vec{c} = 0$$

$$\Rightarrow \quad \vec{B}.\vec{a} = 0,\ \vec{B}.\vec{b} = 2\pi \text{ and } \vec{B}.\vec{c} = 0$$

$$\Rightarrow \quad \vec{C}.\vec{a} = 0,\ \vec{C}.\vec{b} = 0 \text{ and } \vec{C}.\vec{c} = 2\pi$$

Choosing $\vec{A}, \vec{B}$ and $\vec{C}$ as the primitive translation vectors of the reciprocal lattice (also known as reciprocal axes) we can construct the reciprocal lattice points or reciprocal lattice vectors given by

$$\vec{G} = h\vec{A} + k\vec{B} + l\vec{C}$$

where h, k, l are intergers and define the coordinate of the point in the reciprocal space.

The crystal lattice is a lattice in real or ordinary space whereas reciprocal lattice is a lattice in the reciprocal space or associated $\vec{k}$ -space or Fourier space. The wave vector $\vec{k}$ is always drawn in Fourier space. Vectors in the crystal lattice have dimensions of length i.e., $[L^1]$ and vectors in the reciprocal lattice have dimensions of $\frac{1}{(\text{Length})}$ i.e., $[L^{-1}]$.

Properties of reciprocal lattice:

(i) The direct lattice is the reciprocal lattice to its own reciprocal lattice. Simple cubic lattice is self reciprocal whereas BCC and FCC lattices are reciprocal of each other.

(ii) Each point in a reciprocal lattice corresponds to a particular set of parallel planes of the direct lattice.

(iii) The distance of a reciprocal lattice point from an arbitrarily fixed origin is inversely proportional to the interplanar spacing of the corresponding parallel planes of the direct lattice.

(iv) The unit cell of the reciprocal lattice is not necessarily a parallelopiped. We deal with the Wigner Seitz cell of the reciprocal lattice which constitutes the Brillouin zone.

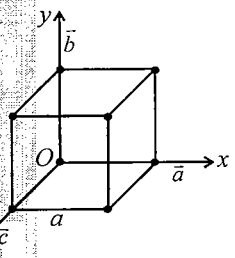
(v) The volume of a unit cell of the reciprocal lattice is inversely proportional to the volume of the corresponding unit cell of the direct lattice.

Reciprocal lattice of simple cubic lattice: Let the primitive translation vectors of a simple cubic cell be $\vec{a}, \vec{b}$ and $\vec{c}$. We can write the vectors as

$$\vec{a} = a\hat{i}, \vec{b} = a\hat{j} \text{ and } \vec{c} = a\hat{k}$$

The primitive translation vectors of the reciprocal of SC lattice are given by

$$\vec{A} = 2\pi \frac{(\vec{b}\times\vec{c})}{\vec{a}.(\vec{b}\times\vec{c})} = 2\pi\left(\frac{a\hat{j}\times a\hat{k}}{a^3}\right) = \frac{2\pi}{a}\hat{i}$$



Similarly, $\vec{B} = 2\pi \frac{(\vec{c}\times\vec{a})}{\vec{b}.(\vec{c}\times\vec{a})} = 2\pi\left(\frac{a\hat{k}\times a\hat{i}}{a^3}\right) = \frac{2\pi}{a}\hat{j}$

and $\vec{C} = 2\pi \frac{(\vec{a}\times\vec{b})}{\vec{c}.(\vec{a}\times\vec{b})} = 2\pi\left(\frac{a\hat{i}\times a\hat{j}}{a^3}\right) = \frac{2\pi}{a}\hat{k}$

Simple cubic lattice has side a and its reciprocal lattice has side $\left(\frac{2\pi}{a}\right)$.

Reciprocal of simple cubic lattice is also a simple cubic lattice with cube side $\left(\frac{2\pi}{a}\right)$ i.e., simple cubic lattice is self reciprocal lattice.

Volume of primitive unit cell of reciprocal lattice

$$V_R = \vec{A}\cdot(\vec{B}\times\vec{C}) \Rightarrow V_R = \left(\frac{2\pi}{a}\right)^3$$

Volume of reciprocal unit cell of any lattice is equal to $V_R = \frac{(2\pi)^3}{V_d}$, where V_d = volume of direct unit cell.

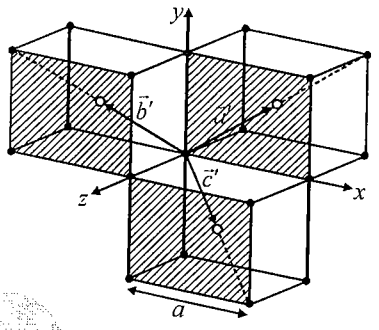
Reciprocal lattice of body centered cubic lattice:

Let the primitive translation vectors of a body centered cubic lattice are

$$\vec{a}' = \frac{a}{2}\left(\hat{i}+\hat{j}-\hat{k}\right),\ \vec{b}' = \frac{a}{2}\left(-\hat{i}+\hat{j}+\hat{k}\right),\ \vec{c}' = \frac{a}{2}\left(\hat{i}-\hat{j}+\hat{k}\right)$$

The primitive translation vectors of the reciprocal of body centered cubic lattice are given by

$$\vec{A} = 2\pi\frac{\vec{b}'\times\vec{c}'}{\vec{a}'.\left(\vec{b}'\times\vec{c}'\right)} = 2\pi\left(\frac{\frac{a^2}{2}\left(\hat{i}+\hat{j}\right)}{a^3/2}\right) = \frac{2\pi}{a}\left(\hat{i}+\hat{j}\right)$$



Similarly, $\vec{B} = 2\pi\frac{\vec{c}'\times\vec{a}'}{\vec{b}'.\left(\vec{c}'\times\vec{a}'\right)} = 2\pi\left(\frac{\frac{a^2}{2}\left(\hat{j}+\hat{k}\right)}{a^3/2}\right) = \frac{2\pi}{a}\left(\hat{j}+\hat{k}\right)$

$$\vec{C} = 2\pi\frac{\vec{a}'\times\vec{b}'}{\vec{c}'.\left(\vec{a}'\times\vec{b}'\right)} = 2\pi\left(\frac{\frac{a^2}{2}\left(\hat{k}+\hat{i}\right)}{a^3/2}\right) = \frac{2\pi}{a}\left(\hat{k}+\hat{i}\right)$$

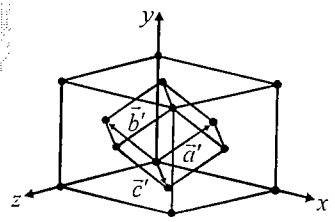
Hence, reciprocal of body centered cubic lattice is face centered cubic lattice with side $\left(\frac{4\pi}{a}\right)$.

Volume of reciprocal unit cell $V_R = \vec{A}\cdot\left(\vec{B}\times\vec{C}\right) \Rightarrow V_R = 2\left(\frac{2\pi}{a}\right)^3$.

Reciprocal lattice of face centered cubic lattice: Let the primitive translation vectors of a face centered cubic lattice are $\vec{a}' = \frac{a}{2}\left(\hat{i}+\hat{j}\right),\ \vec{b}' = \frac{a}{2}\left(\hat{j}+\hat{k}\right),\ \vec{c}' = \frac{a}{2}\left(\hat{k}+\hat{i}\right)$

The primitive translation vectors of the reciprocal of face centered cubic lattice are given by

$$\vec{A} = 2\pi\frac{\vec{b}'\times\vec{c}'}{\vec{a}'.\left(\vec{b}'\times\vec{c}'\right)} = 2\pi\left(\frac{\frac{a^2}{4}\left(\hat{i}+\hat{j}-\hat{k}\right)}{\frac{a^3}{4}}\right) = \frac{2\pi}{a}\left(\hat{i}+\hat{j}-\hat{k}\right)$$



Similarly, $\vec{B} = \frac{2\pi}{a}\left(-\hat{i}+\hat{j}+\hat{k}\right)$ and $\vec{C} = \frac{2\pi}{a}\left(\hat{i}-\hat{j}+\hat{k}\right)$

Hence, reciprocal of face-centered cubic lattice is body centred cubic lattice.

X-RAY DIFFRACTION:

What are X-rays:

Like visible light, X-rays are also a form of electromagnetic radiation having wavelength of the order of 1 Å. In other words, we can say that X-rays represent a relatively high energy form of radiation. In electromagnetic spectrum, X-rays lie between ultraviolet and gamma rays.

Production of X-rays:

The X-rays are produced when a stream of high energy charged particles, e.g. a beam of electrons are allowed to strike a metal target (such as Molybdenum, Copper or Tungsten) of high melting point. In this process electrons loose some of their energy which is converted into electromagnetic radiations, i.e. some part of energy will be converted into electromagnetic radiations of high frequency according to formula $\Delta E = h\nu$. These processes give continuous X-rays.

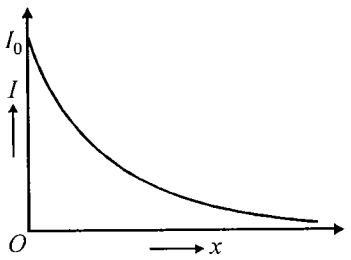
The wavelength of the X-rays can be given by $\Delta E = h\nu_{max} = \frac{hc}{\lambda_{min}}$.

$$\Rightarrow \quad \lambda_{min} = \frac{hc}{\Delta E} = \frac{12400}{E(\text{eV})} \text{ Å}$$

$$\Rightarrow \quad \lambda_{min} = \frac{12400}{V(\text{volt})} \text{ Å},$$

where V is the applied potential for accelerating the electrons.

Thus, we get $\lambda_{min} \propto \frac{1}{V}$.



Absorption of X-rays:

When X-rays of intensity (I_0) are passed through a material of absorption coefficient μ, the intensity of the transmitted beam is given by $I_t = I_0 e^{-\mu x}$, where x is the thickness of the material. Thus the intensity of X-ray beam decreases exponentially with the thickness of the absorbing material.

X-ray diffraction:

Similar to other electromagnetic radiatons, X-rays also interact with the electron cloud of the atoms. Since X-rays have shorter wavelengths, X-rays are scattered by adjacent atoms in crystal. The scattered X-rays interfere and give rise to diffraction phenomena. The atoms of crystal acts as a diffraction center for the X-rays like a three-dimensional diffraction grating. The X-ray diffraction pattern produced by this method can give us the information about the internal arrangement of atoms in crystals.

Consider parallel lattice planes equidistant from one another in a crystal structure, separated by a distance d. Suppose a beam of X-rays AB of wavelength λ is incident on a layer of atoms in one cleavage plane P of a crystal at a glancing angle θ at B and after reflection goes along BC. The ray DE incident on cleavage plane Q after reflection goes along EF. Draw BG perpendicular on DE and BH perpendicular on EF, then BG is the incident wave front and BH the reflected wave front.

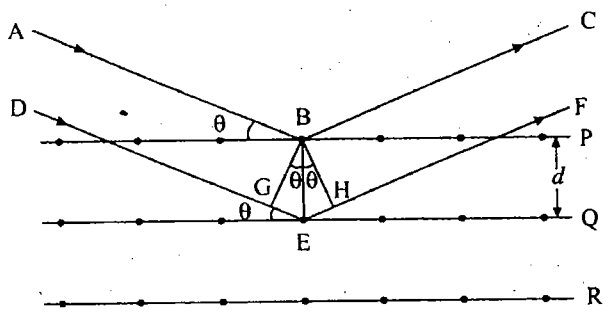
Path difference between the two wave fronts = GE + EH

Now, in ΔBGE, $GE = BE\sin\theta = d\sin\theta$ and in ΔBEH, $EH = BE\sin\theta = d\sin\theta$

Therefore, Path-difference, $GE + EH = 2d\sin\theta$

If this path-difference is an integral multiple of wavelength λ, constructive interference will take place between the reflected beams. Thus the intensity will be maximum if

$$\therefore \quad 2d\sin\theta = n\lambda \qquad \text{where } n = 1, 2, 3, \ldots\ldots\ldots$$



Bragg's reflection of X-rays from the atomic planes

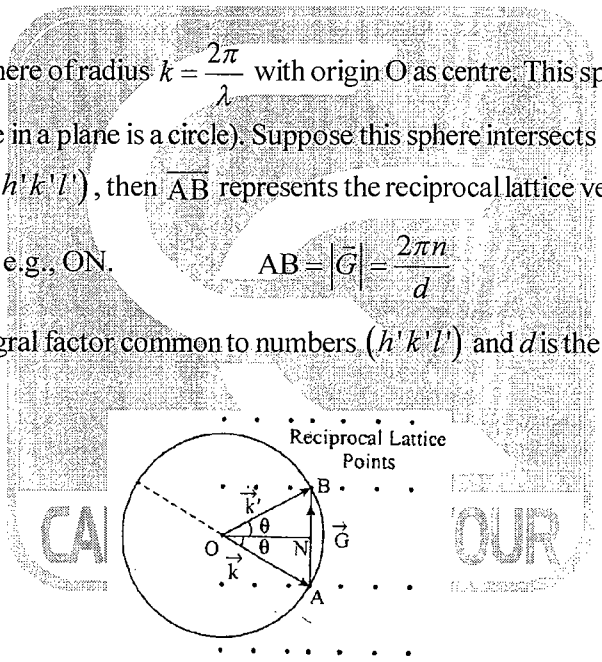
For $n = 1$, we get the first order spectrum and for $n = 2$, we get the second order spectrum and so on. This phenomenon is similar to that of the optical diffraction with diffraction grating with only the difference that here various parallel planes of the crystal act as lines in the grating. The above equation $2d \sin\theta = n\lambda$ is Bragg's equation and represents Bragg's law.

Physical significance of Bragg's law : The interpretation of Bragg's law gives that the diffraction intensities can be stronger only at certain values of θ for specific values of λ and d. Therefore, a knowledge of experimentally observed diffraction angles and wavelength of X-rays can provide the information about the interplanar spacing of the crystal. Thus, the space, shape and orientation of the unit cell can be determined from any diffraction experiment using monochromatic X-rays.

Ewald construction:

Ewald construction is a geometrical construction to get a relation between wave vector $\vec{k}$ and the direction of incident X-ray beam with the help of reciprocal lattice and thus derive Bragg's law in vector form. Draw a vector $\overrightarrow{OA} = \vec{k} = \frac{2\pi}{\lambda}$ in the direction of incident X-ray beam originating at O and terminating at any reciprocal lattice point A. Draw a sphere of radius $k = \frac{2\pi}{\lambda}$ with origin O as centre. This sphere is known as reflex sphere (The section of this sphere in a plane is a circle). Suppose this sphere intersects some point B of the reciprocal lattice whose indices are $(h'k'l')$, then $\overrightarrow{AB}$ represents the reciprocal lattice vector $\vec{G}$ and is normal to some set of direct lattice planes e.g., ON. $AB = |\vec{G}| = \frac{2\pi n}{d}$

where n is the largest integral factor common to numbers $(h'k'l')$ and d is the interplanar spacing for the set of parallel planes.



Ewald construction in the reciprocal lattice

Bragg's diffraction condition in terms of reciprocal lattice:

As the wave vector $\overrightarrow{OA} = \vec{k}$ is the incident wave vector, $\overrightarrow{OB} = \vec{k}'$ is the diffracted (or reflected) wave vector and is given by

$$\vec{k}' = \vec{k} + \vec{G} \qquad \text{...(i)}$$

This shows that scattering changes only the direction of the wave vector $\vec{k}$ and the scattered wave vector differs from the incident wave vector by a reciprocal lattice vector $\vec{G}$.

For the diffraction, the magnitude of $(\vec{k} + \vec{G})$ must be equal to the wave vector $\vec{k}'$.

i.e. $$\vec{k}'^2 = (\vec{k} + \vec{G})^2 = (\vec{k} + \vec{G}) \cdot (\vec{k} + \vec{G}) = \vec{k}^2 + 2\vec{k} \cdot \vec{G} + \vec{G} \cdot \vec{G}$$

But the magnitude of $\vec{k}'$ = magnitude of $\vec{k}$ ($\because$ wavelength of incident and scattered beam is same)

$\Rightarrow \quad \cdot \vec{k}'^2 = \vec{k}^2$

Hence, equation (i) becomes,

$$2\vec{k} \cdot \vec{G} + \vec{G} \cdot \vec{G} = 0 \quad \Rightarrow 2\vec{k} \cdot \vec{G} + G^2 = 0$$

$$\Rightarrow \quad \left(\vec{k} + \frac{\vec{G}}{2}\right) \cdot \vec{G} = 0$$

This is Bragg's law in vector form and gives the Bragg's diffraction conditions in terms of reciprocal lattice vector $\vec{G}$.

Brillouin zones : A Brillouin zone is the locus of all those $\vec{k}$- values in the reciprocal lattice which are Bragg reflected. For example, let us construct the Brillouin zones for a simple cubic structure with lattice parameter a. The primitive translation vectors of the simple cubic structure are

$\vec{a} = a\hat{i},\ \vec{b} = a\hat{j}$ ... (i)

The corresponding translation vectors of the reciprocal lattice are

$$\vec{A} = \left(\frac{2\pi}{a}\right)\hat{i},\ \vec{B} = \left(\frac{2\pi}{a}\right)\hat{j}$$

The reciprocal lattice vector is given by

$$\vec{G} = h\vec{A} + k\vec{B} = \frac{2\pi}{a}\left(h\hat{i} + k\hat{j}\right) \quad \text{... (ii)}$$

The Bragg's law in the reciprocal lattice is given by

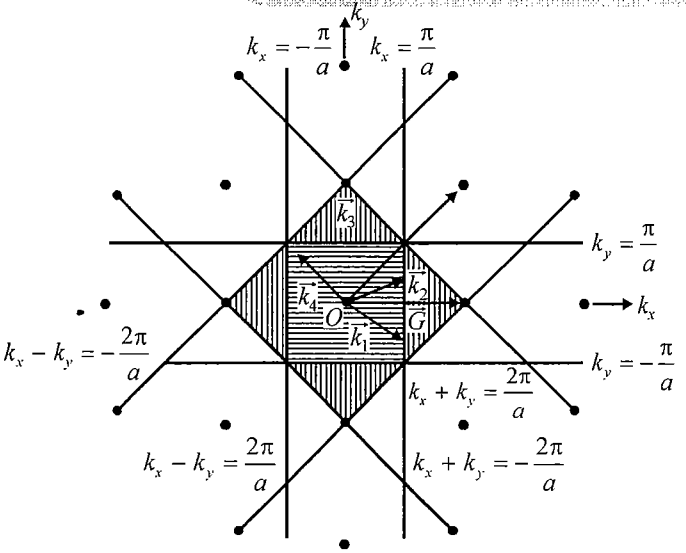
$2\vec{k} \cdot \vec{G} + G^2 = 0$ ... (iii)

$$\Rightarrow \quad 2\left(\frac{2\pi}{a}\right)\left[\left(k_x\hat{i} + k_y\hat{j}\right) \cdot \left(h\hat{i} + k\hat{j}\right)\right] + \frac{4\pi^2}{a^2}\left(h^2 + k^2\right) = 0$$

$$\Rightarrow \quad \frac{4\pi}{a}\left[hk_x + kk_y\right] = -\frac{4\pi^2}{a^2}\left(h^2 + k^2\right)$$

$$\Rightarrow \quad hk_x + hk_y = -\frac{\pi}{a}\left(h^2 + k^2\right) \quad \text{... (iv)}$$

CAREER ENDEAVOUR



Brillouin zones of a square lattice in its reciprocal lattice

The $\vec{k}$ - values which are Bragg reflected are obtained by the all possible combinations of h and k.

For $h = \pm 1$ and $k = 0$, equation (iv) gives $k_x = \pm\frac{\pi}{a}$ and k_y is arbitrary.

Also, for $h = 0$ and $k = \pm 1$, equation (iv) gives $k_y = \pm\frac{\pi}{a}$ and k_x is arbitrary.

Thus, we get a square of side $\left(\frac{2\pi}{a}\right)$ bounded by $k_x = \pm\frac{\pi}{a}$ and $k_y = \pm\frac{\pi}{a}$. This square is called as first Brillouin zone. Hence, the first zone of a square lattice of side a is a square of side $\frac{2\pi}{a}$.

In addition to this set of lines, some other sets of lines are also possible which satisfies equation (iv). For example, $h = \pm 1$ and $k = \pm 1$, equation (iv) gives the following set of lines $\pm k_x \pm k_y = \frac{2\pi}{a}$. The area bounded by these four lines are called as second Brillouin zone. Similarly, we can construct other Brillouin zones. The boundaries of Brillouin zones represent the loci of $\vec{k}$ - values that are Bragg reflected. Hence we consider them as reflecting planes. The boundaries of the first zone represent the reflecting planes for the first order reflection and those of the second zone represent the reflecting planes for the second order reflection, and so on. A $\vec{k}$ - vector that does not terminate at a zone boundary can not produce Bragg reflection. So, we can say that Brillouin zone pattern can be used to determine the X-ray diffraction pattern of a crystal and vice-versa. The Brillouin zones for a three dimensional cubical lattice are constructed using the generalized equation

$$hk_x + kk_y + lk_z = -\frac{\pi}{a}\left(h^2 + k^2 + l^2\right)$$

where a is the length of the cube edge.

Thus, we get first Brillouin zone as a cube of side equal to $\left(\frac{2\pi}{a}\right)$. The second zone is formed by adding pyramids to each face of the cube (first zone) as triangles are added to the square in two-dimensions, and so on.

CAREER ENDEAVOUR

Electron diffraction : Similar to X-rays, a beam of electron can also be used to characterize the crystal structure. A beam of electron emitted from a hot wire (as a result of thermionic emission) is accelerated through vacuum under a very large electrode potential (V) and is then passed through a collimator before it gets diffracted by the specimen to be studied.

The energy (E) acquired by these electrons will depend on accelerating voltage (V), thus, $E = eV$ ... (i)

where e is electronic charge.

According to de-Broglie hypothesis, the wavelength of the moving electrons can be given by

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}} = \frac{h}{\sqrt{2meV}} \qquad \text{... (ii)}$$

where m is the mass of electrons.

Putting $m = 9.11 \times 10^{-31}$ kg and $e = 1.6 \times 10^{-19} C$, the wavelength λ will be $\lambda = \left(\frac{150}{V}\right)^{1/2}$ Å $= \frac{12.24}{\sqrt{V}}$ Å.

Thus, wavelength of moving electron is inversely proportional to the square root of the accelerating voltage and hence can be varied easily to get the desired wavelength of electron beam.

Thus, if we apply the accelerating voltage (V) such that the wavelength of the electron beam become equal to the order of atomic distances, then we can use electron beam diffraction.

Neutron diffraction:

Neutron diffraction is other diffraction technique that can be employed at the place of X-ray and electron diffraction.

The wavelength of neutron is given by

$$\lambda_n = \frac{h}{p} = \frac{h}{\sqrt{2m_n E}} \qquad \ldots (i)$$

Since neutrons are neutral particles, so the neutrons can not be accelerated by applying potential. The neutrons can be accelerated thermally. The kinetic energy of a neutron is given by

$$E = \frac{3}{2} k_B T \qquad \ldots (ii)$$

Thus, the wavelength $\lambda_n = \frac{h}{\sqrt{3m_n k_B T}} \Rightarrow \lambda_n \propto \frac{1}{\sqrt{T}}$.

If temperature of the neutrons is selected such that the wavelength of neutron becomes equal to the order of atomic distances, then these neutrons can be used in diffraction phenomena.

Diffraction intensity:

The different directions of the diffracted X-ray beam obtained according to Bragg's law are governed by the geometry and the size of the unit cell. These directions will not be affected by the arrangement of atoms associated with each lattice point of the crystal structure. However, the complexity of the atomic structure within the unit cell does affect the intensity of the diffracted beam. The intensity depends on the atomic scattering factor, the position of each atom in unit cell. Since, the electrons are the only component that an atom have, therefore, the X-rays scattered from one part of an atom interfere with those scattered from the other part at all angles of scattering. Since, the electrons are distributed throughout the atomic volume, the diffracted beams are obtained as a result of combination of the scattered waves from the electrons of all atoms in the unit cell. This process involves two distinct contributions.

1. Scattering from electrons in the same atom. This gives the atomic scattering factor f.
2. The summation of this scattering from all atoms in the unit cell. This gives the geometrical structure factor S.

Intensity of diffracted beam can be written as $I = F^2 = f^2 S^2$, where S is geometrical structure factor, f is atomic scattering factor and F is diffracted X-ray beam amplitude.

Thus, $F = fS$. The atomic scattering factor (f) is defined as $f = \frac{\text{diffracted amplitude by an atom}}{\text{diffracted amplitude by an electron}}$.

If the atoms are assumed to be points only, then the atomic scattering factor is just equal to the number of electrons present (i.e., the atomic number Z).

Examples: (1) the atomic scattering factor of Na ($Z = 11$) will be larger in comparison to Li ($Z = 3$).

(2) In NaCl, the atomic scattering factors of Na^+ and Cl^- ions are different. The Cl^- ion has more atomic scattering factor.

Thus, the higher scattering factor means higher number of electrons and intensity.

The geometrical structure factor (S) is given by $S = \frac{\text{diffracted amplitude by an unit cell}}{\text{diffracted amplitude by an atom}}$.

Mathematically, $S = \sum_j e^{2\pi i (u_j h + v_j k + w_j l)}$, where (h, k, l) is miller indices of the plane and (u_j, v_j, w_j) is position coordinate of the j^{th} atom.

(1) Simple cubic structure : For simple cubic structure, the effective number of atoms per unit cell is one. Let us take the position coordinate of this atom as origin (0, 0, 0).

Thus, geometrical structure factor $S = \sum_j e^{2\pi i(u_j h + v_j k + w_j l)} = e^{2\pi i(0h+0k+0l)} = e^0 = 1$

$\Rightarrow \quad S = 1$

The diffracted X-ray beam amplitude becomes $F = fS = f(1) = f$ and intensity of diffracted beam $(I) = f^2$. Hence, all the $(h\ k\ l)$ values are allowed. In other words we can say that all the reflection peaks are allowed in simple cubic structures.

(2) Body centered cubic structures:

For BCC structure, the effective number of atoms per unit cell is equal to 2. We can take the position coordinates of these atoms as (0, 0, 0) and $\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)$.

Thus, the geometrical structure factor (S) can be given as

$$S = \sum_j e^{2\pi i(u_j h + v_j k + w_j l)} = e^{2\pi i(0h+0k+0l)} + e^{2\pi i(\frac{1}{2}h + \frac{1}{2}k + \frac{1}{2}l)} = 1 + e^{\pi i(h+k+l)}.$$

For $(h+k+l)$ = even, $S = 1+1 = 2$

For $(h+k+l)$ = odd, $S = 1-1 = 0$

Thus, the amplitude of the diffracted X-ray beam will be given by

$F = 2f$ [For $(h+k+l)$ = even]

$= 0$ [For $(h+k+l)$ = odd]

and the intensity of diffracted beam becomes

$(I) = (2f)^2 = 4f^2$, for $(h+k+l)$ = even

$= 0$, for $(h+k+l)$ = odd

Thus, the reflection peaks (1 1 0), (2 0 0), (2 1 1), (2 2 2),... are allowed for BCC structure and reflection peaks (1 0 0), (1 1 1), (3 0 0),... are absent.

(3) Face centered cubic structure:

For FCC structure, the effective number of atoms per unit cell is equal to 4. Let the position coordinates of these atoms are (0, 0, 0), $\left(\frac{1}{2}, \frac{1}{2}, 0\right)$, $\left(0, \frac{1}{2}, \frac{1}{2}\right)$, $\left(\frac{1}{2}, 0, \frac{1}{2}\right)$.

Thus, the geometrical structure factor (S) can be given as

$$S = \sum_j e^{2\pi i(u_j h + v_j k + w_j l)} = 1 + e^{\pi i(h+k)} + e^{\pi i(k+l)} + e^{\pi i(h+l)}$$

The geometrical structure factor S will be equal to 4 if all the h, k, l are either even or odd and zero otherwise. Thus, the amplitude of the diffracted X-ray beam will be given by

$F = 4f$, h, k, l are either all odd or all even

$= 0$, otherwise

and the intensity of diffracted beam becomes

$I = (4f)^2 = 16f^2$, h, k, l are either all odd or all even

$= 0$, otherwise

Hence, reflection planes (1 1 1), (2 0 0), (2 2 0), (3 1 1),... are allowed (i.e. present) for FCC structure and reflection planes (1 0 0), (1 1 0), (2 1 1),... are not allowed (i.e. absent).

(4) Diamond cubic structure:

For diamond cubic structure, only those reflection peaks will be allowed for which

(i) $(h\,k\,l)$ are all odd, or

(ii) $(h\,k\,l)$ are all even and also $h+k+l=4n$ where $n=1,2,3,...$

i.e. if $(h\,k\,l)$ all are even and $(h+k+l)$ is divisible by 4 then this peak will be present in diamond cubic structure.

(5) Rock salt (NaCl) structure:

The selection rules for NaCl structure is same as FCC structure. There is only one exception. For KCl, the selection rules of NaCl structure is not valid.

SOLVED PROBLEMS

A beam of X-rays is incident on a BCC crystal. If the difference between the incident and scattered wave vectors is $\vec{K}=h\hat{x}+k\hat{y}+l\hat{z}$ where $\hat{x}, \hat{y}, \hat{z}$ are the unit vectors of the associated cubic lattice, the necessary condition for the scattered beam to give a Laue maximum is

(a) $h+k+l=$ even (b) $h=k=l$

(c) h, k, l are all distinct (d) $h+k+l=$ odd **[JEST-2012]**

Sln. For BCC crystal, the effective number of atoms is two; one occupies a corner position and the other occupies the center of the cube. If the coordinates of the corner atom be arbitrary taken as (0, 0, 0), then the coordinates of the other become $\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right)$. Since, both atoms are identical, the diffraction amplitude (F) can be given as

$$F(hkl)=f\sum_j e^{2\pi i(u_j h+v_j k+w_j l)}=f(1+e^{i\pi(h+k+l)})$$

Above equation gives zero for odd value of $(h+k+l)$ and will give non-zero value for even values of $(h+k+l)$.

Hence, for BCC crystal, the necessary condition for the scattered beam to give a Laue maximum is $h+k+l=$ even.

Hence, correct option is (a).

If $\vec{k}$ is the wavevector of incident light $|\vec{k}|=\frac{2\pi}{\lambda}$, λ is the wavelength of light, and $\vec{G}$ is reciprocal lattice vector, then the Bragg's law can be written as

(a) $\vec{k}+\vec{G}=0$ (b) $2\vec{k}\cdot\vec{G}+G^2=0$ (c) $2\vec{k}\cdot\vec{G}+k^2=0$ (d) $\vec{k}\cdot\vec{G}=0$ **[JEST-2016]**

Sln. The Bragg's law in terms of reciprocal lattice vector G)

$\therefore\ \vec{k}'=\vec{k}+\vec{G}$

$\Rightarrow\ \vec{k}'\cdot\vec{k}'=(\vec{k}+\vec{G})\cdot(\vec{k}+\vec{G})\ \Rightarrow\ |\vec{k}'|^2=|\vec{k}|^2+2\vec{k}\cdot\vec{G}+|\vec{G}|^2$

$\because\ |\vec{k}'|^2=|\vec{k}|^2$

$\therefore\ 2\vec{k}\cdot\vec{G}+G^2=0$

Hence, correct option is (b).

CAREER ENDEAVOUR

3. The second order maximum in the diffraction of X-rays of 0.20 nanometer wavelength from a simple cubic crystal is found to occur at an angle of thirty degrees to the crystal plane. The distance between the lattice planes is

(a) 1 Angstrom (b) 2 Angstrom (c) 4 Angstrom (d) 8 Angstrom **[JEST-2012]**

Soln. Given : $n = 2$, $\lambda = 0.20$ nanometer, $\theta = 30°$, $d = ?$

From Bragg's law : $2d \sin\theta = n\lambda$

$\Rightarrow \quad 2 \times d \times \sin 30° = 2 \times 0.20$ nanometer

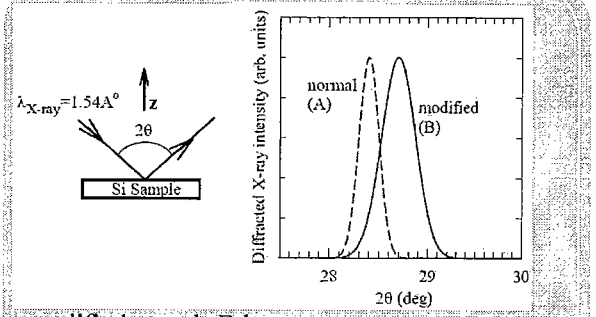
$$\Rightarrow \quad d = \frac{2 \times 0.20}{2 \times \sin 30°} = \frac{2 \times 0.20}{2 \times \frac{1}{2}} = 0.40 \text{ nanometer}$$

$\Rightarrow \quad d = 4$ Angstrom

i.e., the distance between the lattice planes is 4 Angstrom.

Hence, correct option is (c).

4. The figure below shows the Bragg diffraction pattern for X-rays of wavelength 1.54 Å incident on two crystalline silicon thin film samples A and B. The dashed line corresponds to a normal sample A and the continuous line corresponds to another sample B, which is modified due to differences in the growth conditions.



These plots suggest that the modified sample B is

(a) stretched in all directions by 3%

(b) compressed in all directions by 3%

(c) stretched in the z-direction by 1 % and possibly compressed in x and y directions

(d) compressed in the z-direction by 1 % and possibly stretched out in x and y directions

[TIFR-2011]

CAREER ENDEAVOUR

Soln. From Bragg's law : $2d \sin\theta = n\lambda$

For sample (A), the maximum occurs at 28°24′ and

For sample (B), the maximum occurs at 28°42′.

For modified sample B, the diffracted X-ray intensity peak shifted by 10′ i.e., θ increases. Hence, the distance between planes will decrease in the z-direction.

From Bragg's law $2d \sin\theta = \lambda \Rightarrow \frac{\partial d}{d} + \cot\theta\, \partial\theta = 0 \quad (\because \lambda_{\text{X-ray}}$ is fixed)

$$\Rightarrow \quad \frac{\partial d}{d} = -\cot\theta\, d\theta = -(\cot 28°18') \times (18') = -1.857 \times (5.233 \times 10^{-3}) = -9.7 \times 10^{-3}$$

$$\therefore \quad \frac{\partial d}{d}\% = -9.7 \times 10^{-3} \times 100 = -0.97\% \approx -1\%.$$

Therefore, the distance between planes will decrease i.e., the modified sample is compressed in z-direction by 1 %. The sample may be streched out in x and y directions.

Hence, correct option is (d).

Metallic copper is known to form cubic crystals and the lattice constant is measured from X-ray diffraction studies to be about 0.36 nm. If the specific is 63.5, one can conclude that

(a) the crystal are of simple cubic type
(b) the crystals are of BCC type
(c) the crystals are of FCC type
(d) the crystals are a mixture of FCC and BCC types
(e) there is insufficient data to distinguish between the previous options

[TIFR-2012]

Sln. Given, $a = 0.36$ nm

Specific gravity $= \rho = 8.96\ \text{g/cm}^3 = 8.96 \times 10^3\ \text{kg/m}^3$

Avogadro number $(N_A) = 6.023 \times 10^{26}$, $n = ?$

Therefore, from equation

$$\rho = \frac{nM}{N_A a^3} \Rightarrow n = \frac{\rho \cdot N_A \cdot a^3}{M} = \frac{8.96 \times 6.023 \times 10^{26} \times (0.36 \times 10^{-9})^3}{63.3} = 0.0397 \times 10^{29} \times 10^{-27}$$

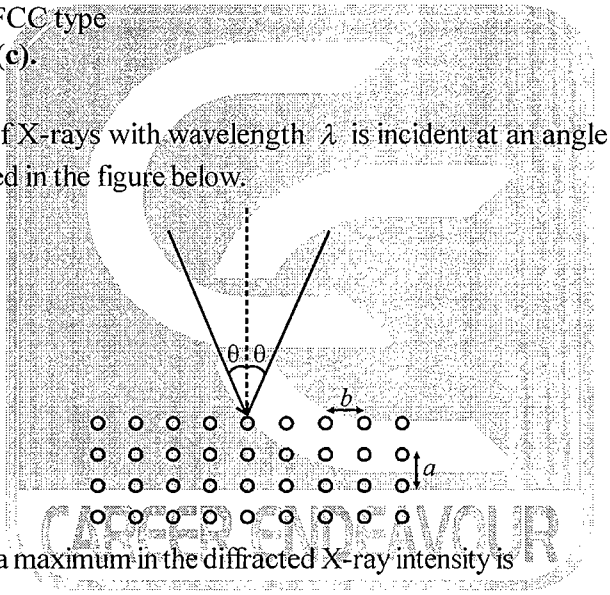
$$= 0.0397 \times 10^2 = 3.97 \cong 4$$

i.e., number of atoms per unit cell = 4.

Hence, the crystals are of FCC type

Hence, correct option is (c).

A monochromatic beam of X-rays with wavelength λ is incident at an angle θ on a crystal with lattice spacings a and b as sketched in the figure below.



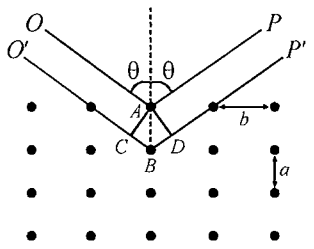
A condition for there to be a maximum in the diffracted X-ray intensity is

(a) $2\sqrt{a^2 + b^2}\ \sin\theta = \lambda$
(b) $2b \cos\theta = \lambda$
(c) $2a \cos\theta = \lambda$
(d) $(a + b) \sin\theta = \lambda$

[TIFR-2012]

Sln. The path difference between OAP and $O'BP' = CB + BD = a\cos\theta + a\cos\theta = 2a\cos\theta$ ...(i)

For maximum intensity, this path difference should be integral multiple of λ.



$\therefore\ 2a\cos\theta = n\lambda$, where $n = 1, 2, 3, \ldots$

Hence, correct option is (c).

7. In a Davisson-Germer experiment, a collimated beam of electrons of energy 54eV, at normal incidence on a given crystal, shows a peak at a reflection angle of 40°. If the electron beam is replaced by a neutron beam, and the peak appears at the same value of reflection angle, then the energy of the neutrons must be

(a) 330 eV (b) 33 eV (c) 0.3 eV (d) 0.03 eV **[TIFR 2013]**

Soln. Energy of electrons (E_e) = 54 eV, energy of neutron (E_n) = ?

Since the angle of reflection is same for electron and neutron.

Therefore, we can use Bragg's law for these cases separately.

For electron : $2d\sin\theta = \lambda_e = \dfrac{h}{\sqrt{2m_eE}}$ ... (i)

For neutron : $2d\sin\theta = \lambda_n = \dfrac{h}{\sqrt{2m_nE}}$ ... (ii)

Since, the crystal is same for both cases and θ is 40°.

From equation (i) and (ii), we get

$$\frac{h}{\sqrt{2m_eE_e}} = \frac{h}{\sqrt{2m_nE_n}} \Rightarrow 2m_eE_e = 2m_nE_n$$

$$\Rightarrow \quad 2\times9.1\times10^{-31}\times54\times1.6\times10^{-19} = 2\times1.6\times10^{-27}\times E_n$$

$$\Rightarrow \quad E_n = \frac{2\times9.1\times10^{-31}\times54\times1.6\times10^{-19}}{2\times1.67\times10^{-27}} = 0.03\,eV$$

Hence, correct option is (d)

8. In the X-ray diffraction pattern of a sodium metal, which has a bcc structure, the missing reflection planes will be **[DU 2016]**

(a) 101 (b) 011 (c) 020 (c) 100

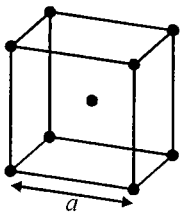
Soln. For BCC structures, $h+k+l$ = even

Here in present case - All the options except (d) has $h+k+l$ = even.

Hence the reflection plane (100) will be missing for BCC structure.

Hence, correct option is (d).

9. The conventional unit cell of a body-centered cubic Bravais lattice is shown in the figure below. The conventional cell has volume a^3. What is the volume of the primitive unit cell ? **[BHU 2012]**



(a) $\dfrac{a^3}{8}$ (b) $\dfrac{a^3}{4}$ (c) $\dfrac{a^3}{2}$ (d) a^3

Soln. Volume of the primitive unit cell of BCC Bravais lattice = $\dfrac{a^3}{2}$.

Hence, correct option is (c).

. The longest wavelength X-ray that can undergo Bragg diffraction in a crystal for a given family of planes of spacing d is **[BHU 2012]**

(a) $\frac{d}{4}$ (b) $\frac{d}{2}$ (c) d (d) $2d$

ln. From Bragg's law $2d \sin\theta = n\lambda$

For longest wavelength, $\theta = 90°$, $n = 1$

Thus, $2d = \lambda$

Hence, correct option is (d).

. The diffraction pattern of copper metal was measured with X-ray radiation of wavelength of 1.315 Å. The first order Bragg diffraction peak was found at an angle 2θ of 50.5°. The d-spacing between the diffracting planes in the copper metal is **[BHU 2012]**

(a) 3.25 Å (b) 1.54 Å (c) 0.41 Å (d) 4.2 Å

ln. $\lambda = 1.315$ Å, $n = 1$, $2\theta = 50.5° = 25.25°$, $d = ?$

From Bragg's law $2d \sin\theta = n\lambda$

$\Rightarrow 2 \times d \times \sin 20.25° = 1 \times 1.315$

$\Rightarrow d = \frac{1.315}{2 \times \sin 25.25°} = 1.54$ Å.

Hence, correct option is (b).

. The continuous X-ray spectrum is the result of the **[BHU 2012]**

(a) photoelectric effect (b) inverse photoelectric effect
(c) compton effect (d) auger effect

ln. The continuous X-ray spectrum is the result of the inverse photoelectric effect.

Hence, correct option is (b).

. The short wavelength limit of X-ray depends upon **[BHU 2013]**

(a) nature of the target (b) potential difference across the X-ray tube
(c) nature of the filament used (d) none of these

ln. The short wavelength $\boxed{\lambda \propto \frac{1}{V}}$. Thus, wavelength depends on potential difference across the X-ray tube.

Hence, correct option is (b).

. The de-Broglie wavelength associated with an electron of mass m and accelerated by a potential V is

(a) $\frac{h}{\sqrt{2mVe}}$ (b) $\frac{\sqrt{2mVe}}{h}$ (c) $\frac{h}{\sqrt{Vem}}$ (d) $\frac{h}{2Vem}$ **[BHU 2013]**

ln. For an electron, the de-Broglie wavelength is given by $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}} = \frac{h}{\sqrt{2meV}}$, where $E = eV$ is the energy of electron.

Hence, correct option is (a).

15. According to Moseley's law, the relation between the atomic number Z and frequency ν is given by **[BHU 2013]**

(a) $\nu \propto (Z-b)$ (b) $\nu \propto Z$ (c) $\nu \propto (Z-b)^3$ (d) $\nu \propto (Z-b)^2$

Soln. According to Moseley's law, the relation between the atomic number Z and frequency ν is given by $\nu \propto (Z-b)^2$

Hence, correct option is (d).

16. In the X-ray diffraction of a set of crystal planes having d equal to 0.18 nm, first order reflection is found to be at an angle of 22°. The wavelength of X-ray is : (sin 22° = 0.208) : **[BHU 2014]**

(a) 0.0749 nm (b) 0.0374 nm (c) 0.749 nm (d) 0.374 nm

Soln. d = 0.18 nm, $n = 1$, $\theta = 22°$

From Bragg's law: $2d \sin\theta = n\lambda$

$\Rightarrow 2 \times 0.18 \times \sin 22° = 1 \cdot \lambda$

$\Rightarrow \lambda = 0.779$ nm

Hence, correct option is (a).

17. During X-ray emission if the voltage is increased : **[BHU 2014]**

(a) minimum wavelength decreases (b) minimum wavelength increases

(c) intensity increases (d) intensity decreases

Soln. Since $\lambda_{min} \propto \frac{1}{V}$

Hence, with increase in voltage, the minimum wavelength decreases.

Hence, correct option is (a).

18. The amplitude of scattering of X-rays scatered by a single atom is generally denoted as **[BHU 2014]**

(a) Structure factor (b) Polarization factor (c) Form factor (d) Fractional coordination

Soln. The amplitude of scattering of X-rays scattered by a simple atom in generally denoted as structure factor.

Hence, correct option is (a).

19. X-ray diffraction can be applied to **[BHU 2014]**

(a) liquids only (b) solid, crystalline materials only

(c) all liquids, solids and crystalline materials (d) gaseous or vapour materials only

Soln. X-ray diffraction can be applied to all liquids, solids and crystalline materials.

Hence, correct option is (c).

20. Characteristic X-rays are the characteristic of which of the following : **[BHU 2014]**

(a) Cathode materials (b) Anode materials

(c) Accelerating voltage (d) Tube current

Soln. Characteristic X-ray are the characteristic of the anode material.

Hence, correct option is (b).

21. FCC lattice is the reciprocal lattice of the **[BHU 2014]**

(a) BCC lattice (b) SC lattice (c) HCP lattice (d) both of the BCC and SC lattices

Soln. FCC lattice is the reciprocal of BCC lattice.

Hence, correct option is (a).

CAREER ENDEAVOUR

2. Because of which property of the crystal X-rays can be diffracted from crystals : **[BHU 2014]**
(a) random arrangement of atoms (b) colour of the crystals
(b) periodic array of atom (d) transparency of crystals

Sol. Due to periodic array of atoms in crystals the X-rays can be diffracted from crystal.
Hence, correct option is (c).

3. According to Mosley's law the frequency of the characteristic X-radiation is proportional to the square of
(a) atomic weight of the element (b) atomic number of the element **[BHU 2014]**
(c) number of neutrons (d) square of atomic number

Sol. From Mosley's law; $\sqrt{\nu} \propto Z$ where ν is frequency and z is atomic number.

$\Rightarrow \nu \propto Z^2$

Thus, the frequency of the characteristic X-radiation is proportional to the square of atomic number of the element.
Hence, correct option is (b).

4. The angle of diffracted X-rays is 90° from the incident direction of X-rays. The interplanar separation of the planes responsible for the diffraction is given by **[BHU 2015]**

(a) $\frac{\lambda}{\sqrt{2}}$ (b) $\frac{\lambda}{2}$ (c) $\sqrt{2}\lambda$ (d) $\frac{\sqrt{3}}{2}\lambda$

Sol. $\theta = 90°,\ n = 1$

From Bragg's law : $2d \sin\theta = n\lambda$

$\Rightarrow 2 \times d \times \sin 90° = 1 \cdot \lambda \Rightarrow 2 \times d \times 1 = \lambda \Rightarrow d = \lambda/2$.
Hence, correct option is (b).

5. Which of the statements about the spectrum of continuous X-rays is false ?
(a) There is no long wavelength cutoff limit **[BHU 2015]**
(b) Continuous X-rays are produced due to deceleration of electrons
(c) The maximum intensity of continuous X-rays depends upon potential difference across the X-ray tube
(d) Short wavelength limit of X-rays depends upon the nature of filament

Sol. Out of above options, only option (d) short wavelength limit of X-rays depends upto the nature of filament, is false.
Hence, correct option is (d).

6. The de Broglie wavelength λ, mass m and charge e of an electron are related to the accelerating potential V by **[BHU 2015]**

(a) $V = \frac{\lambda^2}{2meh^2}$ (b) $V = \frac{h}{2me\lambda^2}$ (c) $V = \frac{h^2}{2me\lambda^2}$ (d) $V = \frac{h^2\lambda^2}{2me}$

Sol. For an electron, $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2me}} = \frac{h}{\sqrt{2meV}}$

Squaring both sides, we get $\lambda^2 = \frac{h^2}{2meV}$

$\Rightarrow V = \frac{h^2}{2me\lambda^2}$

Hence, correct option is (c).

27. The reciprocal lattice of an FCC crystal is **[BHU 2015]**

(a) orthorhombic (b) simple cubic (c) hexagonal (d) BCC

Soln. The reciprocal lattice of an FCC crystal is BCC.

Hence, correct option is (d).

28. Because of which property of the crystals X-rays can be diffracted from the crystals **[BHU 2015]**

(a) colour of the crystals (b) sharp edges of the crystals

(c) periodic array of atoms (d) random arrangement of atoms

Soln. Due to periodic array of atoms in crystal, the X-rays can be diffracted from the crystals.

Hence, correct option is (c).

29. X-rays can be deflected by **[BHU 2015]**

(a) magnetic field only

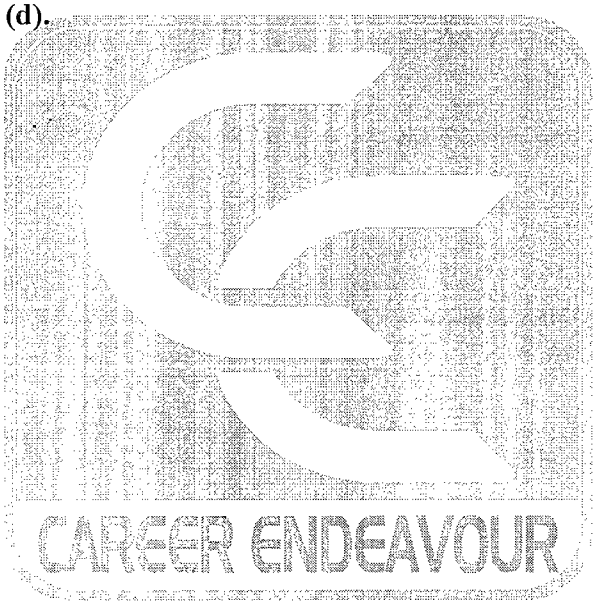
(b) electric field only

(c) simultaneously applying electric and magnetic field

(d) neither by electric nor magnetic field

Soln. X-rays can not be deflected by electric or magnetic field.

Hence, correct option is (d).



PRACTICE SET

(Multiple Choice Questions (MCQ)):

The distance between the adjacent atomic planes in $CaCO_3$ is 0.3 nm. The smallest Bragg scattering for 0.03 nm X-ray is

(a) 2.9° (b) 1.5° (c) 0.29° (d) 5.8°

For a NaCl crystal, the cell-edge $a = 0.563$ nm. The smallest angle at which Bragg reflection can occur corresponds to a set of planes whose indices are

(a) (100) (b) (110) (c) (111) (d) (200)

Silicon has diamond structure with until-cell-edge $a = 0.542$ nm. The interatomic separation is

(a) 0.122 nm (b) 0.234 nm (c) 0.383 nm (d) 0.542 nm

A crystal belongs to a face centered cubic lattice with four atoms in the unit cell. The size of the crystal is 1 cm and unit cell dimension is 1 nm. f is the scattering factor of the atom. The number of atoms in the crystal is

(a) 2×10^{21} (b) 4×10^{21} (c) 2×10^{23} (d) 4×10^{24}

The structure factors for (010) and (200) reflections respectively, in face centered cubic lattice are:

(a) $2f$ and zero (b) zero and $4f$ (c) $2f$ and $2f$ (d) zero and zero

The primitive translation vectors of the body centered cubic lattice are $\vec{a} = \frac{a}{2}(\hat{x} + \hat{y} - \hat{z})$, $\vec{b} = \frac{a}{2}(-\hat{x} + \hat{y} + \hat{z})$ and $\vec{c} = \frac{a}{2}(\hat{x} - \hat{y} + \hat{z})$. The primitive translation vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ of the reciprocal lattice are

(a) $\vec{A} = \frac{2\pi}{a}(\hat{x} - \hat{y})$; $\vec{B} = \frac{2\pi}{a}(\hat{y} + \hat{z})$; $\vec{C} = \frac{2\pi}{a}(\hat{x} + \hat{z})$

(b) $\vec{A} = \frac{2\pi}{a}(\hat{x} - \hat{y})$; $\vec{B} = \frac{2\pi}{a}(\hat{y} - \hat{z})$; $\vec{C} = \frac{2\pi}{a}(\hat{x} + \hat{z})$

(c) $\vec{A} = \frac{2\pi}{a}(\hat{x} + \hat{y})$; $\vec{B} = \frac{2\pi}{a}(\hat{y} + \hat{z})$; $\vec{C} = \frac{2\pi}{a}(\hat{x} - \hat{z})$

(d) $\vec{A} = \frac{2\pi}{a}(\hat{x} + \hat{y})$; $\vec{B} = \frac{2\pi}{a}(\hat{y} + \hat{z})$; $\vec{C} = \frac{2\pi}{a}(\hat{x} + \hat{z})$

The structure factor of a single cell of identical atoms (form factor f) is given by

$$S_{hkl} = f\sum_j \exp\left[-i2\pi\left(x_j h + y_j k + z_j l\right)\right]$$

where (x_j, y_j, z_j) is the coordinate of an atom, and h, k, l are the miller indices. Which one of the following statements is correct for the diffraction peaks of body-centered cubic (bcc) and face-centered cubic (fcc) lattices ?

(a) bcc : (200); (110); (222)
fcc : (111); (311); (400)

(b) bcc : (210); (110); (222)
fcc : (111); (311); (400)

(c) bcc : (200); (110); (222)
fcc : (111); (211); (400)

(d) bcc : (200); (210); (222)
fcc : (111); (211); (400)

8. Consider X-ray diffraction from a crystal with a face centered cubic (fcc) lattice. The lattice plane for which there is no diffraction peak is
(a) (2, 1, 2) (b) (1, 1, 1) (c) (2, 0, 0) (d) (3, 1, 1)

9. A metal with body centered cubic (bcc) structure shows the first (i.e. smallest angle) diffraction peak at a Bragg angle of $\theta = 30°$. The wavelength of X-ray used is 2.1 Å. The volume of the unit cell of the metal is
(a) 26.2 (Å)^3 (b) 13.1 (Å)^3 (c) 9.3 (Å)^3 (d) 4.6 (Å)^3

10. A lattice has the following primitive vectors (in Å), $\vec{a} = 2\left(\hat{j} + \hat{k}\right)$, $\vec{b} = 2\left(\hat{k} + \hat{i}\right)$, $\vec{c} = 2\left(\hat{i} + \hat{j}\right)$. The reciprocal lattice corresponding to the above lattice is
(a) bcc lattice cube edge of $(\pi/2)$ Å^{-1} (b) bcc lattice with cube edge of (2π) Å^{-1}
(c) fcc lattice with cube edge of $(\pi/2)$ Å^{-1} (d) fcc lattice with cube edge of (2π) Å^{-1}

11. The Bragg angle corresponding to the first order reflection from (111) plane in a crystal is 30°. When *X*-ray of wavelength 0.175 nm are used, the interatomic spacing is
(a) 0.6062 nm (b) 0.3031 nm (c) 0.20205 nm (d) 0.1010 nm

12. The volume of primitive unit cell defined by axes $\left(\hat{k} + \hat{i}\right)$ Å, $\left(\hat{i} + \hat{j}\right)$ Å and $\left(\hat{j} + \hat{k}\right)$ Å is
(a) 1 Å^3 (b) 3 Å^3 (c) 2Å^3 (d) 4Å^3

13. If 50 kV is the applied potential in an X-ray tube, then the maximum wavelength of X-rays produced is
(a) 0.2 nm (b) 2 nm (c) 0.2 Å (d) 2 Å

14. The primitive translation vectors of the face centered cubic (fcc) lattice are

$$\vec{a}_1 = \frac{a}{2}\left(\hat{j} + \hat{k}\right),\ \vec{a}_2 = \frac{a}{2}\left(\hat{i} + \hat{k}\right),\ \vec{a}_3 = \frac{a}{2}\left(\hat{i} + \hat{j}\right).$$

The primitive translation vectors of the fcc reciprocal lattice are

(a) $\vec{b}_1 = \frac{2\pi}{a}\left(-\hat{i} + \hat{j} + \hat{k}\right);\ \vec{b}_2 = \frac{2\pi}{a}\left(\hat{i} - \hat{j} + \hat{k}\right);\ \vec{b}_3 = \frac{2\pi}{a}\left(\hat{i} + \hat{j} - \hat{k}\right)$

(b) $\vec{b}_1 = \frac{\pi}{a}\left(-\hat{i} + \hat{j} + \hat{k}\right);\ \vec{b}_2 = \frac{\pi}{a}\left(\hat{i} - \hat{j} + \hat{k}\right);\ \vec{b}_3 = \frac{\pi}{a}\left(\hat{i} + \hat{j} - \hat{k}\right)$

(c) $\vec{b}_1 = \frac{\pi}{2a}\left(-\hat{i} + \hat{j} + \hat{k}\right);\ \vec{b}_2 = \frac{\pi}{2a}\left(\hat{i} - \hat{j} + \hat{k}\right);\ \vec{b}_3 = \frac{\pi}{2a}\left(\hat{i} + \hat{j} - \hat{k}\right)$

(d) $\vec{b}_1 = \frac{3\pi}{a}\left(-\hat{i} + \hat{j} + \hat{k}\right);\ \vec{b}_2 = \frac{3\pi}{a}\left(\hat{i} - \hat{j} + \hat{k}\right);\ \vec{b}_3 = \frac{3\pi}{a}\left(\hat{i} + \hat{j} - \hat{k}\right)$

15. The volume of primitive cell of the fcc reciprocal lattice is
(a) $4\left(\frac{2\pi}{a}\right)^3$ (b) $4\left(\frac{\pi}{a}\right)^3$ (c) $4\left(\frac{\pi}{2a}\right)^3$ (d) $4\left(\frac{3\pi}{a}\right)^3$

(Multiple Select Questions (MSQ)):

The plane(s) that will show allowed reflection in SC, BCC and FCC lattices all are
(a) (200) (b) (222) (c) (111) (d) (110)

The correct statement(s) about the reciprocal lattice is/are
(a) Every reciprocal lattice vector is normal to a set of parallel planes of the crystal lattice.
(b) When no common factor exists among the components of $\vec{G}$ (recirpocal lattice vector), then |G| is inversely proportional to the spacing of the lattice planes normal to $\vec{G}$.
(c) The direct lattice is the reciprocal of its own reciprocal lattice.
(d) The unit cell of the reciprocal lattice need not be a parallelopiped.

The correct statement(s) about the reciprocal lattice is/are
(a) The simple cubic structure is self reciprocal.
(b) The reciprocal lattice for a bcc lattice is an fcc structure.
(c) The reciprocal lattice for an fcc lattice is a bcc structure
(d) The reciprocal lattice for an hcp lattice is a bcc structure

For the X-ray diffraction, the correct statement(s) is/are
(a) Bragg's law is given by $2d\sin\theta = n\lambda$
(b) Bragg's law is given by $2d\sin\theta = (2n+1)\lambda$
(c) For first order reflection $n = 1$
(d) For diffraction, the wavelength of X-ray should be of the order of lattice parameter.

For the X-ray, the correct statement(s) is/are
(a) The intensity of X-ray beam transmitted through a given material decreases exponentially.
(b) The maximum frequency of X-ray corresponding to λ_{min} is $\frac{c}{\lambda_{min}}$.
(c) The short wavelength limit λ_{min} is inversely proportional to the electrode potential.
(d) The diffraction of X-rays is governed by Bragg's law, $2d\sin\theta = n\lambda$.

CAREER ENDEAVOUR

PART-C (Numerical Answer Type (NAT)):

The lattice parameter of copper (fcc) is 3.61 Å. The first order (111) plane appear at an angle of 21.7°. The wavelength of the X-ray (in Å) used is _____

The lattice parameter (in Å) of nickel (fcc) if the Bragg's angle for its (220) reflection is 38.2° and the wavelength of the X-ray used is 1.54 Å, is _____

The glancing angle (in degree) on the plane (110) of a cube of rock salt ($a = 2.81$ Å) corresponding to second order diffraction maximum for the X-rays of wavelength 0.71 Å, is _____

The shortest wavelength (in Å) that is present in the X-rays produced at an accelerating potential of 50 kV is _____

Copper (fcc) has density of 8960 kg/m^3. The unit cell dimension (in Å) is (given the atomic mass of Cu as 63.54 amu) _____

ANSWER KEY						
Multiple Choice Questions (MCQ)						
Questions	1	2	3	4	5	6
Answers	a	c	c	b	b	d
Questions	7	8	9	10	11	12
Answers	a	a	b	a	b	c
Questions	13	14	15			
Answers	c	a	a			
Multiple Select Questions (MSQ)						
Questions	1	2	3	4	5	
Answers	a,b	a,b,c,d	a,b,c	a,c,d	a,b,c,d	
Numerical Answer Type (NAT)						
Questions	1	2	3	4	5	
Answers	1.54	3.54	20.9	0.525	3.61	

CAREER ENDEAVOUR

Chapter 3

MISCELLANEOUS

LATTICE VIBRATIONS

The ability to hold the atoms or ions together is called bonding.

Types of bonding:

The individual atoms of a crystalline solid are held together as one system with different types of bonds existing between them. These bonds are classified into five idealised types.

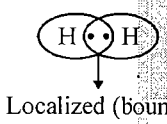
(i) Ionic bonding : Ionic bond is formed by the complete transfer of electron from one atom to another atom.

Example : NaCl. In NaCl, Na has 11 electrons and Cl has 17 electrons. Bond in NaCl crystal is formed by complete transfer of electrons from Na to Cl.

"Ionic bond is formed between electropositive and electronegative atoms". This bond has spherically symmetric charge distribution. Hence, it is not directional in nature.

(ii) Covalent bonding : Covalent bond is formed by sharing of electrons between two atoms

Examples: H_2, diamond, SiC.



In this type of bonding, charge distribution is not spherically symmetric. Charge density is maximum in the direction of bond formation. Hence, this bond is highly directional in nature.

(iii) Metallic bonding : This type of bond is also formed by sharing of electrons but shared electrons pair is not localized. Shared electron pair can move within the solid.

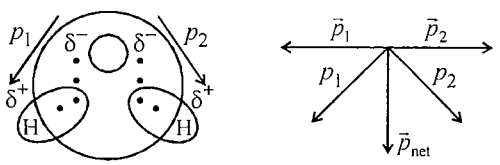
Examples: Cu, Ag, Fe etc.

(iv) Van der Waal bonding : This type of bonding is generally formed between those atoms or molecules which do not have tendency to share or transfer of electrons among them. Van der waal bonding is very weak bonding. This bonding arises due to induced dipole moment. This is slightly directional.

Examples: Inert gases like He, Ne, Ar, solid Argon.

(v) Hydrogen bonding : When a covalent bond is formed between hydrogen atom and highly electronegative atom like oxygen, Cl, etc. then shared electron pair attracted more towards electronegative atoms. This causes the formation of permanent dipole.

Example: H_2O.



Permanent electric dipole moment is the source of H-bonding.

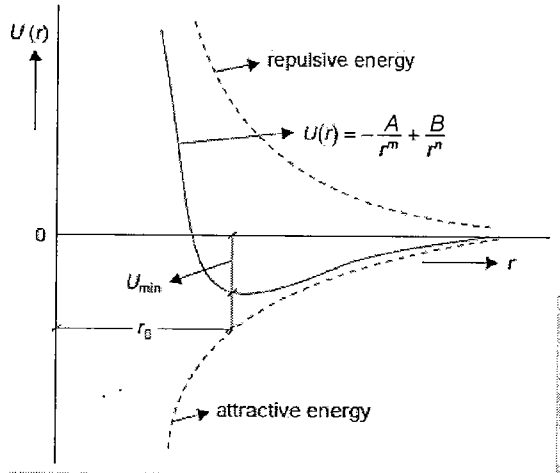
The reason of H-bonding is bond formation between highly electronegative atom and hydrogen. This bond is also directional in nature.

Cohesive Energy of solid :

The cohesive energy of a solid is defined as the energy which will be given out in the process of the formation of a crystal by bringing neutral atoms from infinity to the position of equilibrium separation.

If the potential energy due to attraction U_a is supposed to vary as the m^{th} power of the distance and that due to repulsion U_r as n^{th} power of r, then

$$U_a \propto \frac{1}{r^m} \Rightarrow U_a = -\frac{A}{r^m} \quad \text{and} \quad U_r \propto \frac{1}{r^n} \Rightarrow U_r = \frac{B}{r^n}$$



Therefore, resultant potential or cohesive energy, $U = U_a + U_r = -\frac{A}{r^m} + \frac{B}{r^n}$

where, A and B are constants of proportionality. The cohesive enegy is generally expressed in eV per atom.

The force between the two atoms is given by $F = \frac{-dU}{dr} = \frac{mA}{r^{m+1}} - \frac{nB}{r^{n+1}}$

At $r = r_0$ the attractive and repulsive forces balance each other and $F(r_0) = 0$. In other words at $r = r_0$, we get

$$\frac{mA}{r_0^{m+1}} - \frac{nB}{r_0^{n+1}} = 0 \Rightarrow \frac{m}{n} = \frac{B}{A} \cdot r_0^{m-n} \Rightarrow r_0^{m-n} = \frac{A}{B} \cdot \frac{m}{n}$$

CAREER ENDEAVOUR

Hence, the potential energy at the equilibrium separation r_0 becomes

$$U(r_0) = \frac{-A}{r_0^m} + \frac{B}{r_0^n} = \frac{-A}{r_0^m}\left(1 - \frac{B}{A} r_0^{m-n}\right) = \frac{-A}{r_0^m}\left(1 - \frac{m}{n}\right)$$

Binding or cohesive energy of ionic crystals:

The calculation of binding or cohesive energy of crystals is one of the major problems of the theory of solids and reuire the knowledge of constituents. For ionic crystals the calculation is simple and in further sections we will describe the calculation of binding energy of NaCl crystal.

The calculation is based on the assumption that each ionic crystal is composed of positive and negative ions. Each of the ions have a spherical symmetric distribution as in rare gas atoms. Thus, it can be concluded that the force between the ions depends on the distance between the ions ad is independent of the direction of approach. The force between ions is assumed to be electrostatic in nature. Thus the main contribution to the binding energy arises from electrostatic interaction and is called as Madelung energy.

In ionic crystal there exist two types of interactions, one is long range Coulomb's electrostatic interaction which may be either attractive or repulsive, and the other is short range repulsive interaction.

The coulomb energy of attraction between two ions seperated by a distance r is

$$U_a = -\frac{e^2}{4\pi\varepsilon_0 r} \qquad \ldots (1)$$

In the crystal, each ion interacts with all its neighbouring ions of the same as well as of opposite signs. To take this into account, the net energy of attraction per ion-pair may be written as

$$U_a = \frac{-\alpha e^2}{4\pi\varepsilon_0 r} \qquad \ldots (2)$$

Where, α is the correction factor, called as Madelung constant. For simple crystals α lies between 1.6 and 1.8. For NaCl crystal, $\alpha = 1.7475$ and for CsCl crystal, $\alpha = 1.7626$.

Now, in an ionic crystal as two ions approach each other under coulomb attraction, at a certain distance apart, they begin to repel each other with a force which rise very rapidly with decreae in distance.

$$\therefore \quad U_r = \frac{B}{r^n} \qquad \ldots (3)$$

$$\therefore \quad \text{The total potential energy } U = U_a + U_r = -\alpha\frac{e^2}{4\pi\varepsilon_0 r} + \frac{B}{r^n} \qquad \ldots (4)$$

At equilibrium distance $r = r_0$, U_r will be minimum.

$$\therefore \quad \left.\frac{dU}{dr}\right|_{r=r_0} = 0 = \frac{\alpha^2 e^2}{4\pi\varepsilon_0 r_0^2} - \frac{nB}{r_0^{n+1}}$$

$$\Rightarrow \quad B = \frac{\alpha e^2}{4\pi\varepsilon_0 n} r_0^{n-1}$$

$$\therefore \quad U|_{r=r_0} = -\frac{\alpha e^2}{4\pi\varepsilon_0 r_0} + \frac{\alpha e^2}{4\pi\varepsilon_0 r_0 n} = \frac{-\alpha e^2}{4\pi\varepsilon_0 r_0}\left(1 - \frac{1}{n}\right) \qquad \ldots (5)$$

SOLVED PROBLEMS

The mutual potential energy V of two particles depends on their spatial separation as follows:

$$V = \frac{a}{r^2} - \frac{b}{r};\ a > 0;\ b > 0 \qquad \textbf{[BHU 2012]}$$

For what separation are the particles in static equilibrium ?

(a) $\frac{a}{b}$ (b) $\frac{a}{2b}$ (c) $\frac{a^2}{b}$ (d) $\frac{2a}{b}$

Sln. $V = \frac{a}{r^2} - \frac{b}{r};\ a > 0,\ b > 0$

For equilibrium, $\left.\frac{dV}{dr}\right|_{r=r_e} = 0$

$$\Rightarrow \quad a\left[\frac{-2}{r_e^3}\right] - b\left[\frac{-1}{r_e^2}\right] = 0$$

$$\Rightarrow \quad -\frac{2a}{r_e^3} + \frac{b}{r_e^2} = 0$$

$$\Rightarrow \frac{2a}{r_e^3}=\frac{b}{r_e^2} \Rightarrow r_e=\frac{2a}{b}$$

Hence, correct option is (d).

2. The origin of van der Waal's interaction in molecular crystal is **[BHU 2012]**

(a) nuclear (b) magnetic (c) ionic (d) fluctuating dipolar

Soln. The origin of Van der Waal's interaction in molecular crystal is fluctuating dipolar.

Hence, correct option is (d).

LATTICE VIBRATIONS

Lattice vibration:

A lattice may be regarded as a regular arrangement of atoms in which atoms are joined by elastic massless springs. The motion of any single atom is shared by all the atoms. The lattice can vibrate freely in its normal mode or it may experience forced vibrations. The normal mode vibrations are produced due to its internal energy while forced vibrations produced due to effect of dynamical external forces. The external forces may be of mechanical or electromagnetic type in nature. Normal mode vibrations give informations like thermal properties such as specific heat and thermal conductivity while forced vibrations give informations such as acoustical and optical properties of solids.

Vibrations of one-dimensional monoatomic lattice:

The dispersion relation $(\omega\text{-}k)$ for monoatomic lattice having atoms of mass m is given by

$$\omega=\sqrt{\frac{4C}{m}}\left|\sin\frac{ka}{2}\right|$$

where, C is spring constant and a is lattice parameter.

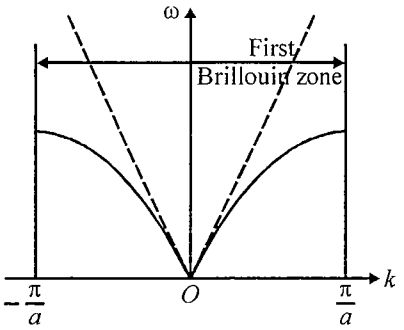
Limiting cases: (a) At lower frequencies or longer wavelength limit i.e. $\lambda\to\infty \Rightarrow k\to 0$,

$$\sin\left(\frac{ka}{2}\right)\to\frac{ka}{2}$$

Thus dispersion relation gives, $\omega=2\sqrt{\frac{C}{m}}\cdot\sin\frac{ka}{2}=2\sqrt{\frac{C}{m}}\cdot\frac{ka}{2}=a\sqrt{\frac{C}{m}}\cdot k$

The phase velocity $(v_p)=\frac{\omega}{k}=a\sqrt{\frac{C}{m}}=v_s$ and the group velocity $(v_g)=\frac{d\omega}{dk}=a\sqrt{\frac{C}{m}}=v_s$

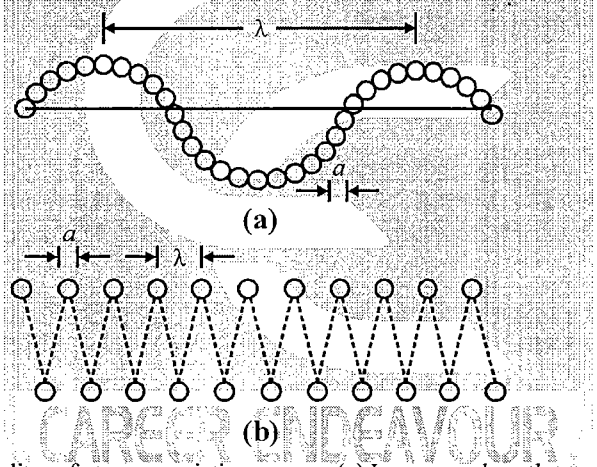
Thus, we can say for monoatomic lattice in the long wavelength limit, phase velocity and group velocity both are same and equal to velocity of sound waves i.e., $v_p=v_g=v_s=a\sqrt{\frac{C}{m}}$



Dispersion relation for a one-dimensional monoatomic lattice

(b) At higher frequencies or smaller wavelength limit, i.e. $\lambda \to 0 \Rightarrow k \to$ higher, thus the phase velocity $\left(v_p\right) = \frac{\omega}{k} = \frac{2}{k}\sqrt{\frac{C}{m}}\sin\frac{ka}{2}$ and group velocity $\left(v_g\right) = a\sqrt{\frac{C}{m}}\cos\frac{ka}{2}$.

Thus, both v_p and v_g are functions of frequency. Thus, for this case, this is called as phenomena of dispersion and the medium is known as dispersive medium. For long wavelength limit, $v_p = v_g = v_s$, i.e. dispersion effects are negligible and medium behaves like a homogeneous continuous medium.



A linear line of atoms consisting a wave (a) Long wavelength case when the motion approaches that of a homogeneous line (b) standing wave

The maximum frequency $\omega = 2\sqrt{\frac{C}{m}}$ will give $k = \frac{\pi}{a}$ or $\lambda = 2a$ and from this, the group velocity $v_g = 0$ and phase velocity $v_p = \frac{2}{k}\sqrt{\frac{C}{m}} = \frac{2a}{\pi}\sqrt{\frac{C}{m}}$.

This confirms that no transfer of signal or energy take place for this frequency limit and the wave behaves like a standing wave.

Vibrations of one-dimensional diatomic lattice:

The dispersion relation $(\omega\text{-}k)$ for monoatomic lattice having atoms of masses m_1 and m_2 is given by

$$\omega^2 = C\left(\frac{1}{m_1}+\frac{1}{m_2}\right) \pm C\sqrt{\left(\frac{1}{m_1}+\frac{1}{m_2}\right)^2 - \frac{4\sin^2 ka}{m_1 m_2}}$$

This is the dispersion relation for a linear diatomic lattice.

In monoatomic lattice, there was only one value of ω for one value of wavevector k. But here in diatomic lattice, for one value of k, we have two different values of ω.

Here, plus (+) sign indicates the frequency corresponding to optical branch and minus (–) sign indicates the acoustical branch.

(I) Optical branch : The dispersion relation for optical branch is

$$\omega_+^2 = C\left(\frac{1}{m_1}+\frac{1}{m_2}\right)+C\sqrt{\left(\frac{1}{m_1}+\frac{1}{m_2}\right)^2-\frac{4\sin^2(ka)}{m_1m_2}}$$

Limiting cases : (1) At lower frequencies, longer wavelength limit i.e., $\lambda \to \infty \Rightarrow k \to 0,\ \sin(ka) \to ka$.

Thus, $$\omega_+^2 = C\left(\frac{1}{m_1}+\frac{1}{m_2}\right)+C\sqrt{\left(\frac{1}{m_1}+\frac{1}{m_2}\right)^2-\frac{4(ka)^2}{m_1m_2}}$$

$$\omega_+^2 = C\left(\frac{1}{m_1}+\frac{1}{m_2}\right)+C\sqrt{\left(\frac{1}{m_1}+\frac{1}{m_2}\right)^2-\frac{4k^2a^2}{m_1m_2}}$$

At $k=0,\ \omega_+^2 = C\left(\frac{1}{m_1}+\frac{1}{m_2}\right)+C\left(\frac{1}{m_1}+\frac{1}{m_2}\right)=2C\left(\frac{1}{m_1}+\frac{1}{m_2}\right) \Rightarrow \omega_+ = \sqrt{2C\left(\frac{1}{m_1}+\frac{1}{m_2}\right)}$

Hence, the value of frequency (ω) is not zero at $k=0$.

(2) At higher frequencies, shorter wavelength limit $\lambda \to 0,\ k \to$ higher, thus $\sin^2(ka) \to 1$

Thus, $\omega_+^2 = C\left(\frac{1}{m_1}+\frac{1}{m_2}\right)+C\sqrt{\left(\frac{1}{m_1}+\frac{1}{m_2}\right)^2-\frac{4}{m_1m_2}} = C\left(\frac{1}{m_1}+\frac{1}{m_2}\right)+C\left(\frac{1}{m_1}-\frac{1}{m_2}\right)=\frac{2C}{m_1}$

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$\Rightarrow \omega_+ = \sqrt{\frac{2C}{m_1}}$

i.e., at $\sin^2(ka)=1 \Rightarrow \sin ka = \sin\frac{\pi}{2} \Rightarrow k=\frac{\pi}{2a}$ (at the boundary of the first Brillouin zone).

The frequency becomes $\omega = 2\sqrt{\frac{C}{m_1}}$.

(II) Acoustical branch : The dispersion relation for acoustical branch is

$$\omega_-^2 = C\left(\frac{1}{m_1}+\frac{1}{m_2}\right)-C\sqrt{\left(\frac{1}{m_1}+\frac{1}{m_2}\right)^2-\frac{4\sin^2 ka}{m_1m_2}}$$

Limiting cases : (1) At low frequencies longer wavelength limit i.e., $\lambda \to \infty \Rightarrow k \to 0,\ \sin(ka) \to ka$.

Thus, $\omega_-^2 = C\left(\frac{1}{m_1}+\frac{1}{m_2}\right) - C\sqrt{\left(\frac{1}{m_1}+\frac{1}{m_2}\right)^2 - \frac{4k^2a^2}{m_1m_2}}$

At $k=0$, $\omega_-^2 = C\left(\frac{1}{m_1}+\frac{1}{m_2}\right) - C\left(\frac{1}{m_1}-\frac{1}{m_2}\right) = 0 \Rightarrow \omega_- = 0$

Hence, the value of frequency (ω) is zero at $k=0$.

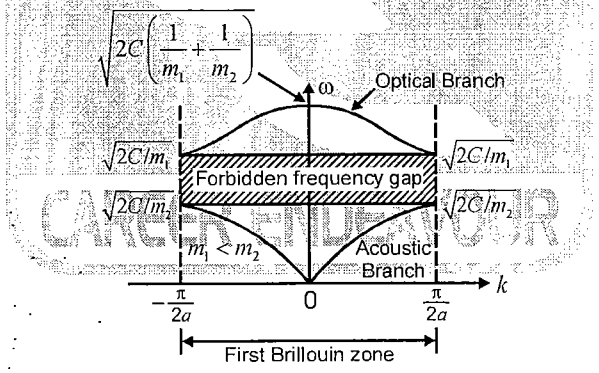
(2) At higher frequencies, shorter wavelength limit i.e., $\lambda \to 0 \Rightarrow k \to$ higher, thus $\sin^2(ka) \to 1$

Thus, $\omega_-^2 = C\left(\frac{1}{m_1}+\frac{1}{m_2}\right) - C\sqrt{\left(\frac{1}{m_1}+\frac{1}{m_2}\right)^2 - \frac{4}{m_1m_2}} = C\left(\frac{1}{m_1}+\frac{1}{m_2}\right) - C\left(\frac{1}{m_1}-\frac{1}{m_2}\right) = \frac{2C}{m_2}$

$\Rightarrow \omega_- = \sqrt{\frac{2C}{m_2}}$

i.e., $\omega_- = \sqrt{\frac{2C}{m_2}}$ for $\sin ka = 1 \Rightarrow \sin ka = \sin\frac{\pi}{2} \Rightarrow k = \frac{\pi}{2a}$

i.e., at the boundary of first Brillouin zone, the frequency of acoustical branch becomes equal to $\sqrt{\frac{2C}{m_2}}$.



Thus, at the zone boundary, the frequency of acoustical branch depend upon the heavier mass (m_2). Therefore, frequency of optical branch and acoustical branch are different at zone boundary. The difference in the frequencies is known as forbidden frequency gap.

Phonons:

Since we know that the energy of a electromagnetic wave is quantized and the quantum of energy is known as photon. In the similar way, the energy of a lattice vibration of an elastic wave is also quantized and the quantum of lattice vibration is called as phonon. All types of lattice vibrations in crytals comprise phonons such as thermal vibrations are thermally excited phonons, sound waves are acoustical phonons and excitations of the branch generate optical phonons.

The energy of phonon is given by $\hbar\omega$, where ω is the angular frequency of a mode of vibration.

If n is the number of phonons in a particular mode of vibration, the total energy of that mode is $n\hbar\omega$, where n can be zero or a positive integer.

SOLVED PROBLEMS

1. Which of the following statements is NOT correct ?
   (a) Acousatic branch of diatomic linear chain is similar to the monoatomic case
   (b) Both group velocity and phase velocity are equal to the velocity of sound in the long wavelength limit
   (c) Under the low wavelength limit, the lattice acts as a low pass filter
   (d) Acoustic and optical modes in diatomic lattice cancel each other at the boundary of first Brillouin zone

Soln. For monoatomic lattice, $\omega = \sqrt{\frac{4c}{m}} \sin\left|\frac{ka}{2}\right|$, where m is mass of atom, c is force constant and k is wave vector.

For long wavelength limit $\lambda \to \infty \Rightarrow k = \frac{2\pi}{\lambda} \to 0$.

Thus, $\omega = \sqrt{\frac{4c}{m}} \sin\left|\frac{ka}{2}\right| = \sqrt{\frac{4c}{m}}\left|\frac{ka}{2}\right|$

Therefore, group velocity $(v_g) = \sqrt{\frac{4c}{m}}\left(\frac{a}{2}\right) = a\sqrt{\frac{c}{m}}$ $\left(\text{From } v_g = \frac{d\omega}{dk}\right)$

Phase velocity $(v_p) = \sqrt{\frac{4c}{m}}\left(\frac{a}{2}\right) = a\sqrt{\frac{c}{m}}$ $\left(\text{From } v_p = \frac{\omega}{k}\right)$

Also sound velocity $(v_s) = a\sqrt{\frac{c}{m}}$

Therefore, for monoatomic lattice, $v_p = v_g = v_s$; for $\lambda \to \infty$ (long wavelength limit)

The maximum frequency for the monoatomic lattice, $\omega_{max} = \sqrt{\frac{4c}{m}} = 2\sqrt{\frac{c}{m}}$

Thus, the monoatomic lattice can pass the frequency lower than $2\sqrt{\frac{c}{m}}$. Higher frequency $(> \omega_{max})$ is not allowed. Thus, the monoatomic lattice acts as a low pass filter. Also, acoustic branch of diatomic linear chain is similar to the monoatomic case.

The option (d) "Acoustic and optical modes in a diatomic lattice cancel each other at the boundary of first Brillouin zone" is incorrect.

Hence, correct option is (d).

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SPECIFIC HEAT OF SOLIDS

The specific heat of solid:

From elementary knowledge, we know that the specific heat is the amount of heat energy that must be supplied to a mole of solid to raise its temperature by one degree. So, when some heat energy is supplied to a solid, its temperature rises and its internal energy is increased. The increase in the internal energy is manifested mainly as (i) an increase in the vibration of atoms about their mean position and (ii) an increase in the kinetic energy of free electrons. The corresponding specific heats are known as the lattice specific heat and the electronic specific heat, respectively. In order to measure the specific heat, either the volume or the pressure must be kept constant. In general, it is measured at constant volume, ie.

$$C_V = \left(\frac{dE}{dT}\right)_V$$

where E is the internal energy and T is the temperature of the system.

The classical model (Dulong-Petit's law):

From the kinetic theory of gases, we know that the kinetic energy of an atom of a monoatomic gas at temperature T is (3/2) $k_B T$, where k_B is the Boltzmann constant. Since an atom has three degrees of (translational) freedom, therefore, the energy (1/2) $k_B T$, is associated with each degree of freedom of the atom. Also, to each translational degree of freedom, there are associated two additional degrees of freedom, because energy can appear as kinetic or potential. If there are N atoms ($=N_A$, Avogadro's number for a monoatomic solid) in a mole of the solid, the total internal energy is

$$3 \times 2 \times N \times \frac{1}{2} k_B T = 3Nk_B T = 3RT \quad \ldots (2)$$

where $Nk_B = R$, the universal gas constant. Therefore, the specific heat at constant volume is

$$C_V = \left(\frac{dE}{dT}\right)_V = 3R = 5.96 \text{ cal} \cdot \text{mol}^{-1} \text{K}^{-1}$$

This is known as Dulong-Petit's law and is found to be generally correct at room temperature and above, for elements of atomic weight greater than 40.

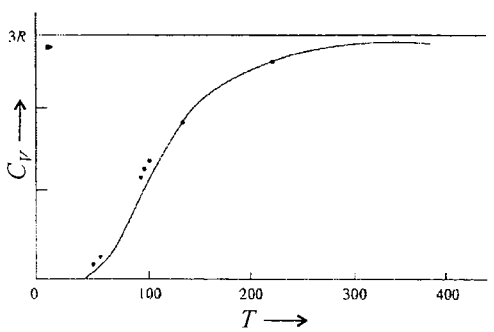
The Einstein model:

Einstein was the first person to resolve the problem of specific heat of solids by the application of Planck's quantum theory. He assumed that a crystalline solid made up of N atoms (per mole) could be regarded as an array of atomic oscillators vibrating in three independent directions (as assumed for the classical calculations) with a constant frequency ν and that the allowed energy states of oscillators are integral multiples by $h\nu$.

The total internal energy of $3N$ harmonic oscillators (N atoms and 3 independent directions) is given by

$$E = \frac{3Nh\nu}{\exp\left(\frac{h\nu}{k_B T}\right) - 1}$$

and the specific heat $C_V = \left(\frac{dE}{dt}\right)_V = 3Nk_B \left(\frac{h\nu}{k_B T}\right)^2 \cdot \frac{\exp\left(\frac{h\nu}{k_B T}\right)}{\left[\exp\left(\frac{h\nu}{k_B T}\right) - 1\right]^2}$



Einstein specific heat curve

The Einstein specific heat curve is fairly close to the experimental curve except at low temperature. In this region, the Einstein specific heat approaches zero more rapidly than the observed values. However, at high temperatures, it approaches the classical value. Let us consider the two temperature ranges separately.

Limiting cases:

(i) High temperature range:

Let us define a quantity $x = h\nu / k_B T$. In this temperature range, $h\nu \ll k_B T$

$$E = 3Nk_B T \text{ and } C_V = \left(\frac{dE}{dT}\right)_V = 3Nk_B = 3R$$

This is same as the classical result.

(ii) Low temperature range:

At low temperatures, $h\nu \gg k_B T$ and hence $e^x \gg 1$.

$$E = 3Nh\nu \exp\left(-\frac{h\nu}{k_B T}\right) \text{ and } C_V = \left(\frac{dE}{dT}\right)_V = 3Nh\nu\left[-\exp\left(-\frac{h\nu}{k_B T}\right)\cdot\frac{h\nu}{k_B T^2}\right]$$

$$= 3Nk_B\left(\frac{h\nu}{k_B T}\right)^2 \exp\left(-\frac{h\nu}{k_B T}\right)$$

This indicates that the exponential term is more important than the term $(h\nu/k_B T)^2$ in determining the variation of C_V with temperature. Thus with decreasing temperature, C_V drops exponentially.

The Debye model:

In 1912, Debye pointed out that the source of error lies in the assumption (that all atomic oscillators vibrate independently at a constant frequency) made by Einstein. According to him, the assumption is not justifiable since all atoms are elastically coupled to its neighbours. Debye therefore, simplified the problem by considering a solid as a continuously vibrating medium (as an elastic continuum), which gives rise to a spectrum of frequencies instead of a single frequency as assumed in the Einstein's model.

The internal energy of the Debye solid is given by

$$E = \frac{9N}{\nu_D^3}\int_0^{\nu_D} \frac{h\nu\nu^2 d\nu}{\exp\left(\frac{h\nu}{k_B T}\right) - 1}$$

Let us suppose that $x = h\nu / k_B T$, so that $d\nu = (k_B T\, dx)/h$. Further, if we suppose that $x_D = (h\nu_D)/k_B T$, above equation becomes

$$E = 9N\left(\frac{k_B T}{h\nu_D}\right)^3 k_B T \int_0^{x_D} \frac{x^3}{e^x - 1} dx$$

Now, we can define the Debye temperature as θ_D according to the equation, $h\nu_D = k_B\theta_D$, then

$$\theta_D = \frac{h\nu_D}{k_B} \text{ and } x_D = \frac{\theta_D}{T}\ ;\quad E = 9Nk_B T\left(\frac{T}{\theta_D}\right)^3 \int_0^{x_D} \frac{x^3}{e^x - 1} dx$$

which does not depend explicitly on the volume and looks similar to the expression obtained for Einstein's model. However, let us examine the dependence of the integral on temperature. For simplicity, we shall consider the following two temperature ranges.

High temperature range:

We assumed earlier that $x = h\nu / k_B T$. In this temperature range, $T >> \theta_D$ and hence $k_B T \gg h\nu$ even when $\nu = \nu_D$, implying that x will be small over the whole range of the integration. So, that

$$\int_0^{x_D} \frac{x^3}{e^x - 1} dx \to \int_0^{x_D} \frac{x^3}{x} dx = \frac{x_D^3}{3} = \frac{1}{3}\left(\frac{\theta_D}{T}\right)^3$$

Therefore, $E = 9Nk_B T \times \frac{1}{3} = 3Nk_B T$

which is the classical result. This implies that in the high temperature range, the specific heat of solids is the same as measured in Dulong and Petit's experiment.

Low temperature range:

In the low temperature range, $T << \theta_D$, so that $k_B T \ll h\nu_D$. In this case, the upper limit of the integration can be replaced by infinity (i.e. $x_D \to \infty$) and hence the above integral becomes

$$\int_0^{x_D} \frac{x^3}{e^x - 1} dx \to \int_0^{\infty} \frac{x^3}{e^x - 1} dx \to \frac{\pi^4}{15}$$

Therefore, $E = 9Nk_B T\left(\frac{T}{\theta_D}\right)^3 \times \frac{\pi^4}{15} = \frac{3}{5}\pi^4 Nk_B T\left(\frac{T}{\theta_D}\right)^3$

and $C_V = \frac{12}{5}\pi^4 Nk_B\left(\frac{T}{\theta_D}\right)^3$

This is a famous expression known as Debye T^3 law for specific heat at low temperature.

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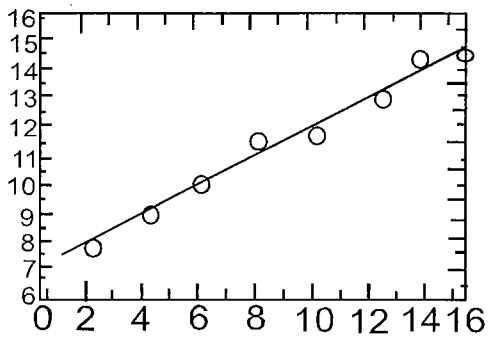
SOLVED PROBLEMS

1. The ratio of the specific heat capacity and temperature, C_v / T, of Cu is plotted as a function of T^2, the square of the absolute temperature, in the graph below: **[JNU 2012]**



The values of γ and β (the coefficients corresponding to the electronic and the vibrational components of the specific heat) are, approximately

(a) $\gamma = 7.0\times10^{-4}\,\text{Jmol}^{-1}\text{K}^{-2}$ and $\beta = 5.0\times10^{-5}\,\text{Jmol}^{-1}\text{K}^{-4}$

(b) $\gamma = 5.0\times10^{-5}\,\text{J mol}^{-1}\,\text{K}^{-2}$ and $\beta = 7.0\times10^{-4}\,\text{J mol}^{-1}\text{K}^{-4}$

(c) $\gamma = 1.4\times10^{-3}\,\text{J mol}^{-1}\,\text{K}^{-2}$ and $\beta = 7.0\times10^{-4}\,\text{J mol}^{-1}\text{K}^{-4}$

(d) $\gamma = 5.0\times10^{-4}\,\text{J mol}^{-1}\,\text{K}^{-2}$ and $\beta = 7.0\times10^{-5}\,\text{J mol}^{-1}\text{K}^{-4}$

Soln. Specific heat varies as, $\frac{C}{T} = \gamma + \beta T^2$

$\Rightarrow$ γ = Intercept = $7 \times 10^{-4}\,\text{J mol}^{-1}\,\text{k}^{-2}$ & β = slope = $\frac{(9-8)\times10^{-4}}{4-2} = 5 \times 10^{-5}\,\text{J mol}^{-1}\,\text{k}^{-4}$

Correct option is (a)

2. The Dulong-Petit law fails near room temperature (300 K) for many light elements (such as boron and beryllium) because their Debye temperature is

(a) >> 300 K (b) ~ 300 K (c) << 300 K (d) 0 K **[JEST-2012]**

Soln. The Dulong-Petit law fails near room temperature (300 K) for many light elements (such as boron and beryllium) because their Debye temperature is larger than 300 K.

Correct option is (a)

3. What is the contribution of the condition electrons in the molar entropy of a metal with electronic coefficient of specific heat ?

(a) γT (b) γT^2 (c) γT^3 (d) γT^4 **[JEST-2014]**

Soln. The contribution of the condition electrons in the molar entropy of a metal with electronic coefficient of specific heat is γT.

Correct option is (a)

4. Debye temperatures of copper and gold are 340K and 170K, respectively. At room temperature, if the specific heat for copper is 1.5 $\text{kJmol}^{-1}\text{K}^{-1}$, the specific heat for gold is about: **[DU 2014]**

(a) 0.19 $\text{kJmol}^{-1}\text{K}^{-1}$ (b) 1.2 $\text{kJmol}^{-1}\text{K}^{-1}$ (c) 1.9 $\text{kJmol}^{-1}\text{K}^{-1}$ (d) 12 $\text{kJmol}^{-1}\text{K}^{-1}$

Soln. Debye temperature for copper $(\theta_D)_{Cu} = 340\ \text{K}$

Debye temperature for gold $(\theta_D)_{Au} = 170\ \text{K}$

$(C_V)_{Cu} = 1.5\ \text{kJ-mol}^{-1}\text{K}^{-1}$

$(C_V)_{Au} = ?$

Since, $C_V \propto \left(\frac{T}{\theta_D}\right)^3 \Rightarrow \frac{(C_V)_{Cu}}{(C_V)_{Au}} = \left[\frac{(\theta_D)_{Au}}{(\theta_D)_{Cu}}\right]^3 = \left(\frac{170}{340}\right)^3 = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$

$\Rightarrow (C_V)_{Au} = 8 \times (C_V)_{Cu} = 8 \times 1.5 = 12$ kJ-mol^{-1}K^{-1}

Hence, correct option is (d).

The Debye temperature of a metal is 450 K. Its Debye frequency in Hz is of the order of

(a) 10^{11} (b) 10^{13} (c) 10^{16} (d) 10^{20}

Soln. Debye temperature of metal $\theta_D = 450$K **[DU 2015]**

Since, $\theta_D = \frac{h\nu_D}{k_B} \Rightarrow \nu_D = \frac{\theta_D \times k_B}{h} = \frac{450 \times 1.38 \times 10^{-23}}{6.6 \times 10^{-34}} = 94.09 \times 10^{11}$ Hz $= 0.94 \times 10^{13}$ Hz.

Thus, the Debye frequency in Hz is of the order of 10^{13}.

Hence, correct option is (b).

According to Dulong-Petit's law, the specific heat of a solid **[DU 2016]**

(a) is proportional to the temperature (b) does not depend on temperature

(c) depends on square of temperature (d) is inversely proportional to temperature

Soln. According to Dulong-Petit's law, the specific heat of a solid

$C_V = 3R$ ... (i)

Thus, the specific heat of a solid does not depend on temperature.

Hence, correct option is (b).

Assume that the heat capacity at constant volume of a metal varies as $aT + bT^3$ at low temperatures. The temperature dependence of entropy is given by **[BHU 2012]**

(a) $aT + bT^3$ (b) $a + bT^2$ (c) $aT + \frac{b}{3}T^3$ (d) $aT^2 + bT^4$

Soln. Since $C_V = aT + bT^3$

$\Rightarrow \left(\frac{dQ}{dT}\right)_V = aT + bT^3$

$\Rightarrow T\frac{dS}{dT} = aT + bT^3$

$\Rightarrow dS = \left(\frac{aT + bT^3}{T}\right)dt = \left(a + bT^3\right)dT$

Integrating on both sides, we get $\Delta S = aT + \frac{bT^3}{3}$.

Hence, correct option is (c).

The specific heat C_V due to free electrons in metals varies as **[BHU 2013]**

(a) $C_V \propto T$ (b) $C_V \propto T^2$ (c) C_V is constant (d) $C_V \propto T^{-1}$

Soln. The specific heat C_V due to free electrons in a metal varies as $C_V \propto T$.

Hence, correct option is (a).

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9. According to Debye model heat capacity $[C_V]_{\text{lattice}}$ at low temperature varies as proportional to

(a) T (b) T^3 (c) T^{-1} (d) T^2 **[BHU 2013]**

Soln. According to Debye model heat capacity $[C_V]_{\text{lattice}}$ at low temperature is given by $[C_V]_{\text{lattice}} \propto T^3$.

Hence, correct option is (b).

10. The electronic specific heats in metals are given by : (R is gas constant, k-boltzmann constant) :

(a) $\frac{kT}{E_0}$ (b) $\frac{R}{E_0}kT$ (c) $\frac{E_0}{R}kT$ (d) $\frac{3E_0}{R}kT$ **[BHU 2014]**

Soln. The electric specific heats in metal are given by $C_e = \frac{R}{Eo}kT$

Hence, correct option is (b).

11. The lattice specific heat at constant volume C_V of a solid at lower temperature depends on temperature T as

(a) $C_V \propto T$ (b) $C_V \propto T^3$ (c) $C_V \propto T^2$ (d) $C_V \propto \frac{1}{T}$ **[BHU 2015]**

Soln. According to Debye theory, the lattice specific heat, $C_V \propto T^3$ (at low temperature).

Hence, correct option is (b).

MAGNETIC PROPERTIES OF SOLIDS

Response of substance of magnetic field:

A magnetic field can be described by either magnetic induction B (or magnetic flux density) or the field strength H. In vacuum, they are related by the equation

$$B = \mu_0 H \qquad \ldots (1)$$

where $\mu_0 = 4\pi \times 10^{-7}\ H - m^{-1}$ and is called the permeability of the free space (vacuum).

When a substance is placed in a magnetic field, it gets magnetized and hence a magnetization M(defined as the magnetic moment per unit volume, i.e. $M = \mu_m / \Delta V$) is produced in it. The magnetic induction inside the substance is given by

$$B = \mu_0 H + \mu_0 M = \mu_0 (H + M) \qquad \ldots (2)$$

where the first term on the right side of equation (2) is due to external field and the second term is due to the magnetization. For an isotropic medium, M and H are parallel vectors and are related to each other according to the relation

$$M = \chi H \qquad \ldots (3)$$

where χ is the susceptibility of the medium and is a scalar quantity. Substituting the value of M from equation (3) into equation (2), we have

$$B = \mu_0 (1 + \chi) H = \mu H \qquad \ldots (4)$$

where $$\mu = \mu_0 (1 + \chi) \qquad \ldots (5)$$

is known as the permeability of the medium. It is often more convenient to use the relative permeability μ_r, which is defined as

$$\mu_r = \frac{\mu}{\mu_0} = (1 + \chi) \qquad \ldots (6)$$

Magnetic susceptibility is defined as $\chi = \frac{M}{H}$.

Classification of magnetic materials:

Magnetic materials can be classified into different categories according to their χ values and the way in which these vary with the magnetic field strength and temperature. They are: (i) Diamagnetic, (ii) Paramagnetic, (iii) Ferromagnetic, (iv) Antiferromagnetic and (v) Ferrimagnetic.

Diamagnetic Materials:

The Diamagnetic property is the result of an induced magnetic moment. This arises when an atom is placed in a magnetic field. The motion of orbital electrons of the atom (analogous to a current flowing in a circuit) gets modified in such a way that weak magnetic moment opposing the field is induced. Also, if χ is negative in equation (3), the direction of magneticzation M is opposite to that of the field H. If M and H behave linearly

then $\chi = \frac{M}{H}$ (for diamagnetic material)

A diamagnetic solid has a tendency to repel the magnetic lines of force due to an external applied field. A superconductor which repels all the lines of force, is an example of a perfect diamagnet.

Diamagnetic substances include inert gases (helium, argon, etc), metals (bismuth, copper, zinc, gold, silver etc.), mercury, water, glass, marble and many other organic compounds.

Diamagnetic effect arises due to change in the orbital motion of electron around the nucleus on the application of external magnetic field. These materials repel in the magnetic field. It is a very weak effect.

Thus, equation (2) $\Rightarrow \mu H = \mu_0 (H + \chi_m H)$

$\Rightarrow \frac{\mu}{\mu_0} = 1 + \chi_m \Rightarrow \mu_r = 1 + \chi_m$

Hence, $\vec{B} = \mu_0 \mu_r \vec{H}$

For diamagnetic material, $\mu_r < 1$ and $\chi_m < 0$ i.e. negative susceptibility.

Therefore, susceptibility of diamagnetic material is negative. Diamagnetic susceptibility of a solid according to quantum mechanics is given by

$$\chi_{dia} = \frac{-\mu_0 NZ}{6m_e} e^2 < r^2 >$$

CAREER ENDEAVOUR

where, N is number of atoms per unit volume, Z is atomic number, m_e is mass of the electron and $< r^2 >$ is average value of r^2 over the particular orbit.

From above equation diamagnetic susceptibility is negative and independent of temperature.

Paramagnetic Materials:

Paramagnetism occur in those solids which have permanent magnetic moment due to spins of unpaired electrons on the application of magnetic field. These magnetic moment aligned in the direction of applied magnetic field. Hence effect is attractive type. This is strong effect as compare to diamagnetic effect.

Molecular oxygen (O_2) has even number of electrons but shows paramagnetic effect (exception).

Paramagnetic suceptability of ions or atoms is given by $\chi_{ions} = \frac{C}{T}$. This is known as Curie's law.

where, C is Curie constant i.e. $\chi_{ions} \propto \frac{1}{T} \Rightarrow \boxed{\chi \propto \frac{1}{T}}$

Ferromagnetic Material :

Like paramagnetism, ferromagnetism is also associated with permanent magnetic moment but ferromagnetic materials are characterize by spontaneous magnetisation (i.e. Non-zero magnetisation even in the absence of magnetic field).

Explanation of ferromagnetism can be given in terms of formation of small regions inside the solid in which magnetisation is saturated. These small regions are called domains.

Suceptability of ferromagnetic material is given by $\chi = \frac{C}{(T - T_C)}$, where T_C is ferromagnetic curie temperature

The Weiss molecular (Exchange) field:

In order to explain the spontaneous magnetization of a ferromagnetic material, Weiss (1907) assumed that there exists an internal molecular field (also called exchange field H_{ex}) acting on a given dipole and is given by

$$H_{ex} = H + \lambda M \qquad \ldots (i)$$

where λ is called Weiss constant (it is independent of temperature) and H is applied field. In a ferromagnet the Curie law holds for exchange field as well. Therefore,

$$\frac{M}{H_{ex}} = \frac{C}{T} \qquad \ldots (ii)$$

Simplifying the equation (i) and (ii), we have

$$M = \frac{CH}{T - C\lambda} \qquad \ldots (iii)$$

Therefore, the ferromagnetic susceptibility becomes,

$$\chi = \frac{M}{H} = \frac{C}{T - C\lambda} = \frac{C}{T - T_C} \qquad \ldots (iv)$$

where $T_C = C\lambda$ is called the Curie temperature and the equation (iv) is known as Curie-Weiss law. The coresponding variation of suscepbitibility with temperature is shown in figure. As long as $T \geq T_C$, this curve and the curve shown in figure are similar. However, as $T \to T_C$. Curive Weiss susceptibility tends to infinity and for $T \leq T_C$ there exists a spontaneous magnetization in the material even in the absence of an external field.

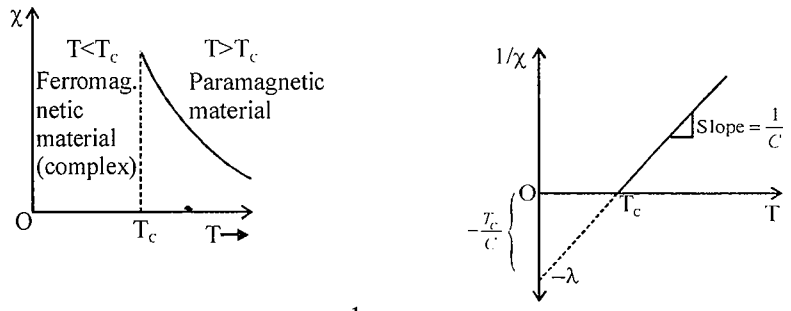


Figure : χ vs T and $\frac{1}{\chi}$ vs T for ferromagnetic materials

SOLVED PROBLEMS

A paramagnetic gas at room temperature is placed in an external magnetic field of 1.5 T (tesla). Each atom of the gas has a magnetic moment $\mu = 1.0\,\mu_B$, where $\mu_B = 9.3\times10^{-24}$ J/T is the Bohr magneton. The difference in energy when an atom is aligned along the magnetic field and opposite to it, is **[JNU 2012]**

(a) 2.8×10^{-23} J (b) 1.4×10^{-23} J (c) 18.6×10^{-24} J (d) 9.3×10^{-24} J

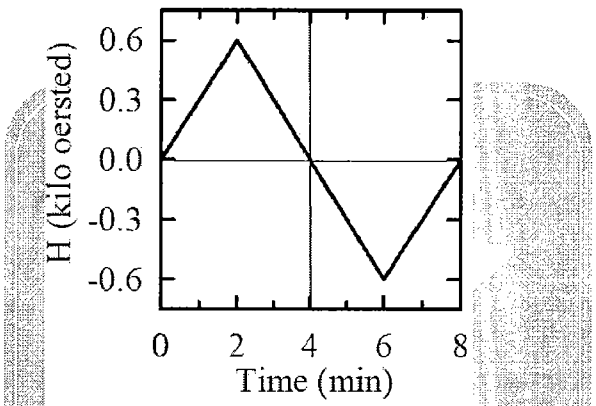
Sln. When atom is along the magnetic field, $E_1 = -\mu B$

when atom is opposite to the magnetic field, $E_2 = +\mu B$

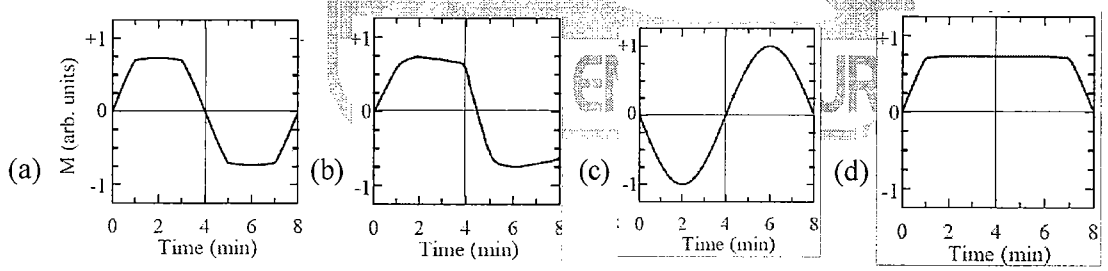
$\therefore \quad \Delta E = E_2 - E_1 = 2\mu B = 2 \times 1.0 \times 9.3 \times 10^{-24} \times 1.5 = 2.8 \times 10^{-23}$ J.

Correct option is (a)

An unmagnetised sample of iron is placed in a magnetic field H which varies with time as shown in the plot below. **[TIFR 2011]**



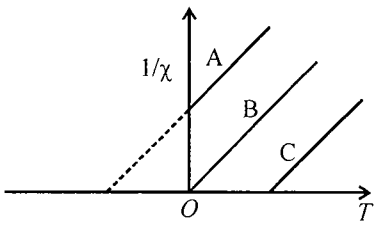
The magnetisation M of this iron sample is continuously measured and also plotted as a function of time. The appearance of this plot will be closest to



Sln. The magnetisation versus time graph is closest to option (b).

Correct option is (b)

The magnetic susceptibility χ of three samples A, B and C is measured as a function of their absolute temperature T, leading to the graphs shown below.



From these graphs, the magnetic nature of the samples can be inferred to be

(a) A : anti-ferromagnet ; B : paramagnet ; C : ferromagnet

(b) A : diamagnet ; B : paramagnet ; C : anti-ferromagnet

(c) A : paramagnet ; B : anti-ferromagnet ; C : ferromagnet

(d) A : anti-ferromagnet ; B : diamagnet ; C : paramagnet **[TIFR-2013]**

Soln. The susceptibility of paramagnet is given by $\chi = \frac{C}{T} \Rightarrow \frac{1}{\chi} = \frac{T}{C}$ ... (i)

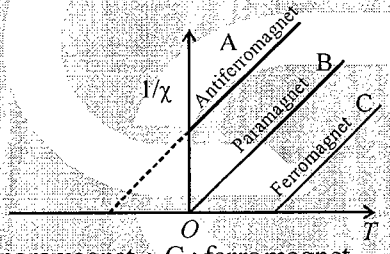
where, C is Curie constant.

The susceptibility of ferromagnet is given by $\chi = \frac{C}{(T-\theta)} \Rightarrow \frac{1}{\chi} = \left(\frac{T-\theta}{C}\right)$ ... (ii)

where, θ is Curie temperature and susceptibility of antiferromagnet is given by $\chi = \frac{C}{(T+T_N)}$, where T_N is Neel temperature.

$\Rightarrow \frac{1}{\chi} = \left(\frac{T+T_N}{C}\right)$

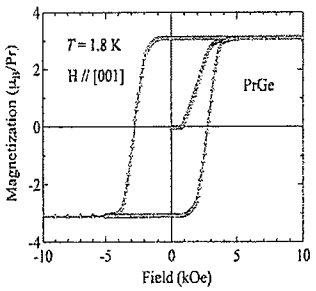
Hence plot of $\frac{1}{\chi}$ versus T for paramagnet ; ferromagnet and antiferromagnet is given by



Thus, A : anti-ferromagnet ; B : paramagnet ; C : ferromagnet

Hence, correct option is (a).

4. The Curie temperature of a single crystal of Pr Ge is known to be 41 K. The magnetization data of this sample is measured at 1.8 K for the magnetic field applied parallel to the (001) direction is shown in the figure to the left. At a temperature of 38 K, the hysteresis loop in the figure will



(a) have the same width (b) decrease in width

(c) increase in width (d) shrink to a line **[TIFR-2013]**

Soln. The magnetization data of the sample at $T = 1.8$ K shows a M-H loop of ferromagnetic type. After Curie temperature $T = 41$ K, this will become paramagnetic type,

At $T = 38$ K, the width of hysteresis loop decreases and above $T = 41$ K, this will shrink to a line

Hence, correct option is (c).

For a paramagnetic material, the energy difference between spin magnetic dipole parallel and anti-parallel to an external field H is (μB is the Bohr magnetron) **[DU 2016]**

(a) $\mu_0 H / 4\pi$ (b) $\mu_B H$ (c) $2\mu_B \mu_0 H$ (d) $\mu_B \mu_0 H$

Soln. For parallel alignment of spin magnetic dipole indirection external field H, the energy $= -\mu_0 \mu_B H \cos 0°$

$E_{\text{parallel}} = -\mu_0 \mu_B H$ ... (i)

For antiparallel alignment of spin magnetic dipole in the direction of external field H, the energy $= -\mu_0 \mu_B H \cos 180°$

$E_{\text{antiparallel}} = -\mu_0 \mu_B H(-1) = \mu_0 \mu_B H$ ... (ii)

From equations (i) and (ii), the energy difference,

$\Delta E = E_{\text{antiparallel}} - E_{\text{parallel}} = \mu_0 \mu_B H - (-\mu_0 \mu_B H) = 2\mu_0 \mu_B H.$

Correct option is (c).

The susceptibility of a diamagnetic material is about **[DU 2015]**

(a) -10^{-5} (b) 10^{-7} (c) 10^{2} (d) 10^{3}

Soln. The magnetic susceptibility of the diamagnetic is always negative. Out of these options -10^{-5} will be magnetic susceptibility.

Hence, correct option is (a).

DIELECTRIC PROPERTIES OF SOLIDS

Dielectrics : Dielectrics are perfect insulators. In dielectrics electrons are very tightly bound to the atoms so that at ordinary temperatures they do not conduct any electric current.

Examples: Solids: glass, porcelin; gases: H_2, N_2; liquids: CH_3Cl, pure water

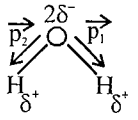
They are of two types.

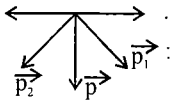
(i) Polar dielectrics

(ii) Non-polar dielectrics

Polar dielectrics are those dielectrics whose molecules contains (are characterized by) the permanent dipole moments. Ex - H_2O, NaCl, KCl.

Non-polar dielectrics are those dielectrics whose molecules do not contain the permanent dipole moments. Ex - Solid Ar.





H_2O: Electronegativity of O is higher than H so it will have more negative charge, so it contain permanent dipole moment. It is the inherent property of molecules.

If we apply external electric field on non-polar dielectrics then it will be polarized and this polarization is called induced polarisation or induced dipole momentum.

Thus, $\vec{p}_{in} \propto \vec{E}$

$$\Rightarrow \boxed{\vec{p}_{in} = \alpha \vec{E}_{loc}} \quad \text{... (1)}$$

where, α is proportionality constant and known as polarizability of atom or molecule and $\vec{E}_{loc}$ is local electric field.

Dipole moment : If we have two equal and opposite charges $+q$ and $-q$, separated by a distance r, then the dipole moment is defined as $p = qr$. The dipole moment is directed from the negative to positive charge.

Polarization: The polarization $(\vec{P})$ is defined as the dipole moment per unit volume i.e., $\vec{P} = \frac{\vec{p}}{\Delta V}$

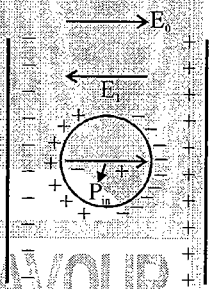
$$\vec{P} = \chi_e \in_0 \vec{E}_{loc} \text{ also } \vec{p}_{in} = \alpha \vec{E}_{loc} \text{ and } \vec{P} = N\vec{p}_{in} = N\alpha\vec{E}_{loc} \quad \ldots (2)$$

where χ_e is electric susceptibility and N is number of dipoles per unit volume.

$$\therefore \quad \chi_e = \frac{N\alpha}{\in_0}$$

Since, $\in_r = 1 + \chi_e \quad \therefore \in_r = 1 + \frac{N\alpha}{\in_0}$

For non-polar dielectrics: If we have a large parallel plate capacitor and we fill a non-polar dielectric between its plates. Suppose external applied field is $\vec{E}_0$. Due to this $\vec{E}_0$, non-polar dielectrics will be polarized then plates occur some bound charge and electric field develop is $\vec{E}_1$ which is less than $\vec{E}_0$.



Thus, resultant electric field, $\vec{E} = \underbrace{\vec{E}_0 + \vec{E}_1}_{\vec{E}_{mac}}$

Macroscopic electric field is average electric field seen between the plates.

If there is a molecule between plates then it will behave as dipole and resultant field on this molecule is different from $\vec{E}_{mac}$ and this electric field is called local field.

$$\vec{E}_{loc} = \vec{E}_0 + \vec{E}_1 + \vec{E}_2 + \vec{E}_3$$

where $\vec{E}_0$ is externally applied electric field, $\vec{E}_1$ is depolarization field, $(\vec{E}_0 + \vec{E}_1)$ is macroscopic field, $\vec{E}_2$ is electric field due to the induce charges on the surface of the cavity (polarization field).

and $\vec{E}_3$ is electric field due to all the charges inside the cavity. Resultant of all these electric fields is the local field. Take a spherical cavity out from the surrounding of molecules. This cavity is called Lorentz cavity. If the dipole is at the centre of the cavity then charges on the cavity will be symmetric about centre.

So, $\boxed{\vec{E}_3 = 0}$ (due to symmetry)

Now, $\vec{E}_{loc} = \vec{E}_0 + \vec{E}_1 + \vec{E}_2$

$\Rightarrow \quad \vec{E}_{loc} = \vec{E}_{mac} + \vec{E}_2 \qquad \left(\because \text{since } E_2 = \frac{\vec{P}}{3\varepsilon_0}\right)$

$\Rightarrow \quad \boxed{\vec{E}_{loc} = \vec{E}_{mac} + \frac{\vec{P}}{3\varepsilon_0}}$

If solid contains N atoms (or molecules) per unit volume, each atoms due to the application of electric field will become dipole, so there will be N dipoles.

$\vec{P} = N\vec{p}_{in}$

$\Rightarrow \boxed{\vec{P} = N\alpha\vec{E}_{loc}}$ ... (3)

From equation (2) and equation (3), we get

$N\alpha\vec{E}_{loc} = \varepsilon_0\chi_e\vec{E}_{mac}$

$\Rightarrow N\alpha\vec{E}_{loc} = \varepsilon_0(\varepsilon_r - 1)\vec{E}_{mac} \qquad (\text{since } \chi_e = \varepsilon_r - 1)$

$\Rightarrow \frac{\vec{E}_{loc}}{\vec{E}_{mac}} = \frac{\varepsilon_0(\varepsilon_r - 1)}{N\alpha}$

$\Rightarrow \boxed{\frac{(\varepsilon_r - 1)}{(\varepsilon_r + 2)} = \frac{N\alpha}{3\varepsilon_0}}$ where, α is electric polarizability and ε_r is dielectric constant.

This relation is known as Classius-Mossotie relation.

This is valid only for non-polar molecules.

Physical significance of Classius-Mossotie relation: Classius-Mossotie relation is a relationships between α and ε_r. α is a microscopic quantity and ε_r is macroscopic quantity (dielectric constant) because α corresponds to dipole moment and ε_r corresponds to polarisation.

For polar molecules, this equation becomes Langvein - Debye equation.

Types of polarisabilities: There are three types of polarisabilities

(i) Electronic polarisability (α_e)

(ii) Ionic polarisability (α_i)

(iii) Dipolar polarisability (α_d)

Thus total polarisability α is sum of electronic, ionic and dipolar polarisabilities.

i.e., $\alpha = \underbrace{\alpha_e + \alpha_i}_{\text{induced}} + \underbrace{\alpha_d}_{\text{Permanent}}$

(i) Electronic polarisability (α_e)**:** This type of polarisability arises due to displacement of electron cloud of an atom relative to its nucleus in the presence of applied electric field.

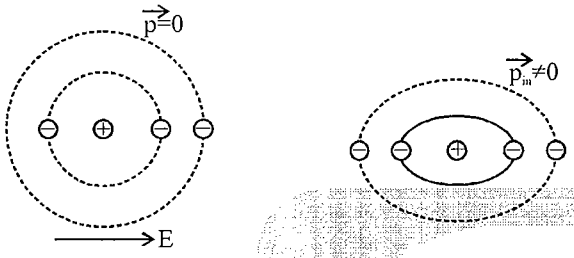
The Electronic polarisability is given by $\alpha_e = 4\pi\varepsilon_0 R^3$, where R is radius of the atom.

This type of polarisability is independent of temperature but depends on the frequency of applied electric field. The frequency dependence of α_e is given by

$$\boxed{\alpha_e = \frac{e^2}{m(\omega_0^2 - \omega^2)}}$$

where m is mass of the electron, e is electronic charge, ω_0 is natural frequency of oscillation of atom and ω is frequency of applied electric field.

Unit of α: In C.G.S. units, α has unit of volume (cm^3), and in M.K.S. units, α is measured in $F - m^2$.



(ii) Ionic polarisability (α_i) :

This type of polarisability arises due to increase or decrease in the bond-length of ions.

This type of polarisability is independent of temperature but depends on the frequency of applied field. The ionic polarisability depends on frequency in following manner. For an ionic solid, the polarisability is given by

$$\boxed{\alpha_i = \frac{e}{\omega_0^2}\left(\frac{1}{m} + \frac{1}{M}\right)}$$

where m is mass of one ion (say Na^+),

M is mass of another ion (say Cl^-),

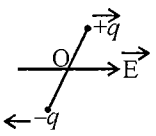
and ω_0 is natural frequency of the vibration of molecule (NaCl).

In general, $\boxed{\alpha_i \simeq \frac{\alpha_e}{10}}$.

(iii) Dipolar polarisability (α_d) :

This type of polarisability arrises due to the permanent dipole moment of the molecule. The dipolar polarisability is given by

$$\boxed{\alpha_d = \frac{p_p^2}{3k_BT}}$$



where, p_p is permanent dipole moment of the molecule, T is temperature and k_B is Boltzmann constant.

From above equation it is clear that α_d depends on both temperature and frequency of the field.

Hence total polarisability,

$\alpha = \alpha_e + \alpha_i + \alpha_{or}$

$$\Rightarrow \boxed{\alpha = \alpha_0 + \frac{p_p^2}{3k_B T}}$$

This relation is known as Langevin - Debye equation. This is valid for only polar molecules.

SOLVED PROBLEMS

Dielectric polarizability is a proportionality constant in the relation between:

(a) Electric dipole moment and local electric field (b) Displacement vector and local electric field

(c) Electric dipole moment and charge density (d) Displacement vector and charge density

Sln. Electric dipole $(\vec{p}) \propto$ local electric field $(\vec{E}_{local})$ **[DU 2014]**

$\Rightarrow \vec{p} = \alpha \vec{E}_{local}$, where α is dielectric polarizability.

Hence, correct option is (a).

What are example of piezo electric materials ? **[BHU 2014]**

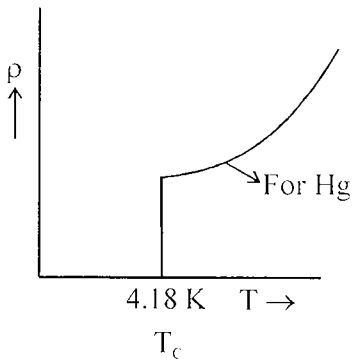
(a) Rochelle salt (b) lead zirconate (c) potassium niobate (d) barium titanate

Sln. Out of the given options, Rochelle salt is the example of piezoelectric materials.

Hence, correct option is (a).

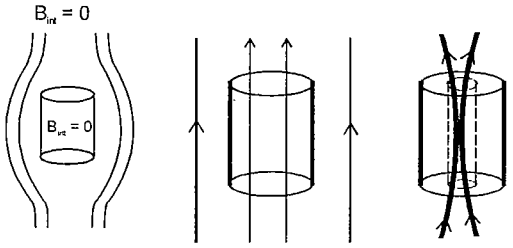
SUPERCONDUCTIVITY

In 1911, Kammerlingh Onnes discovered that the electrical resistivity of Hg (mercury) at low temperature $\simeq 4.18$ K, vanishes (zero) suddenly. This phenomenon was given the name superconductivity. The temperature at which the resistivity disappear (in zero magnetic field) is called the transition temperature or critical temperature (T_c). The transition temperature of mercury is 4.18 K. materials which shows this phenomenon are called superconducters.



Meissner's effect:

This effect is useful to make difference between superconductor and perfect (normal) conductor. When a superconductor and a perfect conductor are placed in an external magnetic field, the magnetic line of the forces do not passes through the superconductor and they are swept away. Thus making the magnetic field (B) inside the superconductor equal to zero. While line of magnetic force passes through the perfect conductor and therefore B is not equal to zero inside the perfect conductor.



As in case of superconductor, magnetic field $B = 0$

In C.G. S. units:

$$\vec{B} = \vec{H} + 4\pi M \Rightarrow \quad \frac{M}{H} = -\frac{1}{4\pi} \Rightarrow \boxed{\chi = -\frac{1}{4\pi}}$$

In M.K.S. units:

$$\Rightarrow \quad \mu_0 (H + M) = 0$$

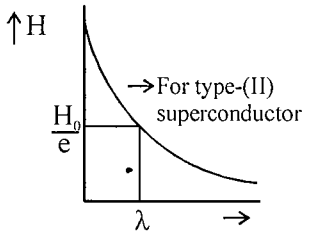
$$\Rightarrow \quad H + M = 0 \quad \frac{M}{H} = -1 \Rightarrow \boxed{\chi = -1}$$

where χ is magnetic succeptibility, H is magnetic field intensity and M is magnetization.
Thus the succeptibility of a super conductor is negative which implies that a superconductor exhibits the properties of perfect diamagnetic susbtances.

Isotope effect:
It has been found that the critical temperature (T_C) of superconductor is found to be proportional to the negative half power of isotopic mass of the element i.e.

$$T_C \propto M^{-1/2} \Rightarrow \boxed{M^{1/2} T_C = \text{constant}}$$

Persistent current:
In the super conductors current contineously flows as long as the material is kept below the critical temperature (T_C). Since the current is very high and the decay formula is $i = i_0 e^{-(R/L)t}$. So once started it continue to flow indenfinitely as long as the temperature and magnetic field remain unchanged. This is called the persistent current.
The magnetic field and surface current penetrate in a superconductor to a depth of 10–100 nanaometer.



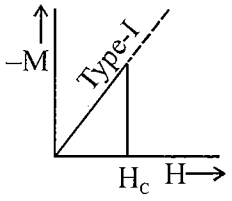
$$H = H_0 e^{-x/\lambda}$$

Similarly, $J_s = J_0 e^{-x/\lambda}$

λ = penetration depth

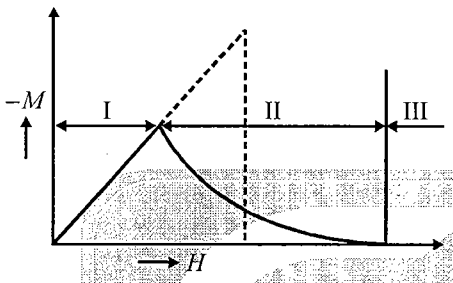
Type-I and type-II supercondutors:
Super conductors are classified into the two categories:
(i) Soft (type-I) superconductor, (ii) Hard (type-II) superconductors

(i) Type-I superconductors: "Superconductor which exhibits complete meissner effect i.e. they are completely diamagnetic are called type-I or soft superconductors". It is disappointing that the critical magnetic fields of superconductors are very low. Since the super current are started, continuously flow without heat loss.



(ii) Type-II superconductors:

These superconductors behaves differently as shown in Figure. For applied field below H_{C1}, the material is diamagnetism and hence the field is H_{C1} is called the lower critical field. Above H_{C1} magnetic flux begins to enter in the superconductor upto an upper critical field H_{C2} at which superconductivity disappear.



$$H_C = \sqrt{H_{C_1} \times H_{C_2}}$$

where, I = Superconducting state
II = Mixed state
III = Normal state

Critical magnetic field:

If a magnetic field is applied parallel to the length of the superconducting wire, the resistance of the wire is suddenly restored at definite magnetic field, which is a function of the temperature and is characteristic of particular metal. This field is known as the critical field magnetic field (H_C). The critical magnetic field H_C is given by the relation

$$H_C = H_{C_0}\left[1-\left(\frac{T}{T_C}\right)^2\right]$$

where H_{C_0} is critical magnetic field at absolute zero.

Penetration depth (λ) :

London described the Meissner effect ($B = 0$) and zero resistivity case of superconductors. He added $E = 0$ for zero resistivity.

As $J = \sigma E$ and $E = \frac{J}{\sigma} = \frac{I}{\infty} = 0$

$B = 0$ according to Meissner effect.

As we know that, $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

But $\vec{E} = 0 \Rightarrow 0 = \frac{-\partial \vec{B}}{\partial t} \Rightarrow \vec{B} \neq 0$

Then London suggested that applied external magnetic field does not suddenly drop to zero at the surface of superconductor but decays exponentially according to the equation.

$$H = H_0 e^{-x/\lambda}$$

where, H_0 is magnetic field intensity at the surface of the superconductor and λ is characteristic length (known as the penetration depth) at which H falls from H_0 to H_0/e.

London equation:

According to the London theory, it is assumed that there are two types of conduction electrons in a superconductor, namely, the super and the normal electrons. At 0 K a superconductor contains only super electrons but as the temperature increases the number of normal electrons increases and above the transition temperature (T_C) all the electrons are normal.

Let n_n, v_n and n_s, v_s is the electrons density, velocity in normal and superconducting state respectively.

Then the average carrier density $(n) = n_n + n_s$.

The current density $\vec{J} = \vec{J}_n + \vec{J}_s = e\, n_n \vec{v}_n + e n_s \vec{v}_s$ ... (1)

For the superconducting electrons

$$m\frac{d\vec{v}_s}{dt} = e\,\vec{E} \Rightarrow \frac{d\vec{v}_s}{dt} = \frac{e\vec{E}}{m} \qquad \left[\text{as } \vec{F} = m\vec{a} = m\frac{d\vec{v}}{dt} = e\,\vec{E}\right]$$

and $\vec{J}_s = e n_s \vec{V}_s \Rightarrow \frac{d\vec{J}_s}{dt} = e\, n_s \left(\frac{e\vec{E}}{m}\right)$

$$\Rightarrow \boxed{\frac{d\vec{J}_s}{dt} = \frac{n_s\, e^2 \vec{E}}{m}} \quad \text{... (2)}$$

The equation (2) is known as the first London equation and it describes the absence of resistivity.

If $\vec{E} = 0$

$$\Rightarrow \frac{d\vec{J}_s}{dt} = 0 \Rightarrow \vec{J}_s = \text{constant}$$

Then it becomes possible to have steady (constant) current in the absence of electric field. This is the superconducting phenomenon.

The corresponding equation for the normal current is

$$\boxed{\vec{J}_n = \sigma_n \vec{E} = \frac{n_n e^2 \tau\, \vec{E}}{m}} \quad \text{... (3)}$$

This equation shows that an electric field is necessary for establishing a steady current in normal metal. If $\vec{E} = 0$, $\vec{J}_n$ becomes zero.

Now by the Maxwell's equation

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \Rightarrow \vec{\nabla} \times \vec{E} = -\mu_0 \frac{\partial \vec{H}}{\partial t} \quad \text{... (4)}$$

Now taking curl of equation (2), we get

$$\vec{\nabla} \times \frac{d\vec{J}_s}{dt} = \frac{n_s\, e^2 \left(\vec{\nabla} \times \vec{E}\right)}{m}$$

$$\frac{\partial}{\partial t}\left(\vec{\nabla} \times \vec{J}_s\right) = \frac{n_s e^2}{m}\left(-\mu_0 \frac{\partial \vec{H}}{\partial t}\right) = -\frac{\mu_0 n_s\, e^2}{m}\frac{\partial \vec{H}}{\partial t} \quad \text{... (5)}$$

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Integrating this equation, we get $\boxed{\vec{\nabla}\times\vec{J}_s \equiv -\frac{n_s e^2 \mu_0 \vec{H}}{m}}$ ... (6)

Equation (6) is second London equation and explains meissner effect $\left(i.e.\ \vec{H}=0 \Rightarrow J_s = \text{constant}\right)$.

Calculation of penetration depth (λ):

Taking curl of equation (6) we get

$$\vec{\nabla}\times\left(\vec{\nabla}\times\vec{J}_s\right) = -\frac{n_s e^2 \mu_0}{m}\left(\vec{\nabla}\times\vec{H}\right)$$

Or, $\vec{\nabla}\left(\vec{\nabla}\cdot\vec{J}_s\right) - \left(\vec{\nabla}\cdot\vec{\nabla}\right)\vec{J}_s = -\frac{n_s e^2 \mu_0}{m}\left(\nabla\times\vec{H}\right)$ (as $\vec{J}_s$ = constant)

$\Rightarrow$ $\nabla^2\vec{J}_s = \frac{\mu_0 n_s e^2}{m}\left(\vec{\nabla}\times\vec{H}\right)$ ... (7)

But $\vec{\nabla}\times\vec{H} = \vec{J} + \frac{\partial\vec{D}}{\partial t} = \vec{J}_n + \vec{J}_s + \frac{\partial\vec{D}}{\partial T}$

where $\vec{D}$ is the electric displacement vector. For direct current case we can neglect $\vec{J}_n$ and $\frac{\partial\vec{D}}{\partial t}$, so that

$$\vec{\nabla}\times\vec{H} = \vec{J}_s$$

Therefore, equation (7) becomes

$$\nabla^2\vec{J}_s = \frac{\mu_0 n_s e^2}{m}\vec{J}_s = \frac{\vec{J}_s}{\lambda^2} \quad \text{... (8)}$$

where $\boxed{\lambda^2 = \frac{m}{\mu_0 n_s e^2}}$ ... (9)

λ has the dimension of length and is called the penetration depth.

The solution of equation (8) is

$$\boxed{J_s = A e^{x/\lambda} + B e^{-x/\lambda}} \quad \text{... (10)}$$

The first factor of this equation shows that J_s increases as x increases which is contrary to the fact. Then it can be left.

Therefore, equation (10) gives $J_s = B e^{-x/\lambda}$

at $x = 0,\ J_s = J_0 \Rightarrow B = J_0$

$\Rightarrow$ $\boxed{J_s = J_0 e^{-x/\lambda}}$ ... (11)

at $x = \lambda,\ J_s = J_0/e$

Thus, λ (penetration depth) is the distance in the superconductor at which the current value falls to $1/e$ of its value at the surface.

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Drawback of London theory:

Though the London theory explains the two conditions $E = 0$ and $B = 0$ characterising the superconducting state but it does not tell us about the process happening inside the superconductors. The process which takes place inside the superconductors is explained by BCS theory.

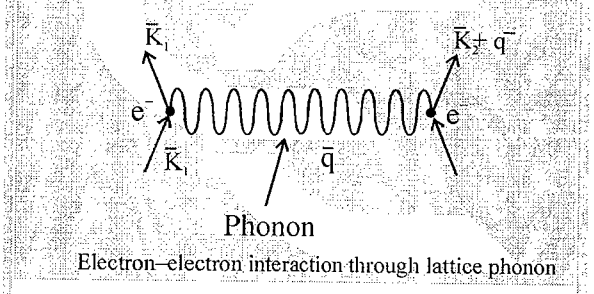
BCS (Bardeen, Cooper and Schrieffer) theory:

This theory explains the microscopic phenomenon which takes place inside the superconductor. It give, the better explanation of the superconductivity than any other theory. This theory involves the following four steps.

(i) Electron-lattice - electron interaction:

When an electron come in contact to the lattice it deforms the lattice and at the same time another electron come in contact to the deformed lattice to take the advantage of lattice deformation. In turns of it the lattice also apply a force on these electrons. Thus, there will be an interaction between electron and electron in which the phonon (lattice vibration) acts as a mediator.

(ii) Cooper pair: The fundamental postulate of BCS theory is that superconductivity phenomenon occurs, when the attractive interaction between two electrons by means of a phonon exchange dominates the usual repulsive coulomb interaction. Two such electrons which interact attractively in the phonon field are called a Cooper pair. These Cooper pairs behave like a single particle. The energy of the pair of electrons in the ground state is less than the energy of the pair in free state (electron separated). The difference of the energy of the Cooper pair and that of the energy of the Cooper pair and that of free electrons is equal to the binding energy of the Cooper pair. At temperature less than T_C electron-lattice-electron interaction is stronger than electron-electron Coulomb repulsion. Pairing is complete at $T = 0$ K and is completely broken at $T = T_C$.



Electron–electron interaction through lattice phonon

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(iii) Existance of Energy gap:

The energy difference between the free state of electrons and the paired state appears as the energy gap at the Fermi surface. The gap is maximum at absolute zero temperature and becomes zero at $T = T_C$.

(iv) The paired electrons (Cooper pair) are not scattered because of their special property of smooth riding over the lattice imperfection without exchanging of energy with them. As a result of this they travel a certain distance in the coupled motion. This distance is called coherence length. It is of the order of 10^{-4} cm. Coherence length $(\xi_o) = \dfrac{\hbar v_F}{\pi E_g}$; where, v_F = Velocity at Fermi-surface.

Expression for coherence length:

The London penetration depth (λ_L) is a fundamental length that characterises a superconductor. Another independent length of equal importance is the coherence length (ξ_0).

Now the modulated wavefunction corresponding to wave function

$$\psi = e^{+ikx} \text{ is } \phi = \frac{1}{\sqrt{2}}\left[e^{i(k+q)x} + e^{ikx}\right] \qquad ..(1)$$

Now the probability density corresponding to ϕ is

$$\phi^*\phi = \frac{1}{\sqrt{2}}\left[e^{-i(k+q)x} + e^{-ikx}\right].\frac{1}{\sqrt{2}}\left[e^{i(k+q)x} + e^{ikx}\right]$$

$$= \frac{1}{2}\left[1+1+e^{-iqx}+e^{iqx}\right] = \frac{1}{2}\left[2+2\cos qx\right]$$

$$\Rightarrow \quad \phi^*\phi = \left[1+\cos qx\right] \qquad \ldots (2)$$

The probability associated with the plane wave is

$$\psi^*\psi = e^{-ikx}\cdot e^{-ikx} = 1$$

The kinetic energy (E) of the wave function $\psi(x) = \frac{\hbar^2 k^2}{2m}$ ... (3)

While the K.E. for the modulated wave function ϕ is

$$\int \phi\left(\frac{\hat{p}^2}{2m}\right)\phi\, dx = \int \phi^*\left(-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\right)\phi\, dx = \int\left[\frac{1}{\sqrt{2}}\left(e^{-i(k+q)x}+e^{-ikx}\right)\right]\left(\frac{-\hbar^2}{2m}\right)\frac{d^2}{dx^2}\left[\frac{1}{\sqrt{2}}\left(e^{i(k+q)x}+e^{ikx}\right)\right]dx$$

$$= +\frac{\hbar^2}{4m}\int\left[e^{-i(k+q)x}+e^{-ikx}\right]\left[(k+q)^2+k^2\right]\left[e^{i(k+q)x}+e^{ikx}\right]dx$$

$$= \frac{\hbar^2}{4m}\left[(k+q)^2+k^2\right]\int\left[e^{-i(k+q)x}+e^{-ikx}\right].\left[e^{i(k+q)x}+e^{ikx}\right]dx$$

$$= \frac{\hbar^2}{4m}\left[2k^2+2kq+q^2\right] \simeq \frac{\hbar^2}{2m}k^2 + \frac{\hbar^2}{2m}kq \qquad (\text{As } q << k)$$

Thus the increase in energy $= \frac{\hbar^2 kq}{2m}$ ... (4)

If this increase in energy exceeds the energy gap E_g, the superconductivity will be destroyed. The critical value of q_0 of the modulation vector is given by

$$\frac{\hbar^2 k_F q_0}{2m} = E_g \qquad \ldots (5)$$

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The intrinsic length (ξ_0) is defined as

$$\xi_0 = \frac{1}{q_0} = \frac{\hbar^2 k_F}{2mE_g} = \frac{\hbar P_F}{2mE_g} \Rightarrow \boxed{\xi_0 = \frac{\hbar v_F}{2E_g}}$$

By BCS theory, $\boxed{\xi_0 = \frac{\hbar v_F}{\pi E_g}}$

$\frac{\lambda}{\xi_0} = k$ and $k > \frac{1}{\sqrt{2}}$, for type-I superconductors and $k < \frac{1}{\sqrt{2}}$, for type-II superconductors.

Properties which change in the superconducting transitions:

(i) All the thermoelectric effects disappear in the superconducting state.

(ii) The specific heat changes discontinuously at the transition temperature in the pressure of magnetic field.

(iii) There is also a small change in volume, in presence of magnetic field.

(iv) The thermal conductivity changes discontinuously when superconductivity is destroyed in a magnetic field.

Properties which do not change in superconducting transitions:

(i) There is no change in the reflectivity (reflection coefficient) of the metal.

(ii) The X-ray diffraction pattern is same.

(iii) The photoelectric properties, remain unchanged.

(iv) The elastic properties do not change.

(v) In the atom of a magnetic field there is no latent heat and hence no change in volume at the transition.

Superconducting band gap: $E_g = 3.53\, k_B T_C \simeq 4\, k_B T_C$

Cooper pair follow Bose-Einstein statistics. Its spin is zero.

Superconductivity is of IInd order phase transition.

Main points of BCS theory of superconductivity:

The basis of a quantum theory of superconductivity was laid by the classic 1957 papers of Bardeen, Cooper and Schrieffer. The BCS theory include:

(i) An attractive interaction between electrons can lead to a ground state seperated from excited state by an energy gap. The critical field, the thermal properties and the most of the electromagnetic properties are consequencies of the energy gap.

(ii) The electron-lattice-electron interaction leads to an energy gap of the observed magnitude. The indirect interaction proceeds when one electron interacts with the lattice and deforms it; a second electron sees the deformed lattice and adjusts itself to take advantage of the deformation to lower its energy. Thus the second electron interacts with the first electron via the lattice deformation.

(iii) The penetration depth of coherence length emerge as natural consequences of the BCS theory.

(iv) The criterion for the transition temperature T_C of an element or alloy involves the electron density of orbitals $D(E_F)$ at the Fermi level and the attractive electron-lattice-electron interaction (U) and is given by

$$T_C = 1.14\,\theta_D \exp\left[-1/UD(E_F)\right]$$

(v) Magnetic flux through the superconducting ring is quantized and the effective unit of charge is2e rather than e. $\phi = n\frac{h}{2e}, n = 1,2,3.....$ The BCS ground state involves pairs of electrons, having opposite wave vectors of opposite spins: $\vec{k}_1 = -\vec{k}_2$ and $s_1 = -s_2$. Such pairs of electrons are called as the Cooper pairs.

SOLVED PROBLEMS

A solid sample has the property that, when cooled below a certain temperature, it expels any small applied magnetic field form within the material. Which of the following best describes this sample in the cooled state ? **[TIFR-2014]**

(a) Paramagnet (b) Diamagnet (c) Ferromagnet (d) Antiferromagnet

Sol. Since, $B_{in} = 0 \Rightarrow \mu_0 (H+M) = 0 \Rightarrow H+M = 0 \Rightarrow \frac{M}{H} = -1 \Rightarrow \chi = -1$

Thus the succeptibility of a super conductor is negative which implies that a superconductor exhibits the properties of perfect diamagnetic susbtances.

Correct option is (b)

Transition temperature T_C and critical field H_C for a superconductor are related as **[DU 2015]**

(a) $H_C = H_0\left[1-\frac{T}{T_C}\right]$ (b) $H_C = H_0\left[\frac{T}{T_C}-1\right]$

(c) $H_C = H_0\left[1-\left(\frac{T}{T_C}\right)^2\right]$ (d) $H_C = H_0\left[\left(\frac{T}{T_C}\right)^2-1\right]$

Sol. Transition temperature T_C and critical field H_C for a superconductor are related as

$$H_C = H_0\left[1-\left(\frac{T}{T_C}\right)^2\right]$$

Hence, correct option is (c).

In superconductors, Cooper pairs are formed due to electron-electron interactions mediated by:

(a) Phonons (b) Photons (c) Excitons (d) Polarons **[DU 2014]**

Sol. In superconductors, Cooper pairs are formed due to electron-electron interactions mediated by Phonons.

Hence, correct option is (a).

Chapter 4

SEMICONDUCTOR DEVICES

Intrinsic and Extrinsic Semiconductors:

Semiconductor Material: The term conductor is applied to any material that will support a generous glow of charge when a voltage source of limited magnitude is applied across its terminals.

An insulator is a material that offers a very low level of conductivity under pressure from an applied voltage source.

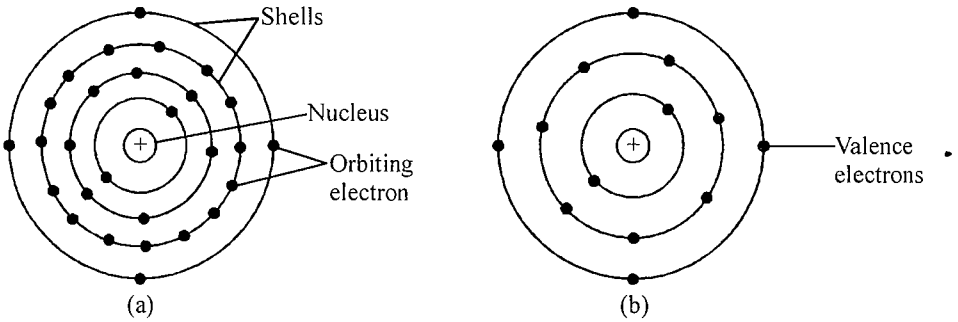
A semiconductor, therefore is a material that has a conductivity level some where between the extremes of an insulator and a conductor.

In a semiconductor current is carried by two types of carries electrons and holes whereas in metals current is carried only by electrons. Silicon (Si) and Germanium (Ge) are the two most commonly used electronic device material. Each atom of Si or Ge forms four covalent bonds with four nearest neighbouring atoms by sharing of valance electrons with opposite spin. Because of this formation of covalent bonds the valance electrons are not available and the crystal behaves as perfect insulator. However at room temperature (~ 300 K) a few of the electrons acquire sufficient kinetic energy from thermal agitation and break their covalent bonds and conduction is made possible. When an electron escape from covalent bond an electron vacancy is created in the bond and such an incomplete bond is called a hole. The hole may behave like a positively charged particle in the conduction of electrocity.

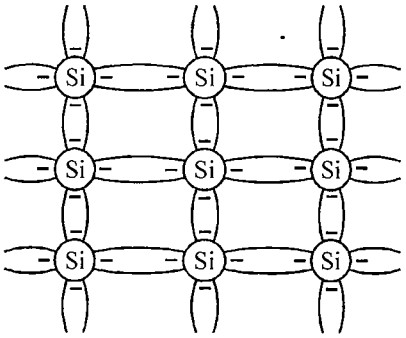
Intrinsic semiconductor materials: Intrinsic materials are those semiconductors that have been carefully refined to reduce the impurities to a very low level essentially as pure as can be made available through modern technology.

As increase in temperature of a semiconductor an result in a substantial increase in the number of free electrons and holes in the material.

Semiconductor material such as Ge and Si that show a reduction in resistance with increase in temperature are said to have a negative temperature coefficient.



Atomic structure (a) Germanium (b) Silicon



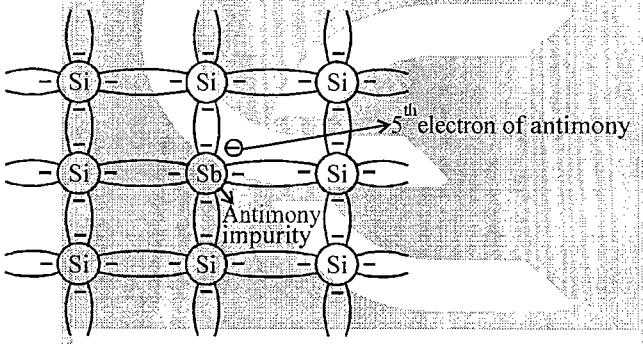
Covalent bonding of the silicon

Extrinsic semiconductor material: A semiconductor material that has been subjected to the doping process is called an extrinsic material. There are two extrinsic materials of immeasurable importance to semiconductor device fabrication *n*-type and *p*-type.

n-type materials: The n-type is created by introducing those impurity elements that have five valance electrons (pentavalent) such as antimony (Sb), arsenic (As), phosphorus (P) etc.

Diffused impurities with five valance electrons are called donor atoms.

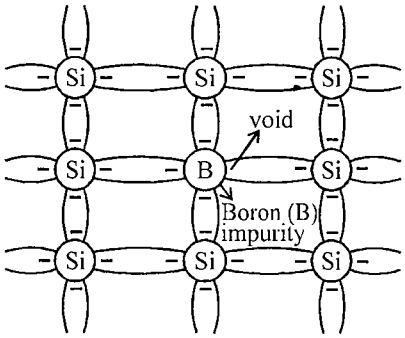
The impurity atoms displace some of the Si/Ge atoms in the lattice. Four of the five valance electrons of the impurity atoms from four covalent bonds with four nearest neighbouring Si atoms. The fifth valance electron remains very loosely bound to the impurity atom. The presence of large number of such loosely bound electron can give rise to high conduction even in room temperature (300 K).



Antimony impurity in n-type material

p-type material: A p-type material is formed by doping a pure germanium or silicon crystal with impurity atoms having three valance electrons. The elements most frequently used for this purpose are boron (B), gallium (Ga) and indium (In).

The impurity atoms displace some of the Si/ Ge atoms in the lattice. The three valance electrons of the impurity atom form three covalent bonds with three nearest neighbouring Si/ Ge atoms and impurity atom lacks by one electron to form the fourth covalent bond with its fourth neighbour. The vacancy that exists in the fourth bond constituting a hole.



Boron impurity in p-type material

"The diffused impurities with three valance electrons are called acceptor atoms".

Note: The resulting p-type material is electrically neutral for the same reasons desemiconductorribed for the n-type material.

Variation of resistivity (ρ) with temperature: Resistivity or specific resistance of material. Unit for (ρ) resistivity = Ωcm or Ωm.

For metal PTC (Positive temperature coefficient)

$$R = \rho.\frac{l}{A} \Rightarrow \boxed{R \propto \rho} \quad P\uparrow T\uparrow R\uparrow$$

In a SEMICONDUCTOR [NTC] $R\downarrow$ with $T\uparrow$

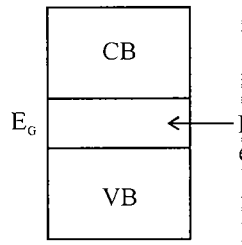
NTC $\rightarrow$ Negative temperature coefficient.

In SEMICONDUCTOR $P\downarrow$ with $T\uparrow$.

Energy Levels: The more distant the electron from the nucleus, the higher the energy state and any electron that has left its parent atom has a higher energy state than any electron in the atomic structure.

Note: Energy gap decreases with the temperature $\boxed{E_G \propto \frac{1}{Tem.}}$

Energy gap $\rightarrow$ E_G or E_g.



Energy band diagram of an element

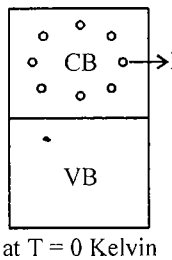
E_{G0} → Energy gap at 0K

	Ge	Si
E_{G0}	0.785eV	1.21eV
E_{G300}	0.72eV	1.1eV

$$\boxed{E_{G(T)} = E_{G0} - B_0 T}$$

B_0 = a material constant = $\frac{eV}{^\circ K}$ unit

Energy band diagram for metals and / or conductors: All metals are good conductors of current i.e. they allow a large flow of current through them.



$E_G = 0 = 0K$

at T = 0 Kelvin

Metallic Bonding:

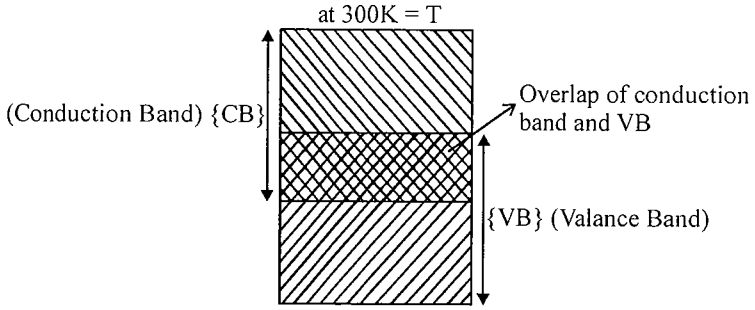
(i) Metals are unipolar, in metals only electrons are available.

(ii) In metals at 0K free electron are available.

(iii) The metals free electron concentration is independent of temperature.

(iv) The e^- concentration in metals is $n = 10^{28}/m^3$, at $t = 300K$.

(v) In metals as temperature is increasing valance sheet band and conduction band will overlap. This overlap increases with temperature.



(vi) Because of overlapping of VB and CB metals will exhibit positive temperature. coefficient of R.

$\{PTC \text{ of } R\}$

i.e. resistance increases with temperature.

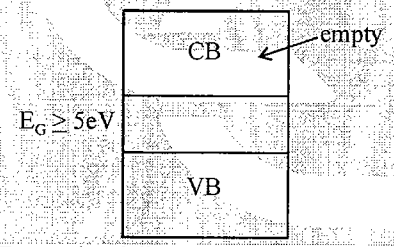
(vii) In metals there is only drift current. Eg. Gold, Silver, Cu, Iron, Al, Platinum and Tungsten.

Drift current: The current that flows through potential difference.

Energy band diagram for insulator:

(i) Insulator are bad conductors of current i.e. they do not allow any flow of current through them.

E_G - large [Energy Gap] $\geq$ 5ev.



Energy band diagram at any temperature 0→400K

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(ii) Charge carrier are zero. Therefore conductivity is zero.

Ionic bonding: Insulators are negative temperature coefficient with temperature.

[NTC of R] i.e. $R\downarrow$ with $T\uparrow$.

E_G slightly decreases with the temperature.

e.g. SiO_2, Air, Glass, Mica, Paper, Cotton, Wood, Plastic, Ciramic, Bakelite, Rubber, Parcelain, Diamond.

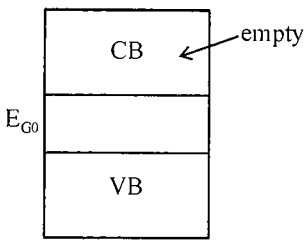
Energy band diagram of SEMICONDUCTOR: In a SEMICONDUCTOR E_G is small

$E_G \rightarrow$
- around 1eV
- 0.7eV to 1.5eV

(i) Exhibit → Covalent bonding

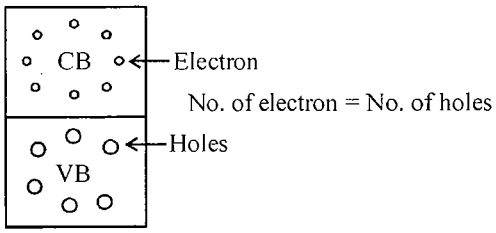
(ii) NTC of R → (R↓ with T↑)

At $t = 0K$

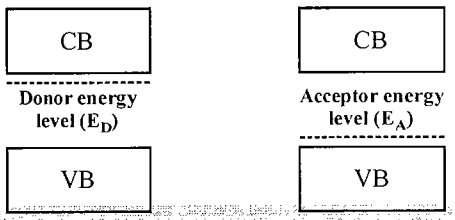


At 0K carrier concentration is zero and therefore conductivity is zero. So all SEMICONDUCTOR are insulators at 0K.

At T = 300K



SEMICONDUCTOR are bipolar {having two types of charges carrier i.e. electron and holes} at room temperature, there will be some conductivity in the semiconductor.



Energy band diagram of a *n*-type semiconductor

Energy band diagram of a *p*-type semiconductor

In case of *n*-type semiconductor the loosely bound valance electrons are formed an energy level close to the conduction band. For Si the separation between that energy level and conduction band is 0.05 eV. These new available levels are essentially discrete level because the impurity atoms are located for apart in the crystal and their interaction is small.

In case of *p*-type semiconductor this energy level is close to valance band for Si the separation is ~ 0.05 eV.

Intrinsic and extrinsic semiconductor:

Electrical properties of Ge and Si:

	Properties	*Ge*	*Si*
(1)	Atomic number	32	14
(2)	Total No. of atoms / cm^3 or Density of atoms	4.421×10^{22}	5×10^{22}
(3)	Ni at 300K ..... atom/cm^3	2.5×10^{13}	1.5×10^{10}
(4)	Intrinsic resistivity Pi at 300K (Ω–cm)	45	2,30,000
(5)	Leakage current I_2	μA	nA
(6)	Maximum operating temperature	75°C	175°C
(7)	Power handling capability	Low	High
(8)	μn {mobility of electrons}	3800 cm^2/v.sec	1500 cm^2/v.sec
(9)	μp {mobility of Hole}	1800 cm^2/v.sec	500 cm^2/v.sec
(10)	Dn {electron diffusion constant at 300K}	99 cm^2/sec	34 cm^2/sec
(11)	Dp {hole diffusion constant at 300K}	47 cm^2/sec	13 cm^2/sec
(12)	E_{G0} {energy gap at 0 Kelvin}	0.785 eV	1.4 eV
(13)	E_{G300} {energy gap at 300 Kelvin}	0.72 eV	1.1 eV

Standard doping level:

) Moderate doping $\rightarrow$ 1 : (10^6 to 10^8) $\rightarrow$ P, N

:) Lightly doping $\rightarrow$ 1 : (10^{11}) $\rightarrow$ P^-, N^-

;) Highly doping $\rightarrow$ 1 : 10^3 $\rightarrow$ P^+, N^+

Note: The minimum doping required to convert intrinsic semiconductor into extrinsic semiconductor is (1 : 10^8).

Any doping concentration below 1 : 10^8 applied the semiconductor will remain intrinsic.

- For 1°C leakage current $\{I_0\}$ approx. increases by 1% in both Germanium and Silicon.
- Leakage current is highly sensitive to temperature.
- Leakage current doubles for every 10°C.
- Silicon have better thermal stability than Ge.

Mass action law: In a semiconductor [intrinsic and extrinsic] under thermal equilibrium the product of electrons [n] and holes [p] is always a constant and is equal to the square of intrinsic concentration (n_i).

$$\boxed{n.p = n_i^2}$$

- This law is mainly used for extrinsic semiconductor to calculate the minority concentration.

Intrinsic concentration: It is a conc. available impure SEMICONDUCTOR at a given temperature. In intrinsic semiconductor.

$$\boxed{n = p = n_i}$$

$$\boxed{n_i^2 = A_0 T^3 e^{-E_G/KT}}$$

where $A_0 \rightarrow$ Amaterial constant K = 8.62 × 10^{-5} eV/°K

In semiconductor {Bipolar}:

$$\boxed{\sigma = nq\mu_n + pq\mu_p}$$

where $\sigma \rightarrow$ conductivity, $n \rightarrow$ No. of electrons, $p \rightarrow$ No. of holes, $\mu_n \rightarrow$ mobility of electrons $\mu_p \rightarrow$ mobility of holes, v $\rightarrow$ charge

CAREER ENDEAVOUR

Problem: A sample of n-type semiconductor has electron density of $6.25\times10^{18}/cm^3$ at 300 Kelvin. If the intrinsic concentration in the sample is $n_i = 2.5\times10^{13}/cm^3$.P =?.

)ln. By mass action law $np = n_i^2$

$$P = \frac{n_i^2}{n} = 1 \times 10^8/cm^3$$

Problem: In Ge the leakage current are 5μ ampere at 10°C. Find its value when the temperature is 25°C.

)ln. $I_0(T_2) = I_0(T_1)\left[2^{\left(\frac{T_2-T_1}{10}\right)}\right] = 5\times[2^{1.5}] = 14.14$ μ ampere

Fermi Energy: The unit of fermi energy is eV.

Fermi energy is define as the maximum energy possessed by an electron at 0K or

Fermi energy is also define as the maximum kinetic energy possessed by the electron at 0 Kelvin.

- Fermi energy = Maximum Kinetic Energy = $\frac{1}{2}mv^2$

Maximum velocity of electron or velocity of electron $\boxed{v=\sqrt{\frac{2E_F}{m}}}$ m/s

- It is the velocity of the electron in terms of fermi energy or
Fermi energy is also define as the energy passed by fastest moving electron at zero Kelvin.

Fermi-Dirac Function : *F*(E): Also {Fermi Dirac probability function}
Fermi-Dirac function $F(E)$ denotes the probability of electron existing as a function of (E), for a metal or a SEMICONDUCTOR.

$$\boxed{F(E)=\frac{1}{1+e^{(E-E_F)/KT}}}$$

where E is a energy possessed by electron in eV. At T = 0K, we get two conditions
when $T=0$

(i) If $E>E_F$, $F(E)=\frac{1}{1+e^{+\infty}}=\frac{1}{1+\infty}=0$

It indicates at T = 0K, No. electron are available with energies $E > E_F$ and the probability of such electron existing is zero.

(ii) If $E<E_F$, $F(E)=\frac{1}{1+e^{-\infty}}=\frac{1}{1+0}=1$ or 100%.

at $t=0$ kelvin, the probability of electron existing is one or 100%. If the energy possessed by the electron is less than E_F.
At $T\neq 0K$ or T > 0K

If $E = E_F$, $F(E)=\frac{1}{1+e^{0}}=\frac{1}{1+1}=\frac{1}{2}=0.5=50\%$

Fermi level is the characteristic level 50% probability of electron existing in the SEMICONDUCTOR. If no for bidden band exist.

- The probability of electron existing in a metal is 1 otherwise 100%.
- In metals $F(E)=1$ or 100%. In a SEMICONDUCTOR the probability of electron existing in $F(E)$ then probability of hole existing in the SEMICONDUCTOR is 1-$F(E)$.

Fermi level in intrinsic semiconductor: In intrinsic semiconductor $n = p$

$$N_c e^{-[E_c-E_F]/KT}=N_v e^{-[E_F-E_v]/KT}$$

Here $N_c=2\left(\frac{m_n kT}{2\pi\hbar^2}\right)^{3/2}$ and $N_v=2\left(\frac{m_p kT}{2\pi\hbar^2}\right)^{3/2}$

$$\frac{N_c}{N_v}=e^{-[E_F-Ev-E_c+E_F]/KT}=e^{[E_c+Ev-2E_F]/KT}$$

Taking natural log on both the side.

$$\log_e \frac{N_c}{N_v} = \frac{E_c + E_v - 2E_F}{KT}, \quad E_c + E_v - 2E_F = KT \log e \frac{N_c}{N_v}$$

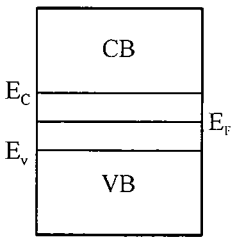
$$\boxed{E_F = \frac{E_c + E_v}{2} - \frac{KT}{2} \log_e \frac{N_c}{N_v}}$$

In intrinsic semiconductor fermi level depends only on temperature. Fermi level is a function of temperature.

Analysis: Let $m_n = m_p$

Case I: then $N_c = N_v$

$$\therefore \log_c \frac{N_c}{N_v} = 0$$

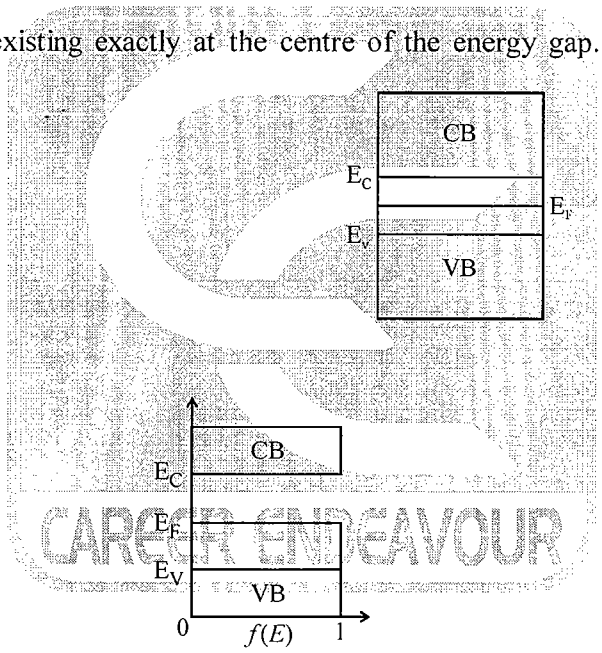


$$\boxed{E_F = \frac{E_c + E_v}{2}}$$

The fermi level is now existing exactly at the centre of the energy gap.

Case II: Let T = 0K

$$\boxed{E_F = \frac{E_c + E_v}{2}}$$



At 0K

$f(E) = 0$ for $E > E_F$

Thus there are no electrons in the conduction band, so the conductivity is zero and intrinsic semiconductor at $T = 0K$ working as an insulator.

Note: In intrinsic SEMICONDUCTOR fermi level will be existing exactly at the centre of the energy gap under the following three conditions.

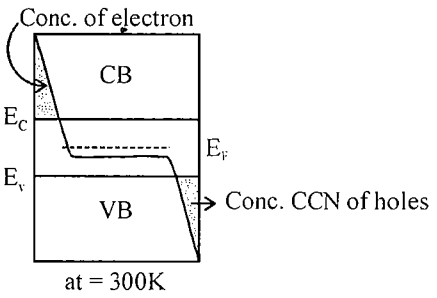
(i) $N_c = N_v$ (ii) $m_n = m_p$ (iii) T = 0K

Case-III: At 300 Kelvin

$$E_F = \frac{E_c + E_v}{2} - \frac{KT}{2} \log \frac{N_c}{N_v}$$

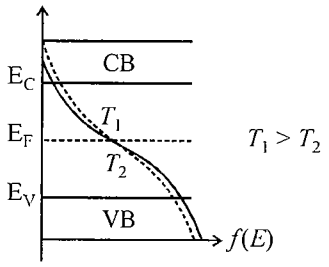
where T = 300K

electron concentration = Hole concentration



- In intrinsic semiconductor at 300K fermi level passes through the centre of the energy gap.
- At 300 Kelvin because of thermal energy a number of covalent bonds will be broken and electrons and holes are created and the conductivity of intrinsic SEMICONDUCTOR will be increasing with the temperature.

Case-IV: Fermi distribution at different temperature:



$$\text{From } f(E) = \frac{1}{1 + e^{(E - E_F)/kT}}$$

We can say that as we rise the room temperature the probability of electron existing is increases as shown in the figure. That explains why the conduction current increases as the room temperature rises.

Fermi level in n-type SEMICONDUCTOR: In n-type SEMICONDUCTOR, $n = N_D$,

$$N_c e^{-(E_c - E_F)/KT} = N_D, \frac{N_c}{N_D} = e^{(E_c - E_F)/KT}$$

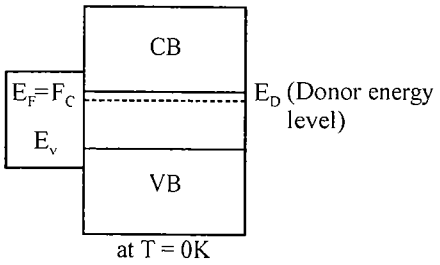
On taking natural log on both the sides.

$$\log \frac{N_C}{N_D} = \frac{[E_C - E_F]}{KT}$$

$$\boxed{[E_C - E_F] = KT \log \frac{N_C}{N_D}} \quad \boxed{E_F = E_C - KT \log_e \frac{N_C}{N_D}}$$

- In *n*-type SEMICONDUCTOR Fermi level depends on temperature and doping concentration.

Case-I: $\boxed{\text{let } T = 0K}$, $\boxed{E_F = E_C}$



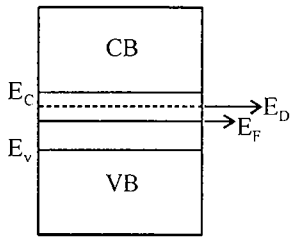
E_F considers with the edge of CB.

- At T = 0K, electron and hole concentration are zero. Therefore, conductivity is zero. Therefore *n*-type SEMICONDUCTOR will be working as an insulator at 0 Kelvin.

Case-II: Let T = 300K

$$E_F = E_C - KT \log \frac{N_C}{N_D}$$

where T = 300K

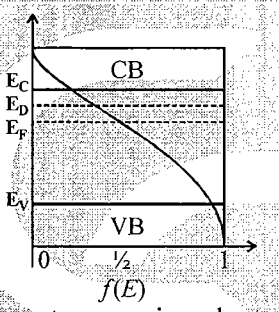


In *n*-type SEMICONDUCTOR at room temperature the fermi level will be existing just below the donor energy level because $E_C - E_D = 0.01$ eV and $E_C - E_F = 0.3$eV.

In n-type SEMICONDUCTOR as temperature is increasing from 0 Kelvin to 300 Kelvin at some intermediate temperature fermi level will be considering with the donor energy level.

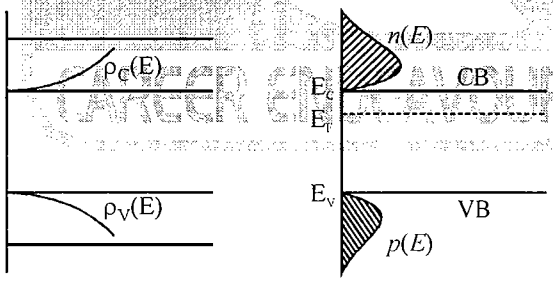
Exact Analysis:

At $T = 300\ K$



For *n*-type semiconductor

Electron concentration in conduction band, $n(E) = \rho_C(E)\, f(E)$, where $\rho_C(E)$ is the density of state in conduction band.



Hole concentration in the valance band, $p(E) = \rho_V(E)\left[1 - f(E)\right]$

Case-III: Mathematical Analysis:

$$E_C - E_F = KT \log e \frac{N_C}{N_D}$$

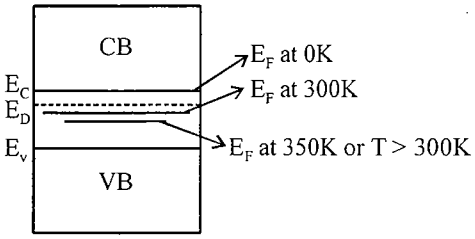
(i) Effect of Temperature: Let temperature ↑ N_C ↑ and let $N_C > N_D$

$E_C - E_F > 0$ or $\boxed{E_C > E_F}$

In type SEMICONDUCTOR as temperature increases fermi level moves away from the conduction band or fermi level moves toward the centre of energy gap.

Hence conductivity decreases with the temperature {above 300 Kelvin}

The position of fermi level for different temperature in the *n*-type SEMICONDUCTOR is given below.



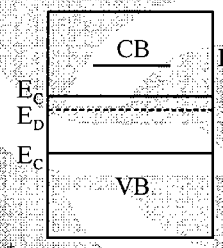
At curie temperature in n-type SEMICONDUCTOR fermi level exist at the centre of energy gap [i.e. at curie temperature *n*-type SEMICONDUCTOR will become intrinsic semiconductor and the conductivity will become small].

Effect of Doping: Let doping increasing i.e. $N_D \uparrow \left\{E_C - E_F = KT \log \frac{N_C}{N_D}\right\}$

let $N_D > N_C$, $E_C - E_F < 0$ or $E_C < E_F$

In *n*-type semiconductor doping increases fermi level moves toward the conduction bond or E_F moves away from the centre of energy gap.

Hence $\sigma \uparrow$ with doping

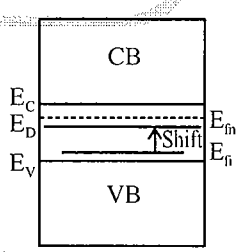


N^+ semiconductor at 300K

In N-type semiconductor as doping increases fermi level takes upward shift. In a lightly doped N-type semiconductor or highly degenerate N-type semiconductor the fermi level exist in the conduction band.

Note:

$$\text{Shift} = KT \log_e \frac{N_D}{n_i} eV$$



N^+ semiconductor at 300K

Fermi level in P-type semiconductor:

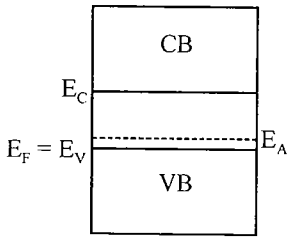
$$P \simeq N_A \quad N_V e^{-[E_F - E_V]/KT} = N_A \Rightarrow \frac{N_V}{N_A} = e^{[E_F - E_V]/KT}$$

taking natural log on both the sides

$$\log_e \frac{N_V}{N_A} = [E_F - E_V]/KT$$

$$[E_F - E_V] = KT \log \frac{N_V}{N_A} \qquad E_F = E_V + KT \log \frac{N_V}{N_A}$$

In p-type semiconductor fermi level is a function of temperature and doping conc.



at, T = 0K

Case I: T = 0K $\boxed{E_F = E_V}$

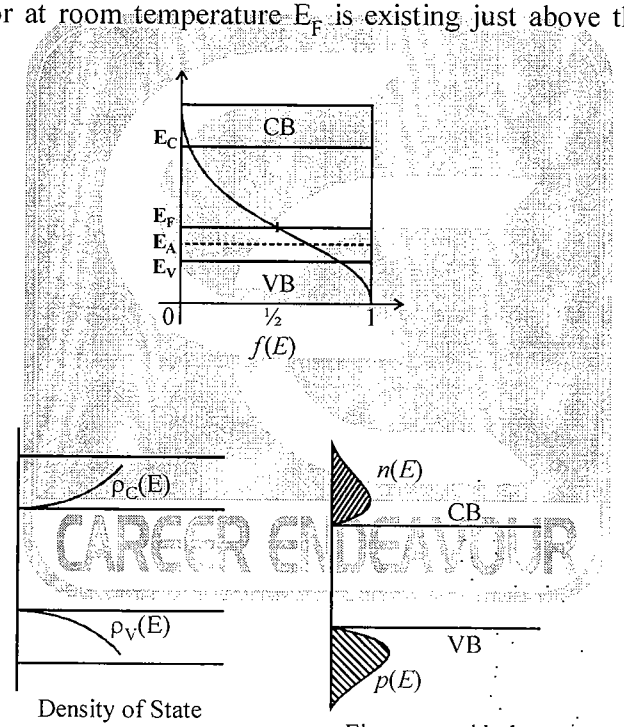
E_F coincides with edge of VB

{electron and holes conc. are zero semiconductor is working like insulator}

At T = 0K, electrons and holes concentration are zero, therefore the conductivity is zero, therefore the p-type semiconductor at 0 Kelvin will be working as an insulator.

Case II: Let T = 300K, $E_F = E_V + KT \log_e \frac{N_V}{N_A}$, when T = 300K

In p-type semiconductor at room temperature E_F is existing just above the acceptor energy level.



Case III: Mathematical analysis : $E_F - E_V = KT \log_e \frac{N_V}{N_A}$

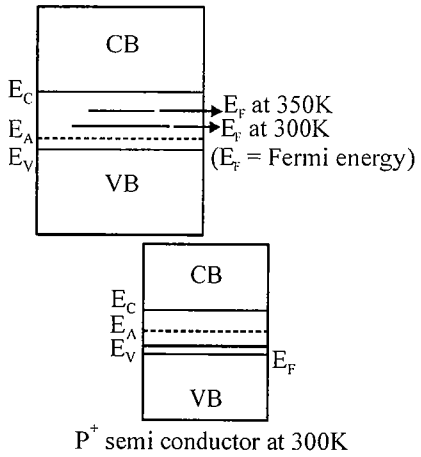
(i) Effect of temperature: As temperature ↑ N_V↑

let $N_V > N_A$, $E_F - E_N > 0$ or $E_F > E_V$

In P-type semiconductor, as temperature increases E_F moves away from V_B or E_F moves towards the centre of energy gap.

Hence $\sigma \downarrow$ with T.

The position of fermi for different temperature is given below:



Effect of doping:

Case III: As doping increases ↑ i.e. N_A ↑

let $N_A > N_V$, $E_F - E_V < 0$ or $E_F < E_V$

In P-type semiconductor as doping N_A increases fermi level moves towards the valency bond or fermi level moves away from the centre of energy gap.

Here σ ↑ with doping

SOLVED PROBLEMS

Fermi level:

1. In an n-type semiconductor the fermi level lies 0.3eV below the conduction band at 300 Kelvin if the temperature is increased to 330K. Find the app. new position of fermi level.

Soln. The position of fermi level below the conduction band depends on (i) Temperature (ii) Donor conc. N_C changes with temperature since N_C value not given neglecting variation of N_C with temperature.

$(E_C - E_F) \propto T$

$(E_C - E_{F_1}) \propto 300$ ... (1)

$(E_C - E_{F_2}) = 330\,eV$ ... (2)

$$(E_C - E_{F_2}) = \frac{330}{300} \times 0.3 = 0.33 \text{ eV}$$

So, the Fermi level will lie 0.33 eV below the conduction band.

2. In a *n*-type semiconductor the fermi level lies 0.4eV below the conduction band. If the conc. of donor atoms (N_D) is doubled. Find the new position of fermi level. Assume KT = 0.03eV

Soln. For N-type semiconductor

$N_D \simeq N_C\, e^{-(E_C - E_F)/KT}$, $N_D \simeq N_C\, e^{-(0.4)/0.03}$ ... (1)

$2N_D = N_C\, e^{-(E_C - E_F)/0.3}$ ... (2)

On dividing (1) and (2)

$$\frac{1}{2} = e^{\frac{(E_C - E_F) - 0.4}{0.03}}, \quad \log\frac{1}{2} = \frac{(E_C - E_F) - 0.4}{0.03} \Rightarrow (E_C - E_F) = 0.03 \log\frac{1}{2} + 0.4 = 0.379\text{eV}$$

3. In a p-type semiconductor the fermi level lies 0.4eV above the valency bond. If the conc. of acceptor atoms is increased by 3 times. Find the position of fermi level. Assume KT = 0.03eV

Soln. $N_A = N_V e^{-(E_F - E_V)/KT}$

$N_A = N_V e^{-(0.4)/0.03}$ ... (1)

$3N_A = N_V e^{-(E_F - E_V)/0.03}$ ... (2)

On dividing (1) and (2)

$$\frac{1}{3} = e^{\frac{(E_F - E_V) - 0.4}{0.03}}, \quad 0.03 \log\frac{1}{3} = (E_F - E_V) - 0.4$$

$(E_F - E_V) = 0.367eV$

In a semiconductor at room temperature the intrinsic carrier concentration and intrinsic resistivity are $1.5\times10^{16}/m^3$ and 2×10^{13} Ωm respectively it is converted into an extrinsic semi conductor with a doping concentration of $10^{20}/m^3$. Find the shift in fermi level due to doping.

Sln. Shift = $KT \log_e \frac{\text{Doping conc.}}{n_i} = 8.62\times10^{-5}\times300 \log_e \frac{(10)^{20}}{1.5\times10^{16}}$ eV

= 0.227 eV

Si is dopped with boron conc. of 4×10^{17} atoms/cm^3 assume $n_i = 1.5\times10^{10}$ atoms/cm^3 and temperature is 27°C compare to undopped Si the fermi level of doped Si is

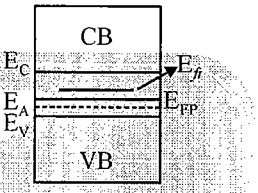
Sln. Shift = $-KT \log_e \frac{N_A}{n_i} = -(8.62\times10^{-5}\times300)\log_e \frac{4\times10^{17}}{1.5\times10^{10}} = -0.442\,eV$

Downward shift of 0.442eV

In p-type semiconductor as doping increases, fermi level takes downward shift.

In a highly doped p-type semiconductor or highly degenerate p-type semiconductor fermi level exist in the valance band.

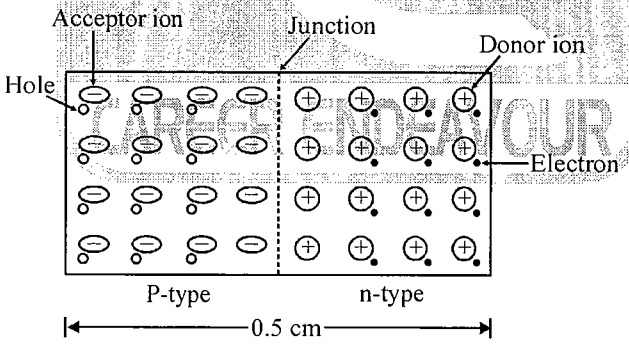
Note: $\text{Shift} = -KT \log_e \frac{N_A}{n_i} - eV$



P-N Junction Diode:

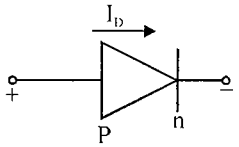
Diode is the simplest and most fundamental non-linear and active circuit element. Just like a resistor. The diode has two terminals, but unlike the register, which has a linear (straight-line) relationship between the current flowing through it and the voltage appearing across it, the diode has a non-linear i-v characteristic.

Consider a crystal of a semiconductor p-n junction is formed if donor impurities are introduced into one side and acceptors into the figure given below.



Donor ion is represented by a plus sign because after 'donating' an electron, it becomes a positive ion. Acceptor ion is indicated by a minus sign because after this atom "accepts" an electron, it converts into a negative ion.

Diode Symbol: Arrow mark indicates direction of forward current.

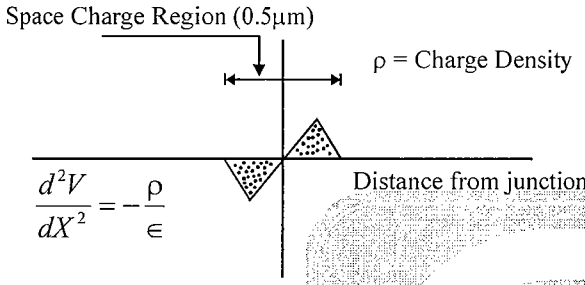


Junction Diode Characteristic:

- Some electron and holes from *n*-type and *p*-type cross into each others region and they get neutralized near junction.
- The unneutralized ions in the neighbourhood of the junction are referred to as uncovered charges.
- Since this region of the junction is depleted of mobile charges, it is called the depletion region, the space-charge region or transition region.
- The thickness of this region is of the order of wavelength of visible light (0.5 μm).
- Within this very narrow space charge layer, there are no mobile carriers.

For Step Graded p-n Junction:

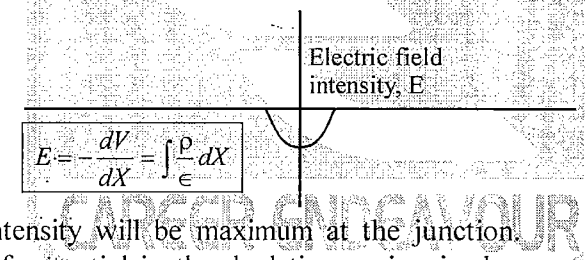
1. **Shape of charge density** ρ.



2. Corresponding field intensity curve is proportional to the integral of charge density curve.

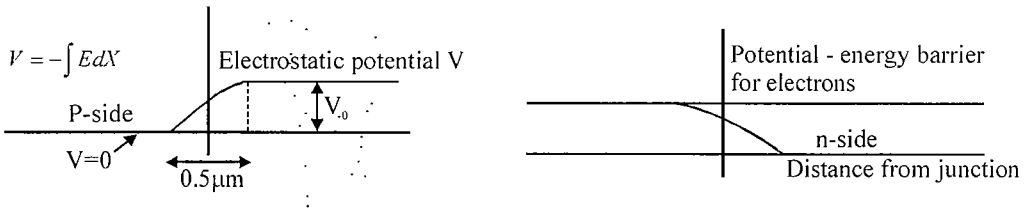
Since, $\frac{d^2V}{dX^2} = -\frac{\rho}{\epsilon} \Rightarrow \frac{d}{dX}\left(\frac{dV}{dX}\right) = -\frac{\rho}{\epsilon} \Rightarrow E = \int_{X_0}^{X} \frac{\rho}{\epsilon} dX, \quad E = -\frac{dV}{dX};$

$E = 0$ at $X = X_0$

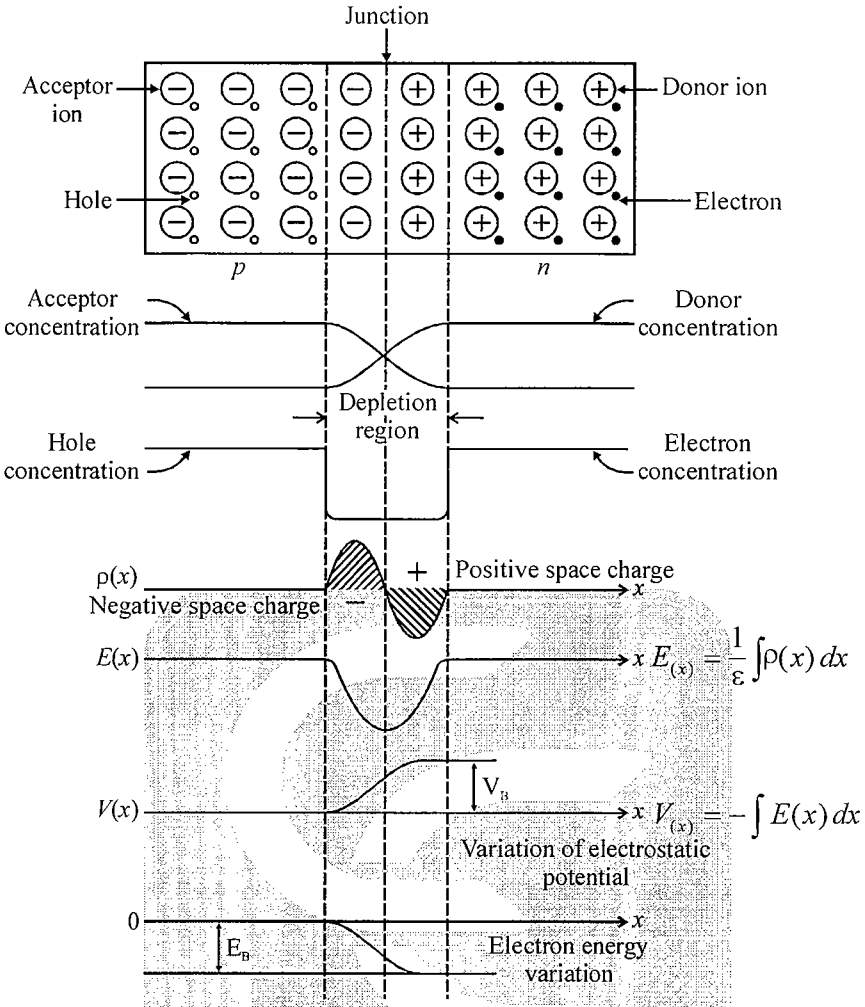


Note: Electric field intensity will be maximum at the junction.

3. **Potential:** Variation of potential in the depletion region is shown as below figure.

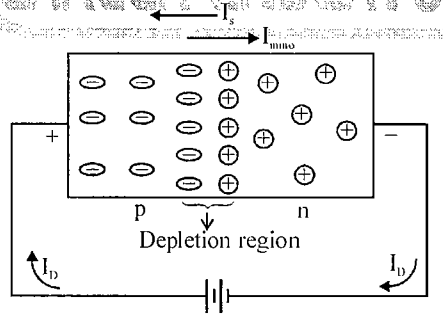


Combination of all these diagram in depletion region shown as

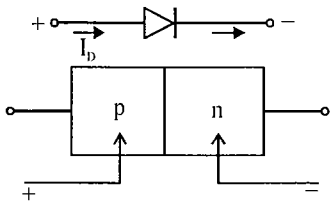


Forward Bias Region:

A forward biased or "on" condition is established when positive terminal of battery is connected to the p-type material and negative terminal to the n-type material as shown in figure.



(a) Internal distribution of charge under forward bias condition.

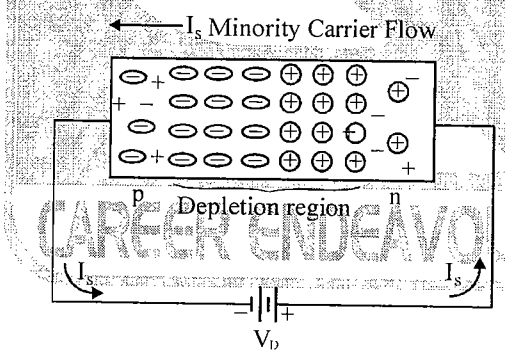


(b) Forward biased polarity.

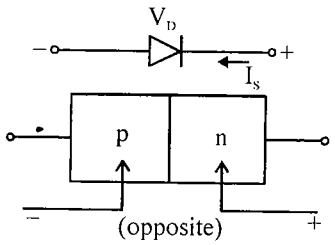
- The application of forward bias potential will "Pressure" electrons in the n-type material and holes in the p-type material to recombine with the ions near the boundary and reduce the width of the depletion region as shown in figure (a).
- The resulting minority carrier flow has not changed in magnitude (since the conduction level is controlled primarily by the limited number of impurities in the material), but the reduction in width of depletion region has resulted in a heavy majority flow across the junction.
- An electron of the n-type material now "sees" a reduced barrier at the junction due to the reduced depletion region and a strong attraction for the positive potential applied to the p-type material.
- As the applied bias increases in magnitude, the depletion region will continue to decreases in width until a flood of electron can pass through the junction, resulting in an exponential rise in current as shown in the forward bias region of the characteristic of figure (b).

Reverse Bias Condition

- If an external potential is applied across the p-n junction such that the positive terminal is connected to the n-type material and negative terminal is connected to the p-type material as shown in figure (a).
- The number of uncovered ions will increase due to the large number of free electrons / hole drawn to the respective potential of the applied voltage. The net effect, therefore is a widening of the depletion region.
- As a result majority carrier can not cross the barrier, effectively reducing the majority current flow to zero.
- The number of minority carriers however, entering the depletion region will not change, resulting in minority carrier flow across the junction.



(a) Internal distribution of charge under reverse bias condition



(b) Reverse bias polarity and direction of reverse saturation current

- The current that exists under reverse bias condition is called the reverse saturation current and is represented by I_s or I_0.

Also called as minority carrier current or leakage current or thermally generated current.

If leakage current is small, the device will have very good thermal stability.

It is highly sensitive to temperature. It doubles for every 10°C rise in temperature for both Si and Ge.

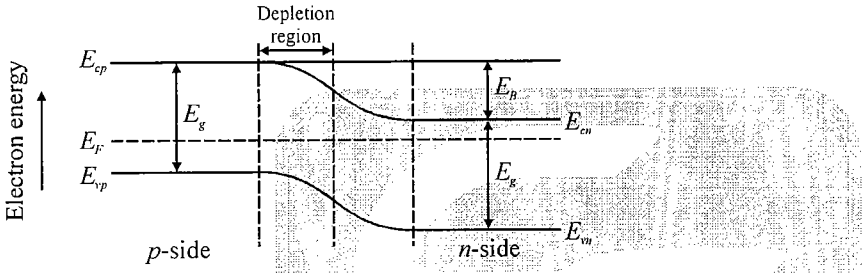
$$\boxed{I_s(T_2) = I_s(T_1)2^{\left(\frac{T_2-T_1}{10}\right)}\text{Amp}}$$

Leakage current for $\boxed{Ge\ diode = \mu A,\ \ Si\ Diode = nA}$

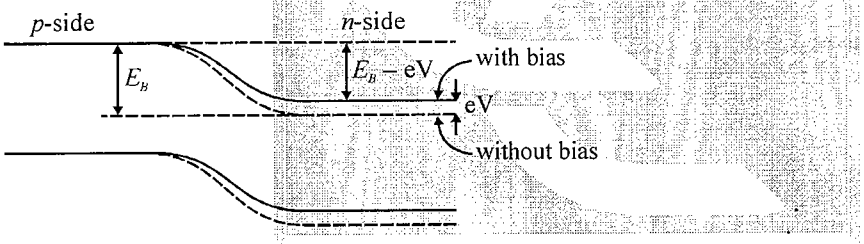
The greatest advantage of Si is smaller leakage currents.

Energy band diagram of a diode:

(*i*) Unbiased

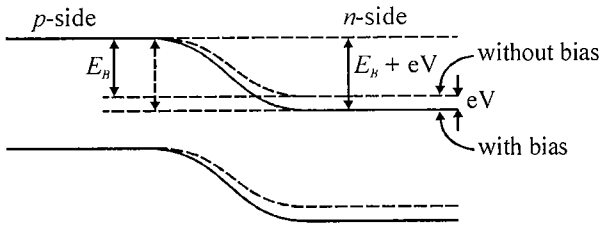


(*ii*) Forward bias



E_B is the energy barrier for the electron across the junction which is reduced to $(E_B - eV)$ when a forward bias V is applied.

(*iii*)Reverse bias



when a reverse bias V is applied the electron's energy barrier increases to $eV + E_B$.

Expression for Diode Current

The general characteristic of a semiconductor diode can be defined by Schokley's equation.

$$\boxed{I = I_0\left(e^{\frac{V_D}{\eta V_T}} - 1\right)}$$

I = Current flowing through the diode

I_0 = Reverse saturation current

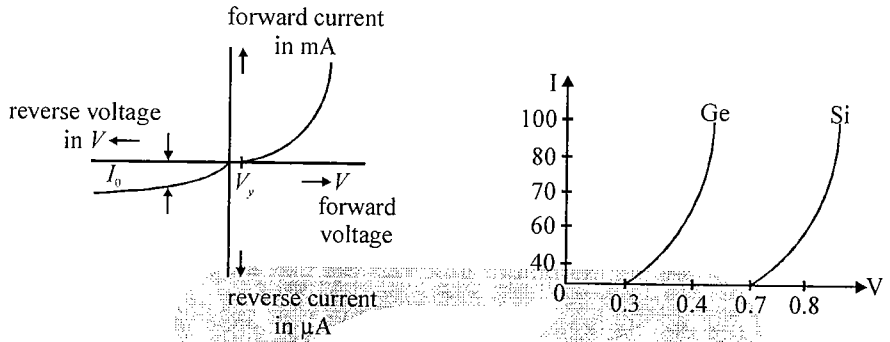
V = Voltage applied across the diode

V_T = Thermal voltage [at room temperature (i.e. T = 300°K) V_T = 25.9mV]

η = Ideality factor which is a function of the operating condition and physical construction.

[For Ge, η = 1 and for Si, n = 2]

Diode V-I Characteristic:



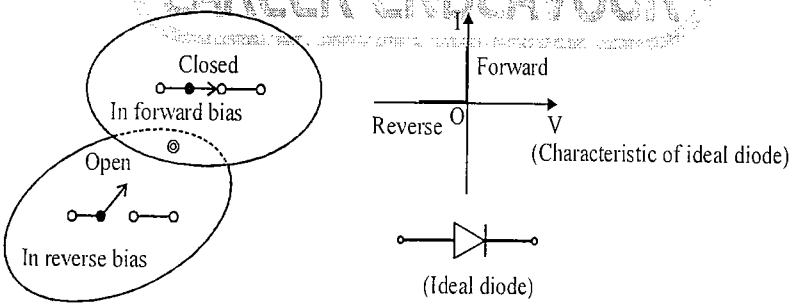
V_y = Cut-in potential

(for Ge = 0.3V and for Si = 0.7V)

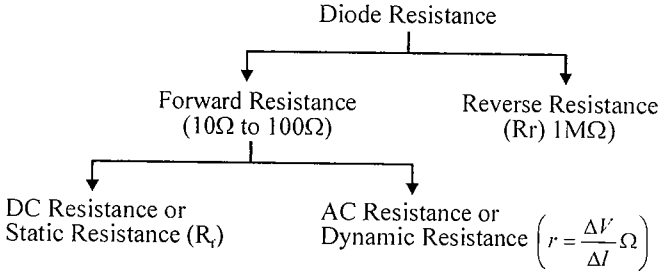
- Cut in voltage (V_y) decreases with temperature for 1°C rise in temperature, it is reduced by 2.5 mV.

Logarithmic Characteristic: $\log I = \log I_0 + 0.434 \frac{v}{\eta V_T}$ at low current log I Vs. V is almost linear corresponding to $\eta = 2$. At high current diode behaves more like a resistor than a diode and current increases linearly rather than exponentially with applied voltage.

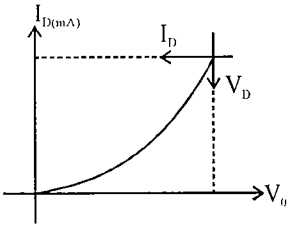
The Ideal Diode:



Diode Resistance:

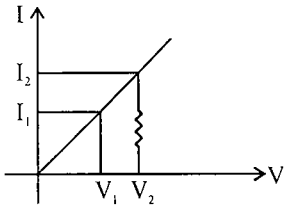


Static / DC Resistance: The static resistance of a diode is defined as the ratio V/I of the voltage to the current.



$$R_D = \frac{V_D}{I_D};$$ Typical $R_D = 80\,\Omega$ (very less)

Dynamic / AC Resistance : For small signal operation current flowing through a diode varies about some dc operating current. The dynamic, or incremental, resistance r is defined as the reciprocal of the slope of the Volt-ampere characteristic, $r = \frac{dV}{dI}$. For a semiconductor diode.



$$r_d = \frac{\eta V_T}{I + I_0}$$

Dynamic conductance $g = \frac{1}{r}$ is $g = \frac{dI}{dV_D} = \frac{I_0 e^{\frac{V_D}{\eta V_T}}}{\eta V_T} = \frac{I + I_0}{\eta V_T}$

For a forward bias greater than a few tenths of a volts, $I >> I_0$, and r is given approximately by

$$\boxed{r = \frac{\eta V_T}{I}}$$

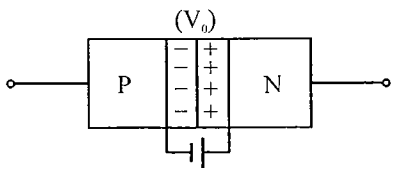
Note: DC resistance > AC resistance

The dynamic resistance of Ge diode with a forward current of 26mA is $1\,\Omega$.

Dynamic resistance in Si diode is more than in Ge diode.

CAREER ENDEAVOUR

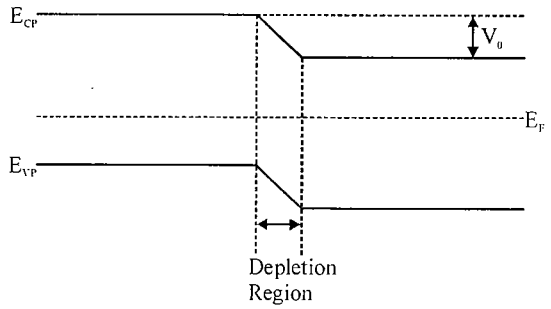
Contact Potential: (V_0)



$$V_0 = \frac{KT}{q}\ln\left(\frac{P_p}{P_n}\right), \quad V_0 = \frac{KT}{q}\ln\left(\frac{n_n}{n_p}\right)$$

If $P_p = N_a^-$ and $\eta_n = N_d^+$

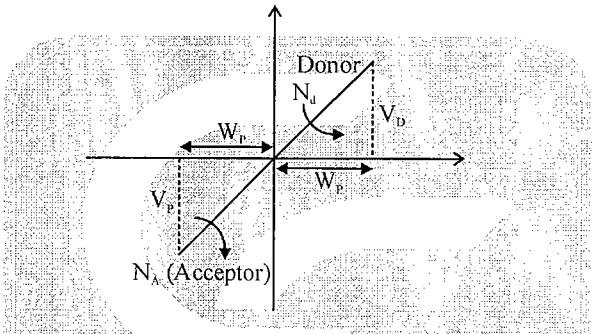
Then $$\boxed{V_0 = \frac{KT}{q}\ln\left(\frac{N_a^- N_d^+}{n_i^2}\right)}$$ $K \rightarrow$ Boltzman's constant



- The contact potential V_0 cannot be read by an voltmeter connected across it because of barrier voltage balance by a metal to semiconductor contact potential at the end of diode.

Potential barrier and width of depletion layer across P-N junction:

The barrier potential V_0 is given by $\boxed{V_0 = |V_P| + |V_n|}$



V_P is the potential full in P-region and V_n is the potential fall in N-region.

Width of depletion layer is given by $\boxed{W = W_p + W_n}$

Since crystal as a whole is electrically neutral the number of charge carriers on both side must be equal i.e.

$$N_a W_p = N_d W_n; \quad W = W_p + \frac{N_a}{N_d} W_p; \quad \boxed{W_p = \frac{W}{\left(1 + \frac{N_a}{N_d}\right)}}; \quad \boxed{W_n = \frac{W}{\left(1 + \frac{N_d}{N_a}\right)}}$$

So, it is clear that if doping is increased on one side then depletion layer width will increase on other side. In case of P–N, junction diode if doping is increased on P-side width of depletion layer will increase on N side.

Value of Width Depletion Layer:

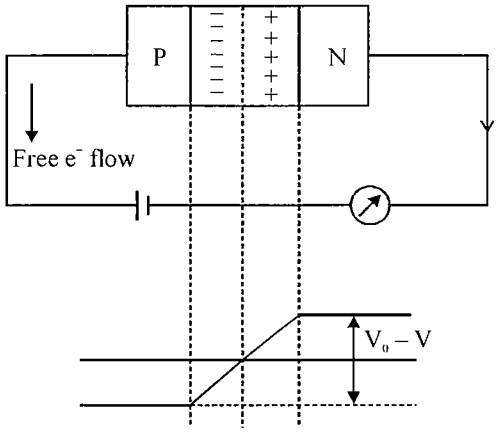
$$W_p = \sqrt{\frac{2\varepsilon V_0}{qN_a\left(1 + \frac{N_a}{N_d}\right)}} \text{ and } W_n = \sqrt{\frac{2\varepsilon V_0}{qN_d\left(1 + \frac{N_a}{N_d}\right)}}$$

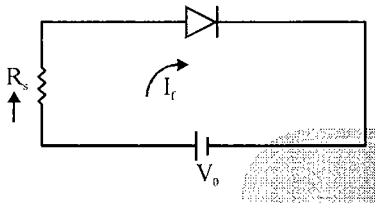
So, total width of depletion layer is given by $W = W_p - W_N$

$$\boxed{W = \frac{2\varepsilon V_0}{q}\left(\frac{N_a + N_d}{N_a N_d}\right)}$$

$\varepsilon = \varepsilon_r \varepsilon_0$; $\varepsilon_0 = 8.854 \times 10^{-12} F/m$, $\varepsilon_r = 11.7$ for Si, $\varepsilon_r = 16$ for Ge

Forward Biased P-N Junction Diode:



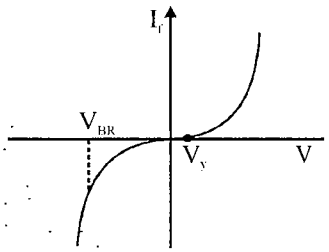


$I_f \rightarrow$ Forward current, $R_s \rightarrow$ Series resistance

When a PN junction is forward biased the width of depletion layer reduces and barrier height reduces. When a diode is properly forward the effect of barrier voltage becomes zero.

$$\boxed{I_f = I_0[e^{\frac{V_d}{\eta V_T}} - 1]} \quad \eta = 2 \Rightarrow \text{Si}, \quad \eta = 1 \Rightarrow \text{Ge}$$

V-I Characteristic:



V_y = 0.3V for Ge = 0.7V for Si

CAREER ENDEAVOUR

Note: $V_y \rightarrow$ Cut in voltage or offset Voltage or threshold voltage

$V_y \rightarrow$ Defined as "minimum forward voltage required above which the forward current flow's through device.

In case of reverse biase current is due to flow of minority carriers. If voltage is further increased in reverse direction then reverse current does not increase. At the breakdown of junction, value of current increases abruptly.

Effect of Temperature on Forward Current:

$$\boxed{I_f = I_0 e^{\frac{V_d}{\eta V_T}}}$$

Forward current is only due to majority carrier concentration is almost independent of temperature.

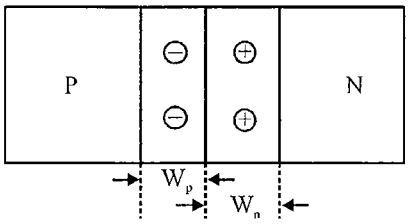
Note: Cut in voltage V_y decreases with temperature. For 1°C increase in temperature V_y decreases

by 2.3 milli volts.

Different Junction in Diode

1. **Step graded junction or Abrupt Junction:**

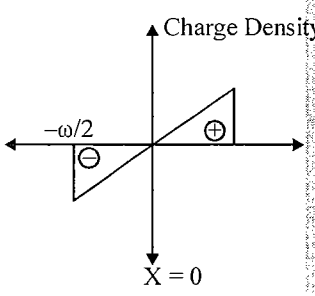
- It has abrupt p-n junction. It refer's to p^+ or PN^+ diode.
- Comparatively faster than normal diode.
- Depletion layer will always penetrate more into lightly doped region consider P^+N diode.



$\omega_N >> \omega_p$; $N_A > N_D$; $\boxed{\omega = \omega_N + \omega_P}$

$\omega = \sqrt{\frac{2\epsilon}{q}\left[\frac{1}{N_a}+\frac{1}{N_d}\right]}.V_0$, $\omega = \sqrt{\frac{2\epsilon}{q}\frac{1}{N_D}V_0}\ (N_A > N_D)$, $\boxed{V_0 = \frac{q\omega^2 N_D}{2\epsilon}}$

Linear Graded Junction: It is P^+ N^+ diode with equal doping level between P and N region.



$\boxed{\omega \propto (V_j)^{1/3}}$ V_j = Junction voltage

CAREER ENDEAVOUR

Different type of Capacitance in P-n Junction:

1. **Space Charge or Transition Capacitance:** When a P-N junction is reversed biased then depletion region acts like an insulator or dielectric material while P and N regions on either side have low resistance and acts as the plate. So this P-N junction in reverse bias may be regarded as a parallel plate capacitor if A is area of parallel plate capacitor and W is width of depletion layer then.

Transition Capacitance $C_T = \frac{\varepsilon A}{W} = C_J$

(a) For step graded junction, $\boxed{W \propto V_J^{1/2}}$

So, $\boxed{C_T \propto V_J^{-\frac{1}{2}}}$

(b) For linear graded junction $\boxed{W \propto V_J^{1/3}}$

So, $\boxed{C_T \propto V_J^{-\frac{1}{3}}}$

$V_k \rightarrow$ Knee voltage, $V_R \rightarrow$ Reversed biased voltage

$K \rightarrow$ Depends upon S.C. material

$n = \frac{1}{2}$ for step graded junction (Alloy junction)

$n = \frac{1}{3}$ for diffused junction / Linearly graded junction

So, $\boxed{C_T \propto (V_k + V_R)^{-1/2}}$ → Step graded / Alloy

$\boxed{C_T \propto (V_k + V_R)^{-1/3}}$ → Linearly graded / Diffused

Diffusion or Storage Capacitance (C_D): Diffusion capacitance dominates in a forward biased diode. It is mainly responsible for storage of minority carriers across the junction.

Mathematical Value: $C_D = \frac{dQ}{dV} = \tau.\frac{dI}{dV} = \frac{\tau}{r_d}$ $\boxed{C_D = \tau.g}$

(r_d = dynamic resistance = $\frac{dV}{dI}$)

Where g_d = dynamic conductance

So, $\boxed{C_D = \frac{\tau.I_f}{\eta.V_T}}$, So $\boxed{C_D \propto I_f}$

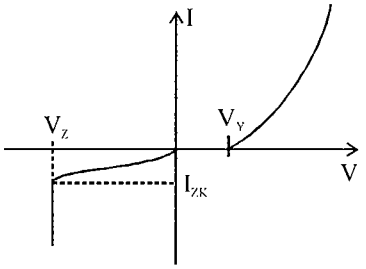
Diffusion capacitance C_D increase linearly with the forward current.

Zener Diode:

Diodes which are designed with adequate power dissipation to operate in breakdown region may be employed voltage reference or constant voltage devices. Such diodes are known as avalanche, breakdown or zener diode.

) Symbol

i) V-I characteristic



$V_z \rightarrow$ Zener voltage or break down voltage, $V_y \rightarrow$ Cut in voltage, $I_{zk} \rightarrow$ Zener current

Zener diode is reversed biased heavily doped Si or Ge P-N junction diode which is operated in breakdown region Si is preferred to Ge due to it's high temperature and current carrying capability. Forward bias V-I characteristic of zener diode is same as ordinary diode. When reverse biased voltage applied to zener diode, on increasing the reverse voltage, zener current is increased greatly from it's normal cut off value. This voltage is V_2 or break down voltage. There are two mechanism of zener break down.

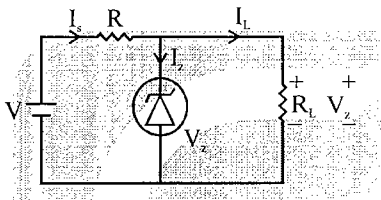
(1) (i) Zener break down / tunneling (ii) Avalanche multiplication.

(i) Zener break down / tunneling: When reverse bias is applied a very strong electric field is set up across the depletion layer and is enough to break or rupture the covalent bonds. Due to rupture of covalent bond large number of electron hole pairs are produced which constituted the reverse saturation current. It occurs in heavily doped p-n junction diode.

(ii) Avalanche break down: It occurs in case of lightly doped p-n junction diode. Here minority carriers accelerated by field applied, they collide with semiconductor atoms in the depletion region. Due to collision with valance electrons covalent bonds are broken and electron hole pairs are generated. These new carriers so produced acquire energy from applied potential and turn produce additional carriers. This forms cumulative process called as avalanche multiplication.

Temperature Characteristic of Zener Diode: If reference voltage is above 6 volt (where physical mechanism involved is avalanche multiplication) temperature coefficient is positive. But below 6 volt where zener breakdown is involved and temperature coefficient is negative.

Explanation of Above Mechanism:



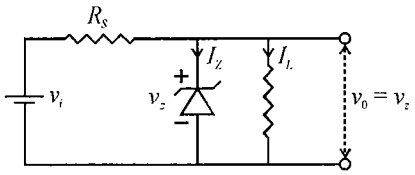
The source V and resistor R are selected in such a manner that diode is operating in breakdown region. Voltage across R_L is V_Z and diode current is I_Z. It have two type of regulation.

Zener Diode as regulator

A typical application of a zener diode as a voltage regulator is shown in the figure. In the breakdown region the the voltage across the zener diode is almost independent of current through it. This fact is very useful to design a voltage regulator. It will maintain a constant output voltage $v_0 = v_z$ independent of variations in load resistance R_L or the variation of input voltage $v_i (> v_z)$ so long as the diode remains in the breakdown region.

CAREER ENDEAVOUR

From the above figure we can write, $I_S = I_Z + I_L$ where, $I_S = \frac{v_i - v_z}{R_S}$ and $I_L = \frac{v_0}{R_L}$.



The minimum values of R_S that prevents the diode from being damaged is given by $R_{S,\min} = \frac{v_{i,\max} - v_z}{I_{ZM}}$,

where I_{ZM} can be obtained from $P_{Z,\max} = V_Z \cdot I_{ZM}$.

$P_{Z,\max}$ is normally specified by the manufacturer which is the maximum power that the zener diode can tolerate.

The maximum permitted value of R_S is set by the condition that the current through the zener diode must not go below I_{ZK} (it is knee current which corresponds to the knee of reverse characteristic curve).

$$R_{S,\max} = \frac{v_{i,\min} - v_z}{I_{L,\max} + I_{ZK}} \simeq \frac{v_{i,\min} - v_z}{I_{L,\max}}$$

Example: what is the maximum permissible current through a 5.6 V, 400 mW zener diode ? If the diode is used in a regulator circuit with maximum input voltage of 15 V, find the maximum value of series resistance that prevents the diode from being damaged.

$P_{Z,\max} = I_{ZM} \cdot v_z$

$$\therefore \quad I_{ZM} = \frac{P_{Z,\max}}{v_z} = \frac{400\text{ mW}}{5.6\text{ V}} = 71.42\text{ mA}$$

current through the zener is maximum when $I_L = 0$. Therezfore, $I_S = I_{ZM}$.

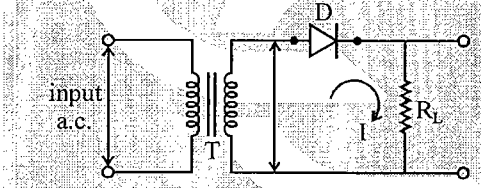
$$\therefore \quad R_{S,\min} = \frac{v_{i,\max} - v_z}{I_{ZM}} = \frac{15 - 5.6\text{ V}}{71.42\text{ mA}} = 132\ \Omega. \qquad \because \ I_{zk} = 0$$

Rectifiers

The device which converts ac power into dc power by virtue of a characteristic permitting appreciable flow of current in one direction only, is called a rectifier. Diode can be used as a rectifying element in two different way.

(1) Half wave rectifier and (2) full wave rectifier.

Half wave rectifier



- In the input a ac voltage applied wave frequency ω and peak current I_0, that it can supply.
- For the positive half of the entire cycle the diode is forward biased whereas for negative half the diode is reverse biased.
- Hence the current through the diode and load R_L will be given by

$i = I_0 \sin \omega t$ for $0 \le \omega t \le \pi$

$= 0$ for $\pi \le \omega t \le 2\pi$

- $$I_{\text{d.c.}} = \frac{1}{2\pi}\int_0^{2\pi} id(\omega t) = \frac{I_0}{\pi} \qquad V_{\text{d.c.}} = \frac{I_0 R_L}{\pi}$$

$$I_{\text{r.m.s.}} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} i^2 d(\omega t)} = \frac{I_0}{2}$$

- An important quantity called efficiency of rectification (x) is defined as

$$\eta = \frac{P_{\text{d.c.}}}{P_{in}} \times 100\,\% = \frac{I_{\text{d.c.}}^2 R_L}{I_{\text{r.m.s.}}^2 (R_f + R_L)} = \frac{40.6}{\left(1 + \frac{R_f}{R_L}\right)}\,\%$$

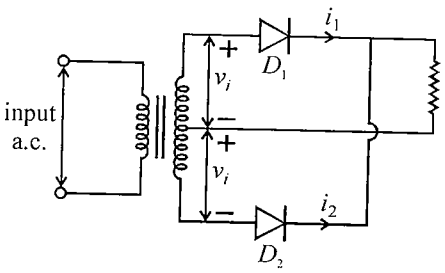
where, R_f is diode forward biased resistance.

- Another important quantity is ripple factor.

$$\nu = \frac{\text{r.m.s. value of a.c. components of load current (or voltage)}}{\text{d.c. value of load current (or voltages)}}$$

$$= \frac{I_{\text{r.m.s.}}}{I_{\text{d.c.}}} = \frac{\sqrt{I_{\text{r.m.s.}}^2 - I_{\text{d.c.}}^2}}{I_{\text{d.c.}}} = \sqrt{\left(\frac{I_{\text{r.m.s.}}}{I_{\text{d.c.}}}\right)^2 - 1} = 1.21.$$

(2) Full wave rectifier



- In this case D_1 is in forward biased when the input is in positive half of a complete cycle and D_2 is forward biased when the input is in negative half.

$$\left.\begin{aligned} i_1 &= I_0 \sin \omega t \\ i_2 &= 0 \end{aligned}\right\} \text{for } 0 \le \omega t \le \pi$$

$$\left.\begin{aligned} i_1 &= 0 \\ i_2 &= -I_0 \sin \omega t \end{aligned}\right\} \text{for } \pi \le \omega t \le 2\pi$$

- $I_{\text{d.c.}} = \frac{2I_0}{\pi}$, $V_{\text{d.c.}} = \frac{2I_0 R_L}{\pi}$

$$I_{\text{r.m.s.}} = \frac{I_0}{\sqrt{2}}$$

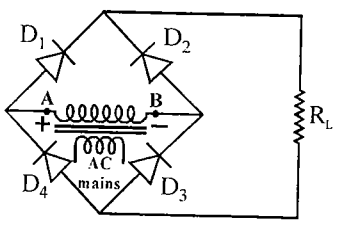
- Efficiency of rectification

$$\eta = \frac{I_{\text{d.c.}}^2 R_L}{I_{\text{r.m.s.}}^2 (R_f + R_L)} \times 100\,\% = \frac{81.2}{\left(1 + \frac{R_f}{R_L}\right)}\,\%$$

- Ripple factor

$$\nu = \sqrt{\frac{(I_{\text{r.m.s.}})^2}{I_{\text{d.c.}}^2} - 1} = 0.48$$

(3) Bridge rectification



- In this case when A is in positive potential with respect to B, diode D_1 and D_3 will conduct, and diode D_2 and D_4 will conduct if the point A become negative with respect to B.

CAREER ENDEAVOUR

- In bridge rectifier circuit a transformer without center tap at the secondary is used.
- The expression of $I_{d.c.}, I_{r.m.s.}, \eta$ and ν all are same except R_f is now changed by $2R_f$, since two diodes are conducting at a time.

Example:

1) A bridge rectifier feeds a load resistance of 2500 Ω from a 30 V (rms) supply. Each diode of the rectifier has a forward resistance of 50 Ω. Calculate

(*i*) the dc load voltage

(*ii*) the ripple voltage at the output.

(*i*) The bridge circuit gives full wave rectification. So, the dc load current is $I_{d.c.} = \frac{2I_0}{\pi}$, where I_0 is the peak load current. Since two diodes in serier conduct simultaneously, we have

$$I_0 = \frac{V_0}{2R_f + R_L} = \frac{30\sqrt{2}}{2 \times 50 + 2500} = 0.0163 \text{ A}$$

$$\therefore \quad I_{d.c.} = \frac{2I_{r.m.s.}}{\pi} = 0.0104 \text{ A}$$

The dc load voltage is $V_{d.c.} = I_{d.c.} R_L = 0.0104 \times 2500 = 26 \text{ V}$

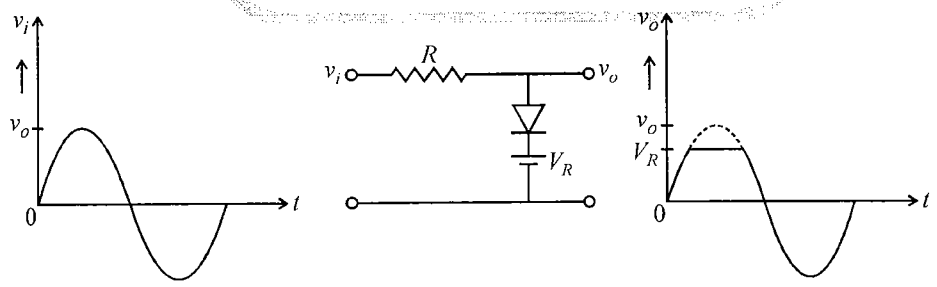
(*ii*) The ripple voltage at the output is $I'_{r.m.s.} R_L = \left(I^2_{r.m.s.} - I^2_{d.c.}\right)^{1/2} R_L$

$$I_{r.m.s.} = \frac{I_0}{\sqrt{2}} = 0.0115 \text{ A}$$

$$\therefore \quad I'_{r.m.s.} R_L = (0.0115^2 - 0.0104^2)^{1/2} \times 2500 = 12.3 \text{ V}$$

Diode clipping and clampping circuit:

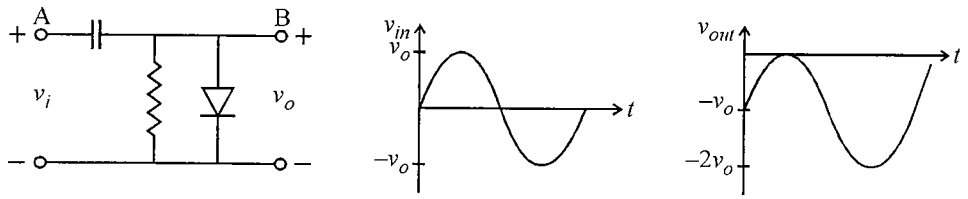
- Diode can be used as wave shaping circuits which can shape the voltage and current waveforms to suit various applications.
- Clipper circuits can 'clip' off a portion of the input signal. It is also referred to as voltage or current limiter amplitude selector or slicer.



In the example shown above the circuit can 'clip' or remove portions of input waveform which lie above the reference voltage V_R. If the input $v_i < V_R$ the diode is reverse biased and does not conduct so open under ideal diode assumption. The input waveform then passes to output as it is. When v_i is greater than V_R, the diode will be forwarded biased and start to conduct which will limit the positive peak of the output voltage approximately to the level of V_R.

• The clampping circuit simply 'clamp' the input signal to a different dc level. This kind of circuit are useful for signal shaping, circuit protection and communications.

A clampping circuit (dc restor) is used to introduce a desired dc level in an ac signal.
For example consider the circuit shown here, it will clamps the positive peaks to zero level.

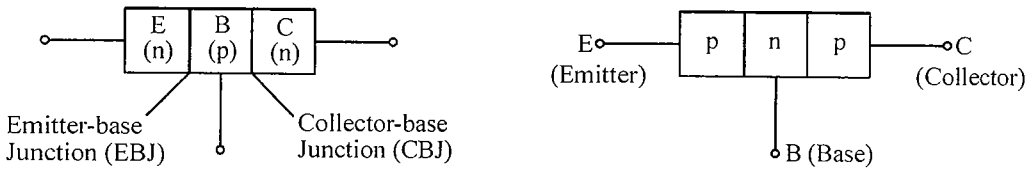


In this case the first quarter cycle charges the capacitor retains all of its charge with a voltage of v_m during subsequent cycles. Then, the output voltage is given by, $v_{out} = v_i - v_c = v_0 \sin \omega t - v_m$. By reserving the diode the negative peaks may be damped at 0 V. This is explained in the figure. Here output is given by,

$$v_{out} = v_i + v_c = v_i + v_o$$

Bipolar Junction Transistors:

- The bipolar junction transistor has three separately doped regions and contains two pn junctions.
- A single p-n junction has two modes of operation forward bias and reverse bias.
- The bipolar transistor, with two P-n junction, therefore has four possible modes of operation, depending on the bias condition of each p-n junction, which is one region for the vergatility of device.
- With three separately doped regions, the bipolar transistor is a three terminal device.
- The basic transistor principle is that the voltage between the two terminals controls the current through the third terminal.
- Current in the transistor is due to the flow of both electrons and holes, hence the name bipolar.
- The n-p-n bipolar transistor contains a thin p-region between two n-regions. In contrast the p-n-p bipolar transistor contains a thin n-region sandwiched between two p-regions.
- The three regions and their terminal connections are called the emitter, base and collector.
- The operation of the device depends, on the two p-n junction being in close proximity, so the with of the base must be very narrow, normally in the range of tenths of a micrometer (10^{-6} m).

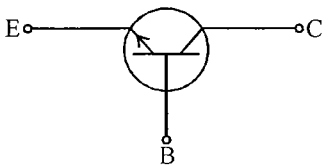


- Of the tree regions, emitter is heavily doped. The next level of doping is collector and base is less than that of collector.

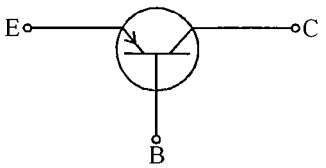
$$\boxed{N_E > N_C > N_B}$$

- The doping of the sandwiched layer is considerably less than that of the outer layers (typically, 10:1 or less)
- Emitter is provided with medium area ($A_C > A_E > A_B$) A; cross-sectional area
- Base is provided with smallest area to reduce the transit time.
- Collector is provided with the largest area to withstand heat dissipation.

Symbol:



(n-p-n transistor symbol)



(p-n-p transistor symbol)

Modes of Operation:

Mode of Operations	J_1 (B-E)	J_2 (C-B)	Applications
Active region	FB	RB	As an amplifier
Saturation region	FB	FB	As an electronic switch
Cut-off region	RB	RB	In digital circuit
Reverse active mode or inverted mode	RB	FB	As an amplifier with voltage and current gain to below (attenuator)

In a transistor

Emitter current - drift current

Base current - recombination current

Collector current - diffusion current

The flow of charge carriers in a transistor between base and collector is due to the diffusion of minority carriers and this action is called "transistor action". Hence, transistor action takes place in the base region.

The total current flowing into the transistor must be equal to the total current flowing out of if (applying Kirchhoff's current law to the transistor as if it were a single node). Hence, the emitter current is equal to the sum of the collector are and base currents.

$$I_E = I_C + I_B$$

The collector current comprise two components the majority and the minority carriers. The minority carrier current is called the leakage current and is given the symbol I_{CO} (I_C current will with emitter terminal open).

$$I_C = I_{C\,majority} + I_{CO\,minority}$$

I_{CO}:

It is also known as collector reverse saturation current or thermally generated current in the transistor.

I_{CO} for

Ge transistor is μA range, Si transistor is nA range.

I_{CO} is very sensitive to temperature.

I_{CO} doubles for every 10°C rise in temperature

For 1°C, I_{CO} approximately increases by 7%.

$$I_{CO(T_2)} = I_{CO(T_1)}\, 2^{\left(\frac{T_2 - T_1}{10}\right)}$$

I_{CO} is independent of collector supply voltage.

- The collector current is less than the emitter current. There are two reasons for this. Firstly, a part of the emitter current consists of holes that do not contribute to the collector current secondly, not all the electrons injected into the base are successful in reaching the collector.

Equation for Emitter Current:

- In a transistor under active region, emitter current is the forward current of emitter diode.

$$\boxed{I_E = I_{CO}\, e^{V_{BE}/nV_T}}$$

- The emitter current exponentially increases with base to emitter voltage of transistor.
- In a transistor under active region $V_{BE} < 1V$.

Typical value 0.2 V, For Si transistor $\boxed{V_{BE} = 0.6V \text{ to } 0.9V}$

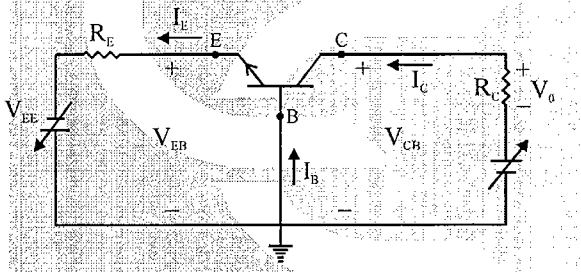
Typical value 0.7V

- V_{BE} reduced by 2.5 mV for 1°C rise in temperature, i.e.

$$\boxed{\frac{\partial V_{BE}}{\partial T} = -2.5\, mV/^\circ C}$$

Common Base Configuration:

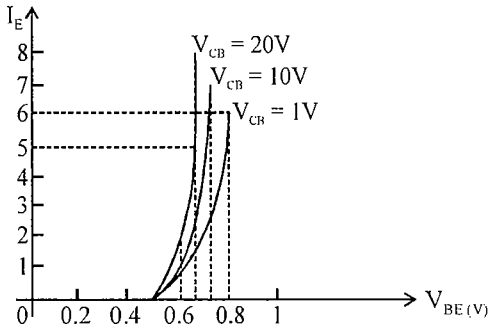
- To fully describe the behaviour of a three terminal devices such as the common base amplifier, requires two set of characteristics one for the 'driving point' or input parameters and the other for the output side.



(n-p-n transistor C-B configuration)

Input Characteristics:

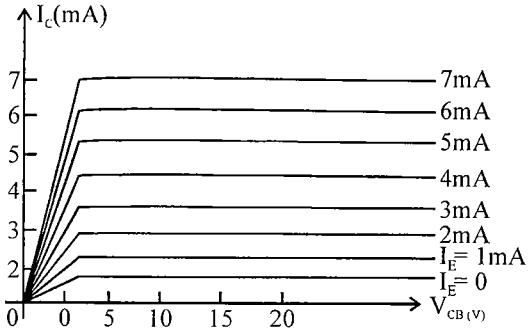
- The input set for the common-base amplifier as shown in figure relates an input current (I_E) to an input voltage (V_{BE}) for various levels of output voltage V_{CB}.



(Input or driving painting characteristic for CB silicon transistor amplifier)

Output Characteristic:

It relates I_C to V_{CB} for various levels of input current I_E as shown.



(Output or collector characteristics for a CB transistor (*n-p-n*) amplifier)

CB transistor is basically a current controlled current source (CCCS).

The curves clearly indicate that a first approximation to the relationship between I_E and I_e in the active region is given by

$$\boxed{I_C \cong I_E}$$

Properties of CB Configuration:

Lowest input resistance ($R_i < 100\Omega$)

Highest output resistance ($R_0 > 1M\Omega$)

Lowest current gain ($\alpha < 1$)

Highest voltage gain

Medium power gain (typical value 68)

Output and input voltages are in phase i.e. phase shift is 0°.

In CB amplifier current gain is loss and therefore bandwidth is large and hence CB amplifier is widely used as high frequency.

Alpha (α):

In the dc mode the levels of I_C and I_E due to the majority carriers are related by a quantity called alpha and defined by the following equation.

$$\boxed{\alpha_{dc} = \left|\frac{I_C}{I_E}\right|} \qquad \boxed{I_C = \alpha I_E + I_{CBO}}$$

Where I_{CBO} is collector to base current when emitter terminal is open.

For ac situations where the point of operation moves on the characteristic curve, an ac alpha is defined by.

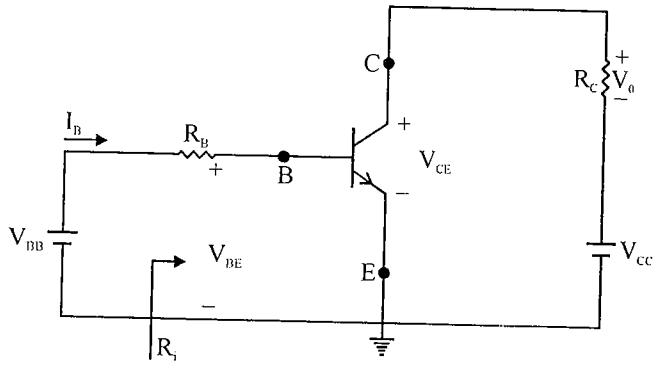
$$\boxed{\alpha_{ac} = \left.\frac{\Delta I_C}{\Delta I_E}\right|_{V_{CB}=\text{Constant}}}$$

The alpha is formally called common base amplification factor on current gain of common base transistor.

Note: The transistor's amplifying action is basically due to its capability of transfer its signal current from a low resistance circuit to high resistance circuit, contracting the two terms transfer and resistor results in the name transistor; i.e.

$$\boxed{\text{transfer} + \text{resistance} \rightarrow \text{transistor}}$$

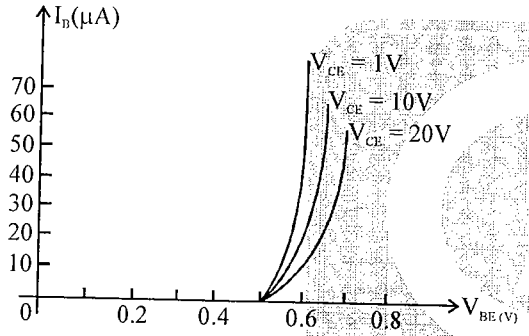
Common-Emitter Configurations:



n-p-n transistor in CE configuration

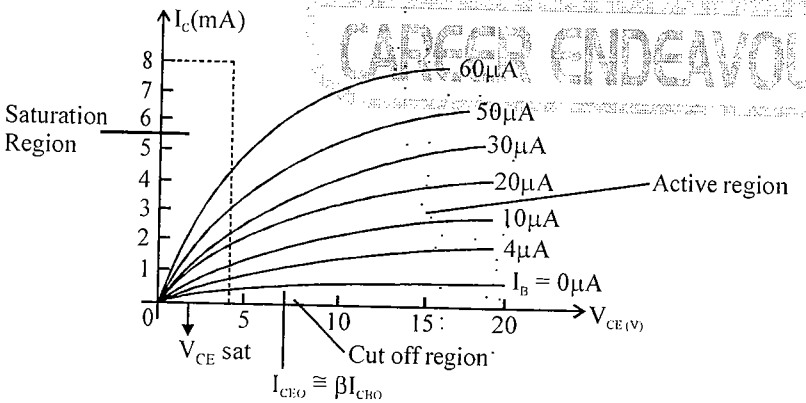
Input Characteristics:

- The input characteristics are a plot of the input current (I_B) versus the input voltage (V_{BE}) for a range of values of output voltage V_{CE}.



Output Characteristics:

- The output characteristics are a plot of the output current (I_C) versus output voltage (V_{CE}) for a range of values of input current (I_B).



Properties:

- Moderate input resistance (around 1kΩ)
- Moderate output resistance (50kΩ to 500 kΩ)
- Moderate current gain (typical value 49)
- Moderate voltage gain
- Highest power gain (typical value 4226)
- Output and input voltages are out of phase shift = 180°.
- It is most common and frequency used amplifier.

Beta (β):

In dc model $\boxed{\beta_{dc} = \frac{I_C}{I_B}}$ ($\beta > 1$) For ac $\boxed{\beta_{ac} = \left.\frac{\Delta I_C}{\Delta I_B}\right|_{V_{CE}=\text{constant}}}$

Range of β is 30 to 300. β in terms of α is $\boxed{\beta = \frac{\alpha}{1-\alpha}}$ $\boxed{\alpha = \frac{\beta}{1+\beta}}$

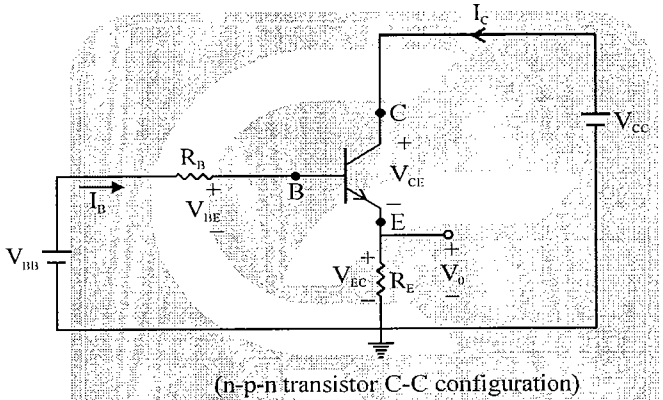
β is called the current gain of transistor in CE mode. It is the most important specification of the transistor.

β is also denoted by h_{fe} and always.

$\boxed{I_C = \beta I_B}$

$I_E = I_C + I_B = \beta I_B + I_B$ $\boxed{I_E = (\beta+1) I_B}$

Common Collector Configuration (C-C):



(n-p-n transistor C-C configuration)

It is popularly known as "Emitter follower".

In common collector, collector resistance must be zero and output voltage is taken across the emitter resistance.

CAREER ENDEAVOUR

Properties:

Highest input resistance (50kΩ to 500 kΩ)

Lowest output resistance (<100Ω)

Highest current gain

Lowest voltage gain (close to unity and ideally one)

Output and input voltages are in phase i.e. phase shift is 0°.

Emitter follower is basically a current controlled voltage source (CCVS).

Applications:

As an audio frequency power amplifier

As a buffer (impedance matching device between high resistance to low resistance)

In designing of voltage sweep circuits.

As an high input resistance devices.

As a "Boot strap emitter follower".

Gamma (γ):

$\gamma = \left|\frac{I_E}{I_B}\right| = 1+\beta$ $\boxed{\gamma = 1+\beta = \frac{1}{1-\alpha}}$ $\boxed{\alpha < \beta < \gamma}$

Note: The output characteristics of transistor in any configuration is divided in three regions.
$\Rightarrow$ Saturation region, $\Rightarrow$ Active region, $\Rightarrow$ Cut-off region

1. Saturation Region:

Minimum I_B required to keep the transistor under saturation region.

$$\boxed{I_{\beta min} = \frac{I_C}{\beta_{dc}}} \text{ or } \boxed{I_{\beta min} = \frac{I_C}{h_{FE}}}$$

Conditions for saturation region:

$I_{B\,min} \leq I_B$ saturation else active

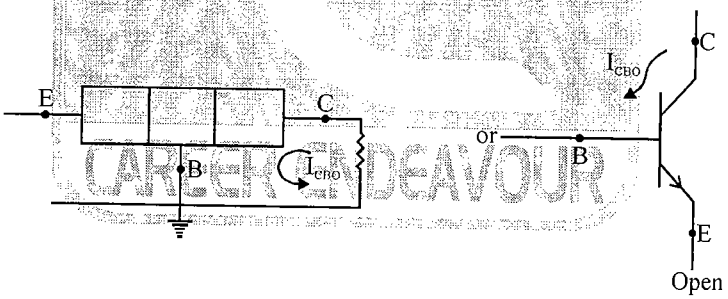
Where $I_{C\,max} = \frac{V_{CC}}{R_C + R_E}$, if $R_E = 0$ $\boxed{I_{C\,max} = \frac{V_{CC}}{R_C}}$

	$V_{BE\ active}$	$V_{BE\ sat}$	$V_{CE\ sat}$
Ge	0.3*V*	0.2*V*	0.1*V*
Si	0.7*V*	0.8*V*	0.2*V*

Note: Theory says under saturation $I_C \geq I_{C\,max}$ but in a practical circuit the maximum collector current is I_{Cmax} under the saturation region i.e. $\boxed{I_C = I_{C\,max}}$

I_{CBO}:

- It is the leakage current passing from collector to base with emitter open circuited, it is also called emitter cut-off current of the transistor.



At low frequencies $\boxed{I_{CBO} = I_{CO}}$

At high frequencies $\boxed{I_{CBO} = I_{CO} + \text{Surface Current}}$

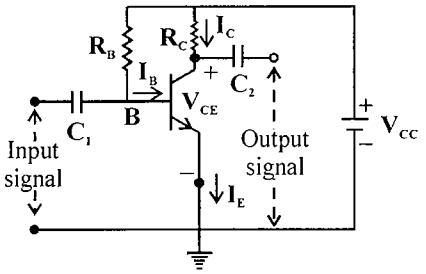
Where surface current is due to skin effect. Skin effect appears only at high frequencies. Surface current is independent of temperature and depends upon only applied voltage. It increases with frequency.

D.C. Biasing Arrangement:

(1) Fixed Bias Arrangement:

Fixed bias arrangement applied to an n-p-n transistor in CE mode is shown in figure below. The current directions of figure are the actual current directions and the voltages are denoted by the standard double subscript notation i.e., the terminal marked by the first subscript is assumed to the positive with respect to the terminal marked by the second subscript. C_1 and C_2 are the so called coupling capacitors. Their reactances are chosen very small at the lowest signal frequency and hence,

they practically offer short circuited path to a.c. but for d.c. they act as open circuit. These capacitors make it possible to connect the input signal source to the amplifier or the output of the amplifier to the input of another amplifier without affecting its d.c. bias. The single d.c. source V_{CC} is used to make the emitter junction forward biased and the collector junction reverse biased. R_B is called base bias resistance and R_C, the load resistance.



Determination of Q-point:

Applying KVL around base-emitter circuit loop we get

$$V_{CC} = I_B R_B + V_{BE}$$

Or, $$I_B = \frac{V_{CC} - V_{BE}}{R_B} \quad \text{... (i)}$$

Usually, $V_{CC} > V_{BE}$ and hence,

$$I_B \simeq \frac{V_{CC}}{R_B} \quad \text{... (ii)}$$

Thus, any change in V_{BE} due to temperature will have practically no effect on I_B.

Since, the base bias current I_B is constant, the circuit is called fixed bias circuit. Applying KVL around the collector-emitter circuit we get

$$V_{CC} = I_C R_C + V_{CE} \quad \text{... (iii)}$$

CAREER ENDEAVOUR

Stability factors:

Since, the Q-piont is chosen in the active region we can write

$$I_C = I_B + (1+\beta) I_{CBO} \quad \text{... (iv)}$$

Using equation (i),

$$I_C = \beta \cdot \frac{V_{CC} - V_{BE}}{R_B} + (1+\beta) I_{CBO} \quad \text{... (v)}$$

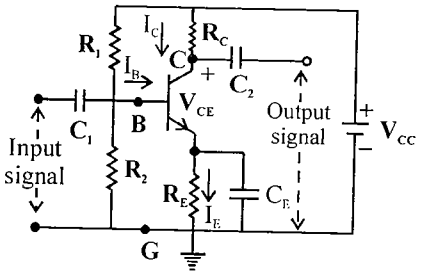
Now, from stability factors,

$$S = \frac{\partial I_C}{\partial I_{CBO}} = 1 + \beta$$

Voltage divider bias or self bias:

The commonly used voltage divider bias arrangement applied to an n-p-n transistor in CE mode is shown in figure below. The current directions shown in figure are the actual conventional current directions. The voltages are denoted by the standard double subscript notations. C_1 and C_2 are the so called coupling or blocking capacitors. Reactances of C_1 and C_2 are chosen very small at the lowest signal frequency and hence they allow easy flow to a.c. but blocks d.c. These capacitors make

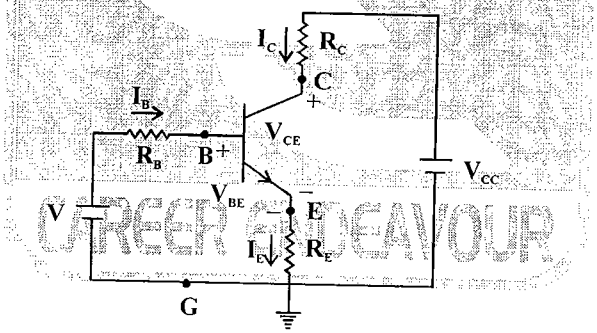
it possible to connect the input signal source or the output of the amplifier to the input of the next stage without affecting its d.c. biasing.



The capacitor C_E is known as by-pass capacitor. Its reactance is chosen very small compared to R_E. It prevents R_E from providing a.c. negative feedback gain loss but d.c. negative feedback remains. In this circuit R1 and R2 act as potential divider. the p.d. across R_E tends to reverse bias the emitter junction while p.d. across across R_2 tends to forward bias it. If temperature increases, I_{CBO} and I_C biased. The p.d. across R_E increases which makes the emitter junction less forward biased. As a result base current I_B decreases. This is turn checks the increase in I_C and it tries to bring I_C back to its original value. Thus, the stabilization of Q-point is maintained automatically by the circuit and for this the circuit is called self bias circuit.

Determination of Q-piont:

To find the d.c. operating point we can neglect the capacitors. Now, applying Thevenin's theorem to the left of the base B and the ground G in figure we above, we get the simplified circuit of figure below.



Thevinin equivalent circuit

$$V = \frac{V_{CC}}{R_1 + R_2} \cdot R_2$$

And $$R_B = \frac{R_1 R_2}{R_1 + R_2} \quad \text{... (1)}$$

Application of KVL around the base circuit yields,

$$V = I_B R_B + V_{BE} + I_E R_E \quad \text{... (2)}$$

Applicatino of KVL around the collector circuit yields,

$$V_{CC} = I_C R_C + V_{CE} + I_E R_E \quad \text{... (3)}$$

Now, we have $I_E = I_B + I_C$. Assuming $I_E \approx I_C$, we can write from equation (3)

$$V_{CE} \approx V_{CE} - I_C (R_C + R_E)$$

Stability factors:

Substituting $I_E = I_B + I_C$ in equation (2), we get

$$V = I_B(R_B + R_E) + V_{BE} + I_C R_E$$

Or, $$I_B = \frac{V - V_{BE} - I_C R_{ER}}{R_B + R_E} \quad \ldots (4)$$

Now, in the active region,

$$I_C = \beta I_B + (1+\beta) I_{CBO}$$

Using, equation (4), we get

$$\left(1 + \frac{\beta R_E}{R_B + R_E}\right) I_C = \frac{\beta(V - V_{BE})}{R_B + R_E} + (1+\beta) I_{CBO} \quad \ldots (5)$$

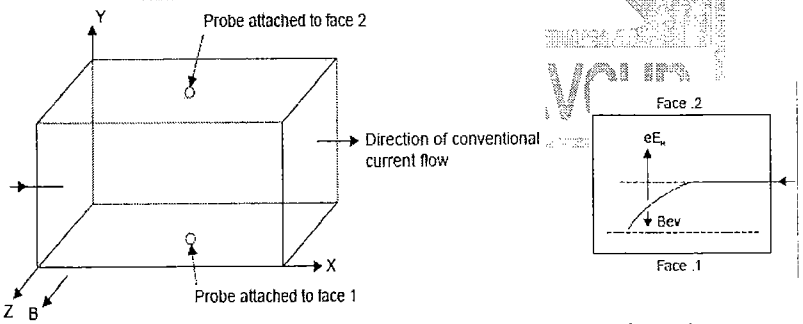
Now,

$$S = \frac{\partial I_C}{\partial I_{CBO}} = \frac{1+\beta}{1 + \frac{\beta R_E}{R_B + R_E}} = (1+\beta) \cdot \frac{1 + \frac{R_B}{R_E}}{1 + \beta + \frac{R_B}{R_E}}$$

Hall effect:

When a semiconductor is placed in a magnetic field $\vec{B}$ (along the z-direction) and a current density $\vec{J}$ (along x-direction) passed through it, a transverse electric field $\vec{E}$ (along y-direction) is set up given by $\vec{E} = R_H \vec{B} \vec{J}$ where R_H is the Hall coefficient or Hall constant.

The Hall effect may be explained with the help of following figure.



Hall effect and motion of electrons in n-type semiconductor

Assuming that the material is an n-type semiconductor, the current flow consists almost entirely of electrons moving from right to left just opposite to the conventional flow of current. If v is the velocity of electrons at right angles to the magnetic field, there is downward force on each electron of magnitude Bev. This causes the electron to be deflected in a downward direction and causes a negative charge to accumulate on the bottom face of the slab. A potential difference is therefore established from top to bottom of the specimen with bottom face negative. This potential difference causes a field E_H in the negative of y-direction, and so there is a force of eE_H acting in the upward direction on the electron. Equilibrium occurs when

$eE_H = eBv$, $E_H = Bv$

If J_x is the current density in the x-direction, then $J_x = nev$ where n is the concentration of current carriers.

Thus, $E_H = \frac{BJ_x}{ne}$; $\frac{E_H}{BJ_x} = \frac{1}{ne}$

The Hall effect is described by means of the Hall coefficient R_H, defined in terms of the current density J_x by the relation $E_H = R_H J_x B \Rightarrow R_H = \frac{E_H}{J_x B} \Rightarrow R_H = \frac{1}{ne}$.

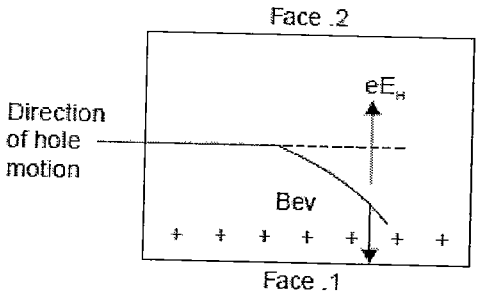
In this case $R_H = \frac{1}{ne}$.

Negative sign is used because the electronic field developed is in the negative y-direction.

i.e., $R_H = \frac{E_H}{J_x B} = -\frac{1}{ne}$.

All the quantities E_H, B and J_x can be measured, and so the Hall coefficient and carrier density n can be found out.

Figure shows the conditions that exist in a p-type semiconductor under identical conditions as that of n-type specimen described above.



Hall effect in p-type semiconductor

Experimental procedure :

The Hall coefficient is determined by measuring the Hall voltage that generates the Hall field. If V_H is the Hall voltage across the sample of thickness t, then $V_H = E_H t$.

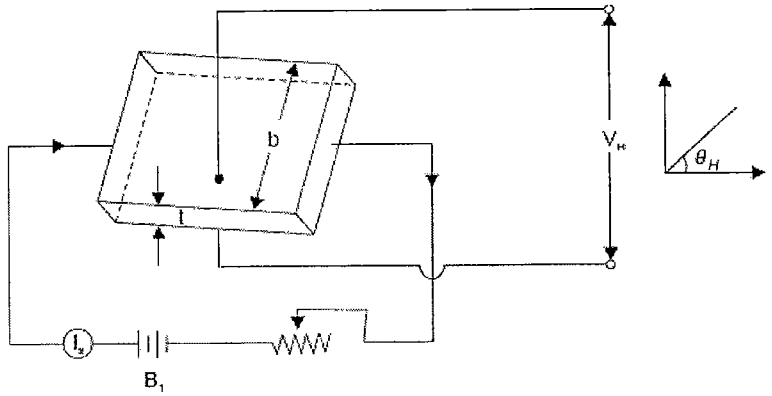
Substituting for E_H for equation $V_H = B t R_H J_x$.

If b is the width of the sample, then its cross-section will be $(b \times t)$ and the current density, $J_x = \frac{I_x}{bt}$.

$V_H = \frac{B t R_H I_x}{bt}$ and $R_H = \frac{V_H b}{I_x B}$

Note the polarity of V_H will be opposite for n and p-type.

A rectangular slab of the given material having a thickness t and width b is taken and a current I_x ampere is allowed to pass through this sample by connecting it to a battery. The slab is then placed between two pole pieces of an electromagnet such that if the current direction coincides with the x-axis, the magnetic flux density B coincides with the z-axis, as shown in the following figure.



Set up for the measurement of Hall voltage

The Hall voltage V_H is then measured by placing two probes at the centres of the bottom and top faces of the sample. If the magnetic flux density is B weber/m^2 and the Hall voltage is V_H volt, then R_H is obtained from above equations in m^3/coulomb. The analysis is given above applies only when the charge carriers are free of attractive forces in the energy bands and when they move with a steady drift velocity v_d. This is not the case in a semiconductor, and a computation of the average speed leads to the conclusion that

$$R_H = -\frac{3\pi}{8ne} = -\frac{1.18}{ne} \text{ instead of } -\frac{1}{ne}.$$

For n-type of material, the conductivity is given by $\sigma_n = ne\mu_n$ and $\mu_n = \frac{\sigma_n}{ne} = -\frac{\sigma_n R_H}{1.18}$.

For p-type material, $\mu_p = \frac{\sigma_p}{pe} = \frac{\sigma_p R_H}{1.18}$.

There is another interesting quantity called the Hall angle (θ_H) defined by the equation, $\tan\theta_H = \frac{E_H}{E_x}$

But $E_H = \frac{J_x B}{ne} = \frac{nev_x B}{ne} = \frac{v_x B E_x}{E_x}$.

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Hence, $\tan\theta_H = \frac{E_H}{E_x} = \frac{v_x B}{E_x} = \mu B$, where μ is the mobility of the charge carriers.

Applications of Hall effect:

(i) Determination of type of semiconductor: For an n-type semiconductor the Hall coefficient is negative where as for a p-type semiconductor it is positive. Thus, the sign of the Hall coefficient can be used to determine whether a given semiconductor is n-type or p-type.

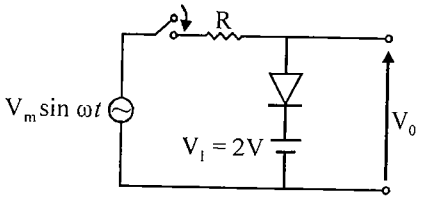
(ii) Calculation of carrier concentration (n).

(iii) Determination of mobility (μ).

(iv) Measurement of magnetic flux-density (B).

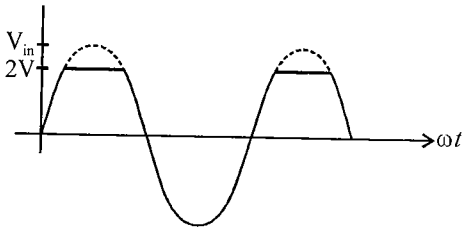
SOLVED PROBLEMS

1. Draw the output wave form of the circuit as shown in figure, when switch is closed.

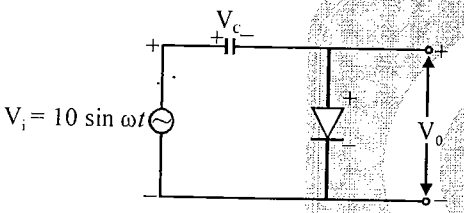


Soln. This is a simple clipping circuit

V_i	Diode	V_0
$V_i \leq 2V$	OFF	V_i
$V_i \geq 2V$	ON	2V

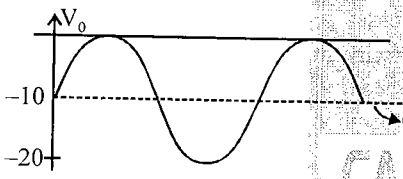


2. Draw the output waveform for the given circuit if the input is sinusoidal.



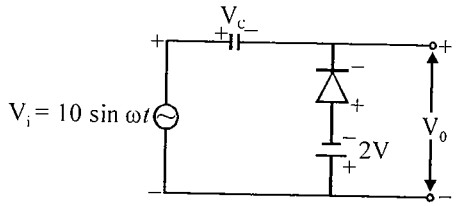
Soln. Apply KVL $V_i - V_C - V_0 = 0$, Capacitor will charge till 10V.

$\because\ V_0 = V_i - V_C,\ V_i > V_C,\ V_0 = V_i - 10$

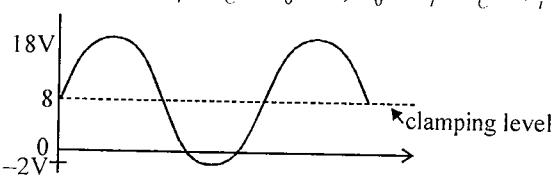


Trick input waveform is shifted to the direction indicated by the diode.

3. Draw the output waveform for the given circuit if the input is sinusoidal.



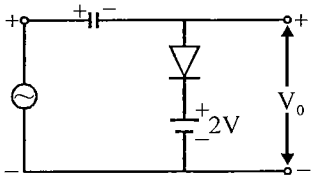
Soln. Apply *kVL*, $V_i - V_C - V_0 = 0,\ V_0 = V_i - V_C = V_i - (-8),\ V_0 = V_i + 8$



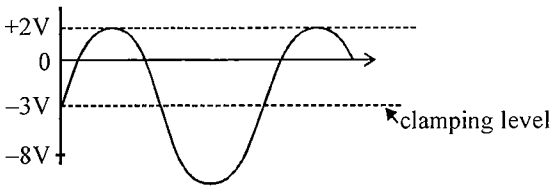
Capacitor will charge when V_i is negative

$-10 + 2 - V_i = 0,\ V_C = -8V$

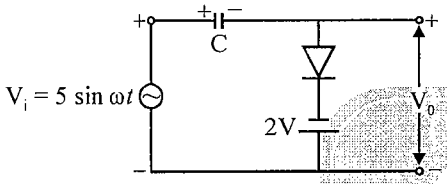
. Draw the output waveform



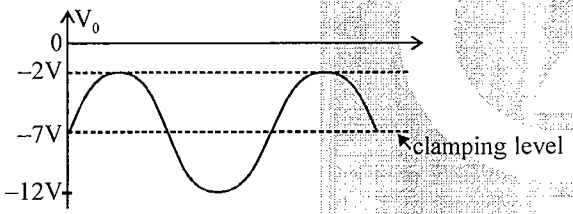
oln. Capacitor will charge when V_i is positive, apply KVL, $5 - V_C - 2 = 0$, $V_C = 3V$



. Draw the output waveform

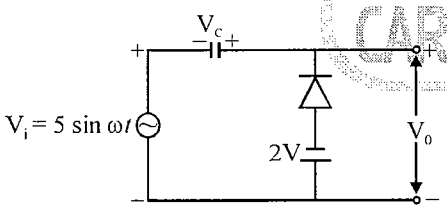


oln. Capacitor will charge when $V_i > -2V$, apply KVL $5 - V_C + 2 = 0$, $V_i = 7V$

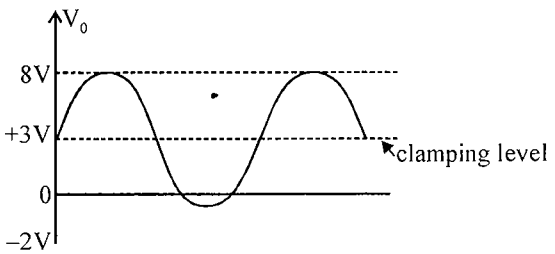


Apply KVL, $V_i - V_C - V_0 = 0$, $V_0 = V_i - V_C$

. Draw the output voltage waveform



oln. Capacitor will charge when V_i is negative. Apply KVL, $-5 + 2 + V_C = 0$, $V_C = -3V$



Apply KVL, $V_i - V_C - V_0 = 0$, $V_0 = V_i - V_C$, $V_0 = V_i + 3$

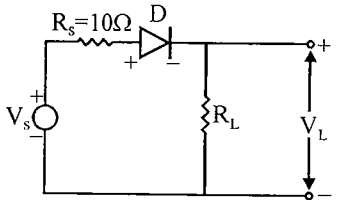
CAREER ENDEAVOUR

7. Find D.C. value and r.m.s. value of signal $X_T = 2 + 4\sin\omega t$

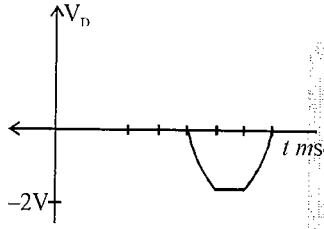
Soln. $X_{D.C.} = 2,\ X_{A.C.} = 4\sin\omega t,\ X_{A.C.(r.m.s.)} = \dfrac{4}{\sqrt{2}} = 2\sqrt{2}$

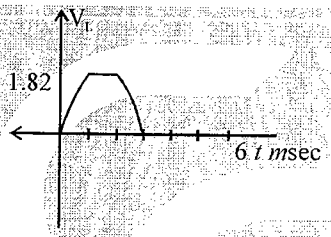
$$X_{T(r.m.s.)} = \sqrt{X_{D.C.}^2 + X_{A.C.(r.m.s.)}^2} = \sqrt{2^2 + (2\sqrt{2})^2} = 2\sqrt{3}$$

8. For the circuit of figure, sketch the waveforms of V_L and V_D if the source voltage V_S is as given in figure. The diode is ideal and $R_L = 100\,\Omega$.



Soln.

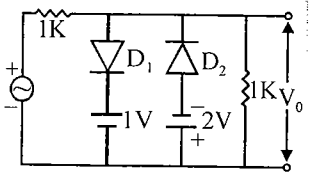




If $V_S \geq 0, D$ conducts, so that $V_D = 0$ and $V_L = \dfrac{R_L}{R_L + R_S} V_S = \dfrac{100}{100+10} V_S = 0.91 V_S$

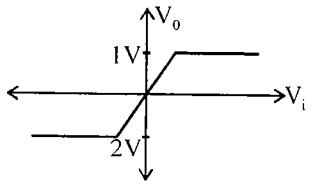
If $V_S < 0$, diode blocks, so that $V_D = V_S$ and $V_L = 0$ sketches of V_D and V_L are shown in figure.

9. Draw the transfer characteristic for the given circuit assuming ideal diodes.



Soln. Case-I: When $V_i > 1, V_1 = D_1 = \text{ON}, D_2 = OFF$ and $V_0 = 1$ volt

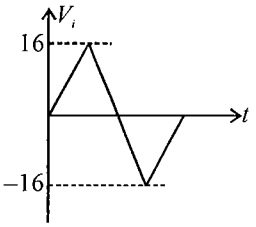
Case-II: $V_i < 2V, D_1 = \text{OFF}, D_2 = \text{ON}$ and $V_0 = -2$ volt



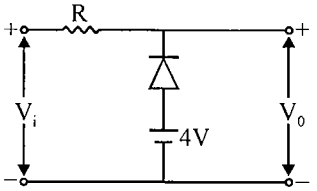
V_i	D_1	D_2	V_0
$V_i < -2V$	OFF	ON	$-2V$
$-2V < V_i < 1V$	OFF	OFF	$V_i/2$
$V_i > 1V$	ON	OFF	1V

Case-III: For, $-2V < Vi < 1V,\ V_0 = \dfrac{V_i}{1+1} \times 1 = \dfrac{V_i}{2}$

10. Determine V_0 for the network of figure.



Vi	Diode	V_0
$V_i > 4V$	ON	4V
$V_i > 4V$	OFF	V_i

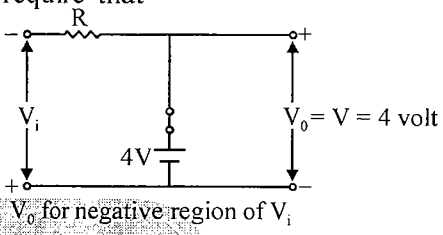


Soln. The polarity of the d.c. supply and the direction of the diode strongly suggest that the diode will be in the (on) state for the negative region of the input signal. For this region the network will appear as shown in figure where the defined terminals for V_0 require that

$V_i > 4V_1$ diode is off (O.C)

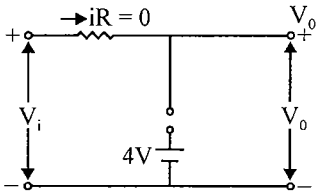
$V_i < 4V_1$ diode is on (short circuit)

For the open circuit state, $V_0 = V_i$

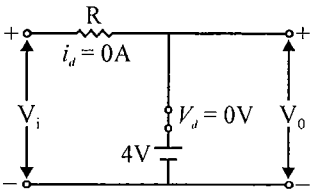


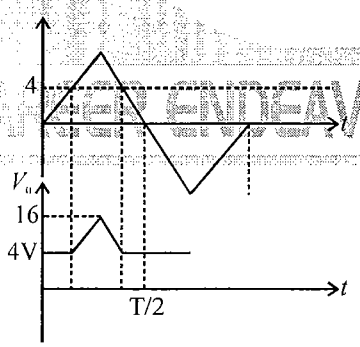
V_0 for negative region of V_i

Case-I: $V_i > 4V$, diode is off (open circuit) $V_0 = V_i$

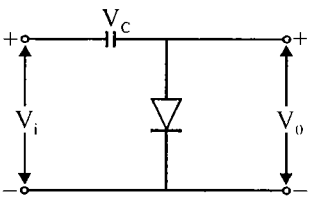


Case-II: $V_i < 4V_1$ diode is on (short circuit), V_0 = 4 volt





11. To find V_0? when (i) $0 < V_i < +5V$ (ii) $V_i = 5V$, $V_C = 5V$ (iii) $-5V < V_i < 0V$



Soln. (i) $0 < V_i < +5V$, Diode is on, it means short circuit

Capacitor charges upto V_C = 5V, then $V_0 = 0$

(ii) V_i = 5V, V_C = 5V

Apply KVL in input loop $V_i - V_C - V_0 = 0$, $V_0 = V_i - V_C = 0$, $V_0 = 5 - 5 = 0V$

(iii) $-5V < V_i < 0V$

Apply KVL $V_i - V_C - V_0 = 0$, $V_0 = V_i - V_C$, $V_0 = V_i - 5V$

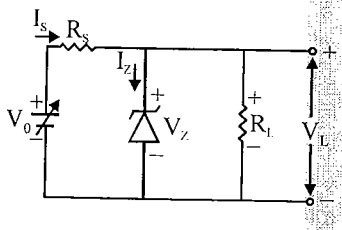
12. $X(t) = 4\sin\omega t + 2\sin(\omega t + 60°)$, find out $X_{r.m.s.}$ and $X_{d.c.}$

Soln. $X_{r.m.s.} = \sqrt{\left[X_{1r.m.s.}^2 + X_{2r.m.s.}^2 + 2X_{1r.m.s.}.X_{2r.m.s.}\cos\phi\right.}$

$$= \sqrt{\left(\frac{4}{\sqrt{2}}\right)^2 + \left(\frac{2}{\sqrt{2}}\right)^2 + 2\left(\frac{4}{\sqrt{2}}\right)\left(\frac{2}{\sqrt{2}}\right)\cos 60°} = \sqrt{8 + 2 + 8 \times \frac{1}{2}} = \sqrt{14}$$

and $X_{DC} = 0$

13. The zener diode in voltage regulator circuit of figure has a constant reverse breakdown voltage $V_Z = 8.2V$, for $75mA \le i_z \le 1A$ if $R_L = 9\Omega$ size R_S so that $V_L = V_Z$ is regulated to (maintained at) 8.2V while V_b varies by ± 10 percent from it's nominal value of 12 volt.



Soln. By Ohm's law, $i_L = \frac{V_L}{R_L} = \frac{V_Z}{R_L} = \frac{8.2}{9} = 0.911A$

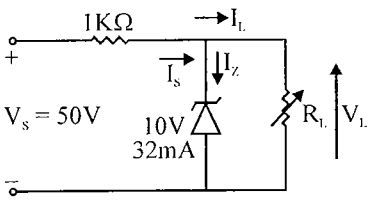
Now an application of KVL gives $R_S = \frac{V_b - V_Z}{i_Z + i_L}$

and ω_0 are to size R_S maximum zener current i_Z at the largest value of V_b.

$$R_S = \frac{(1.1)(12) - 8.2}{1 + 0.911} = 2.62\Omega$$

14. A zener regulated circuit is shown in the figure. Find the range of load resistance, load current for V_L to be maintained constant. Assume zener is ideal zone and also find the maximum power consumed.

Assuming rating of zener is 10V, 32mA



Soln. Zener is ideal, means $R_Z = 0$, $V_Z = V_L = 10V$

$$I_S = \frac{50-10}{1K\Omega} = 40mA \text{ (constant)}$$

(a) R_L is minimum means $I_Z = 0$, $I_{Lmax} = I_S = 400mA$, $V_L = V_Z = I_{Lmax}.R_{Lmin}$

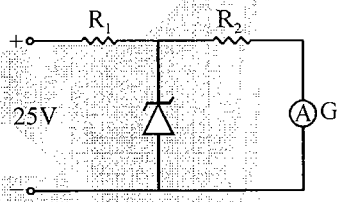
$$R_{Lmin} = \frac{10}{40\times10^{-3}} = 250\Omega$$

(b) R_L is maximum,
mean $I_Z = 32mA = I_{zm}$, $I_{Lmin} = I_S - I_{Zm}$
$V_L = V_Z = I_{Lmin}.\ R_{Lmax}$

$$R_{Lmax} = \frac{V_L}{I_{Lmin}} = \frac{10}{8\times10^{-3}} = 1.25K\Omega$$

P_{max} = maximum power rating = 10 × 32 = 320m watt

5. The complete circuit shown in figure represents a 25dc voltmeter where G is a PMMC galvanometer having full scale deflection current



$I_{fsd} = 200mA$ and $R_G = 500\Omega$

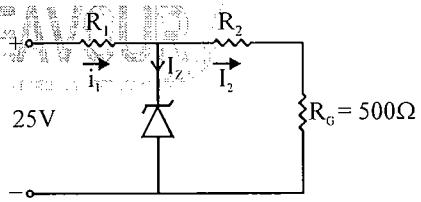
and D is 20V zener diode, find R_1 and R_2.

What is the function of the diode D in this circuit.

Soln. Current through galvanometer

$$I_{fsd} = I_Z = \frac{\text{Zener voltage}}{R_2 + R_G}, \quad 200\times10^{-6} = \frac{20}{R_2 + 500}$$

As zener diode current is not specified let it be assumed zero.



$\therefore I_1 - I_2 = I_z$ or $I_1 = I_2 = 200\mu A$

Also, $I_1 = \frac{25-20}{R_1} = 200\times10^{-6}$, $R_1 = \frac{5\times10^6}{200} = 25k\Omega$

6. In a common-base connection, current amplification factor is 0.9. If the emitter current is 1mA, determine the value of base current

(a) 0.9mA (b) 1mA (c) 0.5mA (d) 0.1mA

Soln. **(d)**

7. Which one of the following statement is correct? In a transistor

(a) I_{CBO} is greater than I_{CEO} and does not depend upon temperature.

(b) I_{CEO} is greater than I_{CO} and doubles for every temperature degree rise in temperature

(c) I_{CBO} is equal to I_{CO} and double for every temperature degree rise in temperature

(d) I_{CEO} is equal to I_{CO} and doubles for every ten degrees rise in temperature

Soln. **(c)**

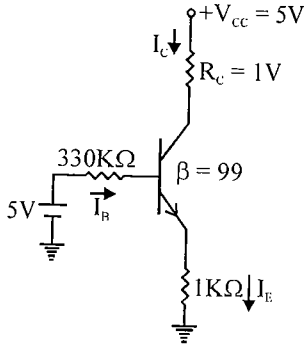
18. Find out Q point?

(a) $3mA, 1V$

(b) $-99mA, 3.02\text{Volt}$

(c) $1.1mA, 3V$

(d) None of these



Soln. Step-I: Assume BJT is active mode

$\therefore V_{BE} = 0.7\text{Volt}$

Step-II: Apply KVL base emitter loop

$5 - 330 \times 10^3 I_B - 0.7 - 1 \times 10^3 I_E = 0$

$4.3 = 330 \times 10^3 I_B + 1 \times 10^3(I_B + I_C) = 330 \times 10^3 I_B + 1 \times 10^3 (I_B + \beta I_B)$

$4.3 = I_B(330 \times 10^3 + 100 \times 10^3),\ 4.3 = I_B(430 \times 10^3)$

$I_B = 10\mu A$

Step-III: $I_{C(\text{active})} = \beta I_B = 99 \times 10 \times 10^{-6} = .99\text{mA}$

Step-IV: Apply KVL in collector-emitter loop for $I_{C(\text{saturation})}$

$5 - I_C \times 10^3 - V_{CE(\text{sat})} - I_E \times 10^3 = 0$

$5 - 0.2 = I_C \times 10^3 + (I_B + I_C) \times 10^3$

$4.8 = I_C(2 \times 10^3),\ I_{C(\text{sat})} = 2.4\text{mA}$

Step-V: $\because I_{C(\text{active})} < I_{C(\text{sat})}$ $\therefore$ BJT in active mode.

Step-VI: $V_{CE} = V_{CC} - I_C (R_C + R_E)$

$= 5 - 0.99 \times 10^{-3} (2 \times 10^3)$

$= 5 - 1.98 = 3.02$

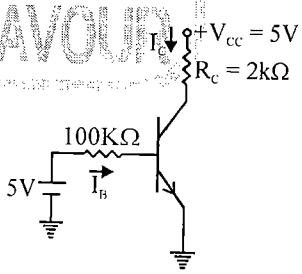
Q point = $I_{C(\text{active})} = 0.99\text{mA}$, $V_{CE\ (\text{active})} = 3.02$

Correct answer is (b)

19. Find out Q-point, where $\beta = 100$

Soln. Step-I: Assuming BJT inactive mode

$\therefore V_{BE} = 0.7$ volt, For Si



Step-II: Applying KVL in base-emitter loop

$5 - I_B \times 100 \times 10^3 - V_{BE} = 0$

$I_B = 4.3 \times 10^{-5}$

Step-III: $I_{C(\text{active})} = \beta I_B = 4.3 \times 10^{-5} \times 100 = 4.3\text{ mA}$

Step-IV: Applying KVL in collector-emitter loop for $I_{C(\text{sat})}$

$V_{CC} - 2 \times 10^3 I_C - V_{CE} = 0$

$\because V_{CE} = 0.2\text{V}$, this is the typically value in saturation mode for Si.

$\therefore 5 - 0.2 = 2 \times 10^3 I_C,\ I_{C(\text{sat})} = 2.4\text{ mA}$

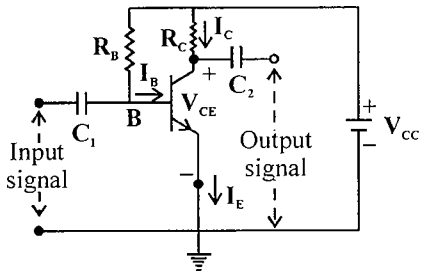
Step-V: $\because I_{C(\text{active})} > I_{C(\text{sat})}$

$\therefore$ BJT is in saturation region

Q-Point: $[I_{C(\text{saturation})} = 2.4\text{ mA}, V_{CE(\text{sat})} = 0.2\text{V}]$

CAREER ENDEAVOUR

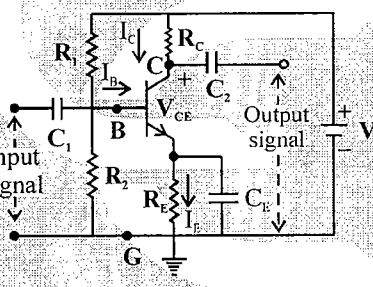
). Consider an n-p-n Si transistor used with fixed bias arrangement as shown in figure below. Assuming $\beta = 100$ and V_{CC} = 10 V, find the values of R_C and R_B required to set up the Q-point at I_C = 5 mA, V_{CE} = 5V. [Given : $I_B >> I_{CBO}$]



)ln. $R_c = \dfrac{V_{CC} - V_{CE}}{I_C} = \dfrac{10V - 5V}{5mA} = 1\,k\Omega$

$I_B \approx \dfrac{I_C}{\beta} = \dfrac{5mA}{100} = 50\,\mu A$ and $R_B = \dfrac{V_{CC} - V_{BE}}{I_B} = \dfrac{10V - 0.7V}{50\,\mu A} = 186\,k\Omega$

1. Consider the voltage divider bias, circuit of figure given below. Assuming $\beta = 260$, V_{CC} = 12V, $R_C = 560\,\Omega$, $R_E = 680\,\Omega$, $R_1 = 22k\Omega$ and $R_2 = 12\,k\Omega$ find the Q-point and the stabiity factor *S*.



)ln. $V = \dfrac{V_{CC}R_2}{R_1 + R_2} = \dfrac{12V \times 12k\Omega}{22k\Omega + 12k\Omega} = 4.24V$

$R_B = \dfrac{R_1 R_2}{R_1 + R_2} = 7.76\,k\Omega$

Now,

$$I_B = \frac{V - V_{BE}}{R_B + (1+\beta)R_E} = \frac{4.24V - 0.7V}{7.76\,k\Omega(1+260) \times 680\,\Omega} = 0.019\,mA$$

$$I_C \approx \beta I_B = 4.86\,mA$$

$$V_{CE} = V_{CC} - I_C(R_C + R_E) = 12V - 4.86\,mA(560\,\Omega + 680\,\Omega) = 5.97\,V$$

From stability factor,

$$S = (1+\beta) \cdot \frac{1 + \dfrac{R_B}{R_E}}{1 + \beta + \dfrac{R_B}{R_E}} = (1+260) \cdot \frac{1 + 7.76\,k\Omega / 680\,\Omega}{1 + 260 + 7.76\,k\Omega / 680\,\Omega} = 11.89$$

22. The rms value of full wave rectified waveform is : **[BHU 2014]**

(a) 0.636 times the peak value (b) 0.707 times the peak value

(c) 0.5 times the peak value (d) 0.373 times the peak value

Soln. We know that,

$$I_{rms} = \frac{I_0}{\sqrt{2}} = 0.707 \text{ times the peak value}$$

Correct option is (b)

23. The relation between current gains β and α of a transistor is : **[BHU 2014]**

(a) $\beta = \frac{\alpha}{(1+\alpha)}$ (b) $\beta = \frac{\alpha}{(1-\alpha)}$ (c) $\beta = \frac{(1+\alpha)}{\alpha}$ (d) $\beta = \frac{(1-\alpha)}{\alpha}$

Soln. We know that, $\alpha = \frac{I_C}{I_E}$; $\beta = \frac{I_C}{I_E}$ and $I_E = I_C + I_B$

$$\Rightarrow \quad \frac{I_E}{I_C} = 1 + \frac{I_B}{I_C} \Rightarrow \frac{1}{\alpha} = 1 + \frac{1}{\beta} \Rightarrow \frac{1}{\beta} = \frac{1}{\alpha} - 1$$

$$\Rightarrow \quad \beta = \frac{\alpha}{1-\alpha}$$

Correct option is (b)

24. The conductivity of Ge doped with phosphorus with impurity concentration $4.41\times10^{14}/\text{cm}^3$. (Given that for Ge the intrinsic carrier concentration is $2.5\times10^{13}/\text{cm}^3$, electron mobility $\mu_s = 3800$ cm²/V-sec and hole mobility $\mu_p = 1800$ cm²/V-sec) is **[BHU 2015]**

(a) 0.27 $(\Omega\text{-cm})^{-1}$ (b) 0.03 $(\Omega\text{-cm})^{-1}$ (c) 0.05 $(\Omega\text{-cm})^{-1}$ (d) 0.42 $(\Omega\text{-cm})^{-1}$

Soln. $\sigma = eNd\mu_e = 1.6\times10^{-19}\times4.41\times10^{14}\times3800 = 0.26(\Omega\text{-cm})^{-1}$

Correct option is (a)

CAREER ENDEAVOUR

25. A transistor in common base configuration has ratio of collector current to emitter current β and ratio of collector to base current α. Which of the following is true ? **[JEST 2016]**

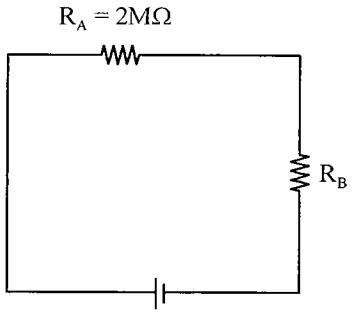
(a) $\beta = \frac{\alpha}{(\alpha+1)}$ (b) $\beta = \frac{(\alpha+1)}{\alpha}$ (c) $\beta = \frac{\alpha}{(\alpha-1)}$ (d) $\beta = \frac{(\alpha-1)}{\alpha}$

Soln. Given, $\beta = \frac{I_C}{I_E}$; $\alpha = \frac{I_C}{I_\beta}$

We know that, $I_C + I_\beta = I_E \Rightarrow 1 + \frac{I_\beta}{I_C} \Rightarrow \beta = \frac{\alpha}{1+\alpha}$

Correct option is (a)

26. In the circuit given below, a person measures 9.0 V across the battery, 3.0 V across the 2MΩ resistor R_A and 4.5 V across the unknown resistor R_B, using an ordinary voltmeter which has a finite input resistance r. Assuming that the battery has negligible internal resistance, it follows that (i) the resistance R_B and (ii) the input resistance r of the voltmeter are, in MΩ **[TIFR 2010]**



(a) $R_B = 3.0, r = 1.0$ (b) $R_B = 2.5, r = 7.5$ (c) $R_B = 4.0, r = 12.0$ (d) $R_B = 4.5, r = 10.0$

oln. According to given condition,

$$\frac{9\times\frac{2r}{2+r}}{R_B+\frac{2r}{2+r}} = 3$$

$$\Rightarrow \quad \frac{6r}{2R_B + R_B r + 2r} = 1 \Rightarrow 4r = 2R_B + R_B r$$

$$\Rightarrow \quad R_B = \left(\frac{4r}{2+r}\right) \quad \ldots (1)$$

And, $$\frac{9\times\frac{R_B r}{R_B + r}}{2+\frac{R_B r}{R_B + r}} = 4.5$$

$$\Rightarrow \quad \frac{2R_B r}{2R_B + 2r + R_B r} = 1 \Rightarrow R_B r = 2R_B + 2r$$

$$\Rightarrow \quad \frac{4r^2}{2+r} = 2\left(\frac{4r}{2+r} + r\right) \quad \left(\text{Putting } R_B = \frac{4r}{2+r}\right)$$

$$\Rightarrow \quad 2r^2 = 4r + 2r + r^2 \quad \Rightarrow r^2 - 6r = 0$$

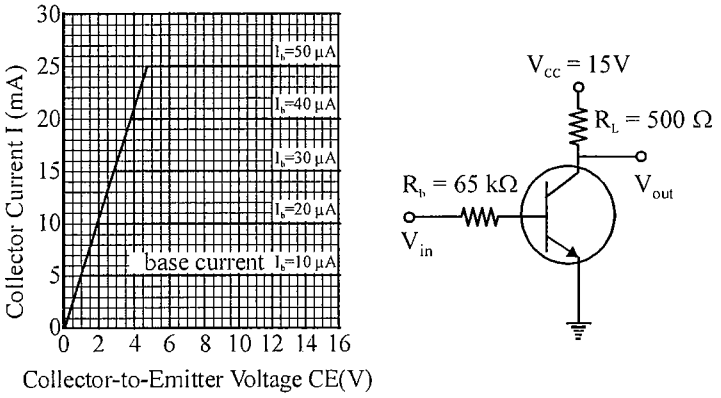
$$\Rightarrow \quad r = 6 \qquad \text{since, } r \neq 0$$

$$\therefore \quad R_B = \frac{4\times 6}{2+6} = 3$$

Correct option is (a)

CAREER ENDEAVOUR

27. A plot of the common-emitter characteristics of a silicon n-p-n transistor is shown below. Given this information, and assuming that there will be a 0.7 V drop across a forward biased silicon p-n junction, the approximate value of the output voltage V_{out} for an input voltage V_{in} = 2 V in the adjacent circuit will be



(a) 4 V (b) 6 V (c) 8 V (d) 10 V **[TIFR 2010]**

(e) 12 V (f) 14 V

Soln. Using KVL input,

$$V_{in} = I_B R_B + V_{BE}$$

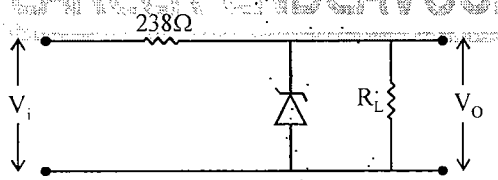
$$\Rightarrow \quad I_B = \frac{(2-0.7)}{65k\Omega} = 20\,\mu A$$

For $I_B = 20\mu A$ from graph we get $I_C = 10\,mA$

Now, $V_{CC} = I_C R_C + V_0 \Rightarrow V_0 = 15 - 10\times 500\times 10^{-3} = 15 - 5 = 10V$

Correct option is (d)

28. The voltage regulator circuit shown in the figure has been made with a Zener diode rated at 15V, 200mW. It is required that the circuit should dissipate 150mW power across the fixed load resistor R_L.



For stable operation of this circuit, the input voltage V_i must have a range **[TIFR 2012]**

(a) 17.5 V to 20.5 V (b) 15.5 V to 20.5 V (c) 15.5 V to 22.5 V

(d) 17.5 V to 22.5 V (e) 15.0 V to 20.5 V

Soln. From figure, we can write,

$$V_L = \frac{V_i R_L}{238 + R_L}$$

to work as voltage regulator this voltage should be greater than V_z.

$$\therefore \quad V_{i\min} = \frac{238 + R_L}{R_L} \times v_z = \frac{238 + 1500}{1500} \times 15 = 17.5\,V$$

$$\left[\text{Given: } \frac{15}{R_L}\times 15 = 150\times 10^{-3}\,\Omega \Rightarrow R_L = 1500\,\Omega\right]$$

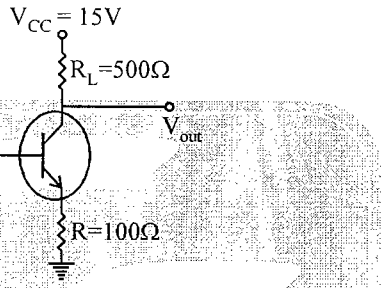
And for $V_{i\,max}$,

$$V_{i\,max} = I_S \times R_s + V_z = \left(I_{z\,max} + I_L\right)R_s + V_z$$

$$= \left(\frac{200}{15}\,mA + 10\,mA\right)\times 238 + V_z = 5.5 + 15 = 20.5\,volt$$

Correct option is (a)

9. The circuit depicted on the right has been made with a silicon n-p-n transistor. Assuming that there will be a 0.7V drop across a forward-biased silicon p-n junction, the power dissipated across the transistor will be, approximately, **[TIFR 2013]**



(a) 53 mW (b) 94 mW (c) 17 mW (d) 67 mW

oln. From input loop,

$$2 = V_{BE} + I_E R_E$$

$$I_E = \frac{2-0.7}{100} = 13\,mA$$

From output loop, $V_{CC} = I_C R_C + V_{CE} + I_E R_E \quad (I_E \simeq I_C)$

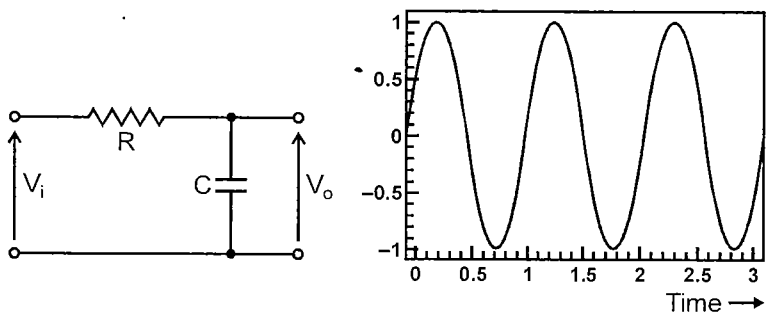
$$V_{CE} = 15 - 600\times 13\times 10^{-3} = 7.2$$

Therefore, power dessipated across the transistor

$$P = 7.2\times 13 \simeq 94\,mW$$

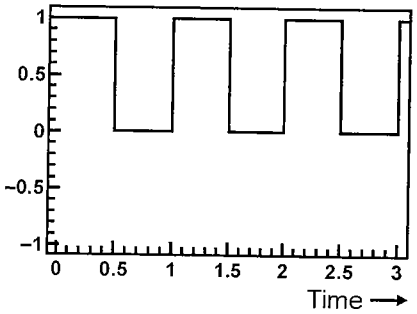
Correct option is (b)

0. For the circuit depicted on the right, the input voltage V_i is a simple sinusoid as shown below, where the time period is much smaller compared to the time constant of this circuit. **[TIFR 2016]**

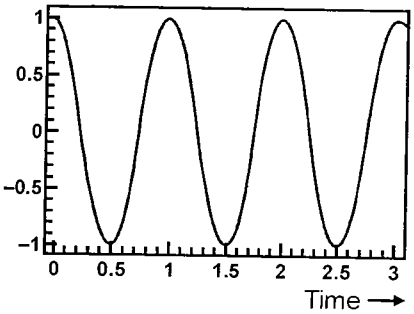


The voltage V_o across C is best represented by

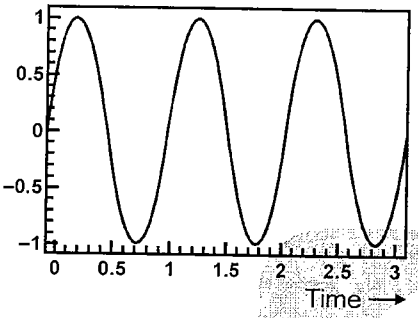
(a)



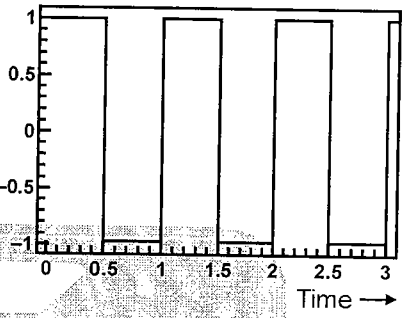
(b)



(c)



(d)



Soln. $\frac{V_0}{Vi} = \frac{1}{1+RCS} = \frac{1}{1+j\omega RC}$

$\frac{V_0}{Vi} = \frac{1}{RCS} \Rightarrow RCS >> 1 \Rightarrow j\omega RC >> 1$

$\omega RC >> 1$

$2\pi fRC >> 1$

$\tau = RC$

$\boxed{\tau >> T}$ (Integrator)

Correct option is (b)

31. Consider a sawtooth waveform which rises linearly from 0 Volt to 1 Volt in 10 ns and then decays linearly to 0V over a period of 100 ns. Find the r.m.s. voltage in units of milliVolt? **[TIFR 2016]**

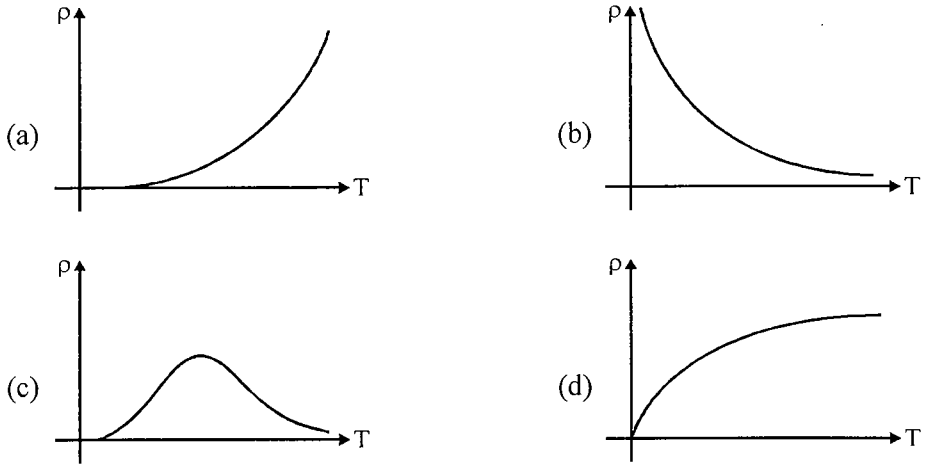
Soln. $V_{rms} = \sqrt{\frac{1}{T}\int_0^T V^2 dt} = \sqrt{\frac{1}{110}\left[\int_0^{10}\left(\frac{t}{10}\right)^2 dt + \int_{10}^{110}\left(\frac{110-t}{100}\right)^2 dt\right]}$

$= \sqrt{\frac{1}{110}\left[\frac{1000}{300} + 100\times\frac{1}{3}\right]} = \sqrt{\frac{1}{3}} = 577 \text{ millivolt}$

Correct answer is (577)

CAREER ENDEAVOUR

2. Which of the following best represents the temperature dependence of the resistivity of an undoped semiconductor ? **[BHU 2012]**



oln. The undoped semiconductor, the resistivity (ρ) decreases with increase temperature (T).
Hence, correct option is (b).

3. The mean kinetic energy of the conduction electrons in metals is ordinary much higher than kT because **[BHU 2012]**
(a) electrons have many more degrees of freedom than atoms do
(b) the electrons and the lattice are not in thermal equilibrium
(c) the electrons form a degenerate Fermi gas
(d) electrons in metals are highly relativistic

oln. The mean kinetic energy of the conduction electrons in metal is ordinary much higher than kT because the electrons form a degenerate fermi gas.
Hence, correct option is (c).

4. Which of the following statements concerning the electrical conductivities at room temperature of a pure copper sample and a pure silicon sample is NOT true ? **[BHU 2012]**
(a) If the temperature of the copper sample is increased, its conductivity will decrease
(b) If the temperature of the silicon sample is increased, its conductivity will increase
(c) The addition of an impurity in the copper sample always decreases its conductivity
(d) The addition of an impurity in the silicon always decreases its conductivity

oln. Out of above options, the option (d) "The addition of an impurity in the silicon always decreases its conductivity" is not type.
Hence, correct option is (d).

5. The Hall effect is used in solid-state physics to measure **[BHU 2012]**
(a) ratio of charge to mass
(b) magnetic susceptibility
(c) the sign of the charge carriers
(d) the width of the gap between the conduction and valence bands

oln. The Hall effect is used in solid state physics to measure the sign of the charge carriers.
Hence, correct option is (c).

36. Lattice forces affect the motion of electrons in a metallic crystal, so that the relationship between the energy E and wave number k is not the classical equation $E = \frac{\hbar^2 k^2}{2m}$, where m is the electron mass. Instead, it is possible to use an effective mass m^* given by which of the following ? **[BHU 2012]**

(a) $m^* = \frac{1}{2}\hbar^2 k\left(\frac{dk}{dE}\right)$ (b) $m^* = \frac{\hbar^2 k}{\left(\frac{dk}{dE}\right)}$ (c) $m^* = \hbar^2 k\left(\frac{d^2k}{dE^2}\right)^{1/3}$ (d) $m^* = \frac{\hbar^2}{\left(\frac{d^2E}{dk^2}\right)}$

Soln. The effective mass (m^*) is given by $m^* = \frac{\hbar^2}{\left(\frac{d^2E}{dk^2}\right)}$

Hence, correct option is (d).

37. In the Kronig-Penney model, discontinuities in E versus k curve occur for **[BHU 2012]**

(a) $k = \frac{n\pi}{a}$ (b) $k = \frac{8n\pi}{a}$ (c) $k = \frac{n^2\pi}{a}$ (d) $k = \frac{n\pi}{4a}$

Soln. In the Kronig-Penney model, discontinuities in E versus k curve occur for $k = \frac{n\pi}{a}$

Hence, correct option is (a).

38. The Fermi energy of silver is 5.51 electron volt. The average energy of the free electrons in silver at 0 °K is given by

(a) 4.205 eV (b) 0.864 eV (c) 3.306 eV (d) 9.425 eV

Soln. $E_f = 5.51\,\text{eV}$

The average energy $\bar{E}_0 = \frac{3}{5}E_f = \frac{3}{5} \times 5.51\,\text{eV} = 3.306\,\text{eV}$

Hence, correct option is (c).

CAREER ENDEAVOUR

39. Which of the following statements is true about the effective mass of electron in crystal ? **[BHU 2013]**

(a) It is positive near the top of energy band (b) It is negative near the bottom of energy band
(c) It is constant through out the band (d) It is negative near the top of energy band

Soln. Out of the above options, "It is negative near the top of energy band" is correct.
Hence, correct option is (d).

40. Indicate the statement about the semiconductors which is false ? **[BHU 2013]**

(a) n-type semiconductors are obtained by doping phosphorus into silicon
(b) The conductivity of all semiconductors always increases with temperature
(c) p-type semiconductors are obtained by doping boron into silicon
(d) Intrinsic semiconductors are insulators at $T = 0$ °K

Soln. Out of the above options, the option (b) the conductivity of all semiconductors always increases with temperature is false.
Hence, correct option is (b).

1. The electrical conductivity of a metal in terms of mass (m), charge (e), collision time (τ) and concentration (n) of electrons is given by **[BHU 2013]**

(a) $\frac{me\tau}{n}$ (b) $mne\tau$ (c) $\frac{ne^2\tau^2}{m}$ (d) $\frac{ne^2\tau}{m}$

Soln. The electrical conductivity is given by $\sigma = \frac{ne^2\tau}{m}$.

Hence, correct option is (d).

2. Which of the statements is false ? **[BHU 2013]**

(a) A semiconductor exhibits negative temperature coefficient of resistivity

(b) In an n-type semiconductor, as the density of donor atoms is increased, the Fermi level shifts towards the valence band

(c) Mobility carriers in a p-type semiconductors are holes

(d) In an intrinsic semiconductors, the Fermi level lies midway in the forbidden gap

Soln. "In an n-type semiconductor, as the density of donor atoms is increased, the fermi level shifts towards the valence band" is false.

Hence, correct option is (b).

3. Electrical conductivity of insulators is in the range : **[BHU 2014]**

(a) $10^{-10}(\Omega - \text{mm})^{-1}$ (b) $10^{-10}(\Omega - \text{cm})^{-1}$ (c) $10^{-10}(\Omega - \text{m})^{-1}$ (d) $10^{-8}(\Omega - \text{m})^{-1}$

Soln. Electrical conductivity of intensity is in the range $10^{-8}(\Omega - \text{m})^{-1}$.

Hence, correct option is (d).

4. Energy band gap size for semiconductors is in the ranges **[BHU 2014]**

(a) 1-2 eV (b) 2-3 eV (c) 3-4 eV (d) greater than 4 eV

Soln. Energy band gap size for semiconductors is in the ranges 1-2 eV.

Hence, correct option is (a).

5. Flow of electron is affected by which of the following : **[BHU 2014]**

(a) thermal vibration only (b) impurity atom only

(c) crystal defects only (d) by all of (a), (b) and (c)

Soln. Flow of electron is affected by normal vibrations, impurity and crystal defects.

Hence, correct option is (d).

6. Fermi energy level for intrinsic semiconductors lies : **[BHU 2014]**

(a) at the middle of the band gap (b) close to the conduction band

(c) close to valence band (d) inside valence band

Soln. Fermi-level for intrinsic semiconductors lies at the middle of the band gap.

Hence, correct option is (a).

7. In one-dimensional periodic chain of atom with lattice parameter 'a' has first brillouin zone at :

(a) a (b) $2a$ (c) π/a (d) $2\pi/a$ **[BHU 2014]**

Soln. First Brillouin zone lies at π/a.

Hence, correct option is (c).

8. Mobility of holes as compared to mobility of electrons in intrinsic semiconductors is : **[BHU 2014]**

(a) equal (b) greater (c) less (d) cannot be defined

Soln. Mobility of holes as compared to mobility of electrons in intrinsic semiconductors is less because the mass of hole is slightly larger than electron so its velocity will be smaller.

Hence, correct option is (c).

49. According to Hall effect if a conducting materials is placed in a uniform magnetic field and a current is passed, voltage is found to develop at : **[BHU 2014]**

(a) parallel to the current
(b) parallel to the magnetic field
(c) perpendicular to the magnetic field and current
(d) 45 degrees to the magnetic field and current

Soln. According to Hall effect, if a conducting material is placed in uniform magnetic field and a current is passed, voltage is found to develop at perpendicular to the magnetic field and current.
Hence, correct option is (c).

50. In the periodic potential model, discontinuities in E versus k curve occur for **[BHU 2015]**

(a) $k=\frac{n\pi}{a}$ (b) $k=\frac{8n\pi}{a}$ (c) $k=\frac{n^2\pi}{a}$ (d) $k=\frac{n\pi}{4a}$

Soln. For periodic potential model, the discontinuous in E versus k curve occur for $k=\left(\frac{n\pi}{a}\right)$.

Hence, correct option is (a).

51. Which of the following statements is true about the velocity of electron in an energy band ?
(a) It is constant throughout the band **[BHU 2015]**
(b) It is maximum at the bottom of the band
(c) It is maximum at the inflection point of the $E(k)$ curve
(d) It is maximum at the top of the band

Soln. Out of above options, only option (c). "It is maximum at the inflection point of the $E(k)$ curve" is true for the velocity of electrons in an energy band.
Hence, correct option is (c).

52. According to the Wiedemann-Franz law, the ratio of thermal to electrical conductivity of any metal is given by **[BHU 2015]**

(a) $\left(\frac{\pi kT}{2e}\right)^2$ (b) $\frac{\pi^2}{3}\left(\frac{k}{e}\right)^2 T$ (c) $\frac{\pi^2}{3}\left(\frac{kT}{e}\right)^2$ (d) $\left(\frac{\pi k}{e}\right)^2 T$

Soln. According to Wiedemann-Franz law, $\frac{k}{\sigma}=\frac{\pi^2}{3}\left(\frac{k_B}{e}\right)^2 T$.

Hence, correct option is (b).

53. Indicate the false statement about the semiconductors **[BHU 2015]**
(a) All intrinsic semiconductors are insulators at T = 0 °K
(b) At very high temperature all semiconductors become intrinsic semiconductors
(c) The conductivity of all semiconductors always increases with temperature
(d) N-type semiconductors are obtained by doping phosphorus into silicon

Soln. Out of the given options, the option (c) the conductivity of all semiconductors always increases with temperature.
Hence, correct option is (c).

54. The density of carriers in a pure semiconductor is proportional to **[BHU 2015]**

(a) $\exp\left[-\frac{E_g}{kT}\right]$ (b) $\exp\left[-\frac{2E_g}{kT}\right]$ (c) $\exp\left[-\frac{E_g}{kT^2}\right]$ (d) $\exp\left[-\frac{E_g}{2kT}\right]$

oln. The density of carriers in a pure semiconductor is proportional to $\exp\left[-\frac{E_g}{2k_BT}\right]$.

Hence, correct option is (d).

5. The reverse saturation current in germanium *p-n* diode is of the order of **[BHU 2015]**
(a) nA (b) μA (c) mA (d) ampere

oln. The reverse saturation current in germanium *p-n* junction diode is of the order of microampere (μA)
Hence, correct option is (a).

6. The Fermi level in an *n*-type semiconductor at 0 °K **[BHU 2015]**
(a) lies below the donor level
(b) lies in the conduction band
(c) lies half-way between the conduction band and donor level
(d) coincides with the intrinsic Fermi level

oln. The Fermilevel in an *n*-type semiconductor at 0 °K lies half-way between the conduction band and donor level.
Hence, correct option is (c).

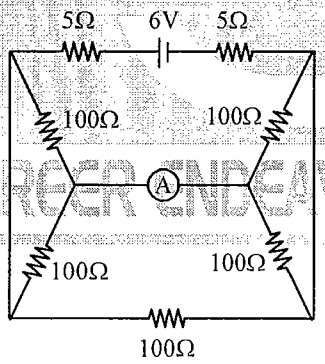
7. If the Fermi energy of a metal at 0 °K, 10 eV, the mean energy of the electrons in the metal at 0 °K is
(a) 6 eV (b) 5 eV (c) 1.5 eV (d) 2 eV

oln. The average energy of electrons in the metal at 0 °K, **[BHU 2015]**

$$\bar{E} = \frac{3}{5}E_f = \frac{3}{5} \times 10 \text{ eV} = 6 \text{ eV}.$$

Hence, correct option is (a).

8. The current read by the ammeter (A) in the circuit given below is **[TIFR 2011]**

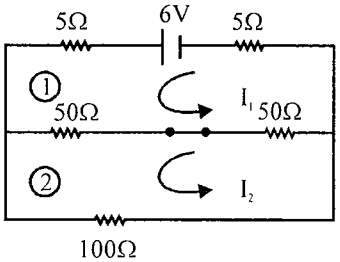


(a) 27.3 mA (b) 100.0 mA (c) 54.5 mA (d) 50.0 mA

oln. ⇒ 100Ω and 100Ω are in parallel

⇒ Simplify circuit

⇒ Ammeter used to measure current, make short circuit



Apply mesh analysis

Loop (1) KVL:

$$6 = 5I_1 + 50(I_1 - I_2) + 50(I_1 - I_2) + 5I_1$$

$$6 = 110I_1 - 100I_2 \quad \ldots (1)$$

Loop (2) KVL:

$$100I_2 + 50(I_2 - I_1) + 50(I_2 - I_1) = 0$$

$$200\,I_2 - 100\,I_1 = 0$$

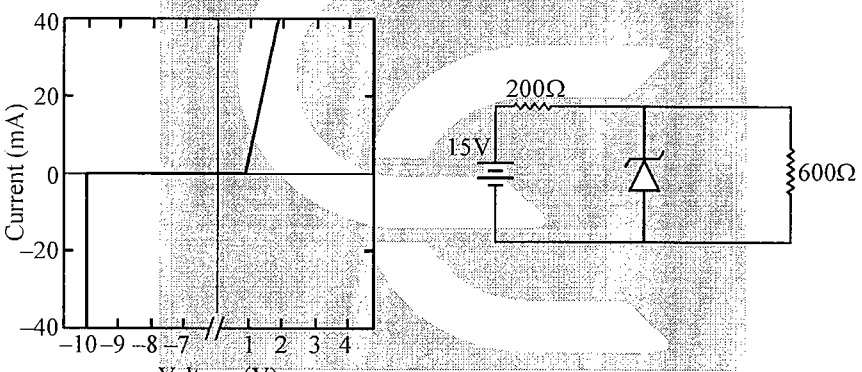
$$I_1 = \frac{1}{10};\quad I_2 = \frac{1}{20}$$

Ammeter reading = $I_1 - I_2$

$$I_1 - I_2 = 50\text{ mA}$$

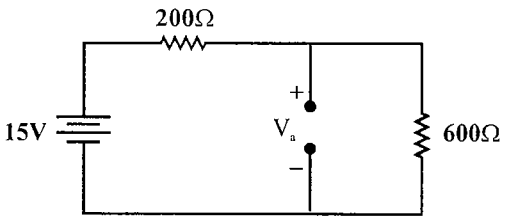
Correct option is (d)

59. The figure on the right shows the current-voltage characteristics of a diode over a range of voltage and current where it is safe to operate the diode. When this diode is used in the circuit on the extreme right, the approximate current, in mA, through the diode will be **[TIFR 2013]**



(a) 0 (b) 8.3 (c) 16.7 (d) 25

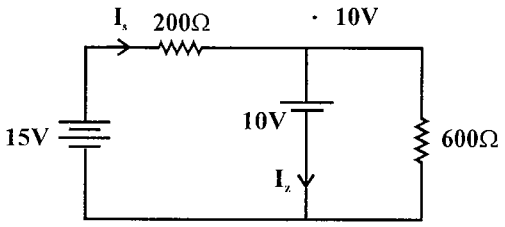
Soln.



$V_a = ?$, find V_a.

$$V_a = \frac{15 \times 600}{800}\text{V} \Rightarrow V_a = 11.25\text{V}$$

$\Rightarrow$ $V_a \geq V_z$ means zener on (since, V_z = 10 V from figure)



$$I_{600\Omega} = \frac{10}{600} \Rightarrow I_{600\Omega} = 16.67\ \text{mA}$$

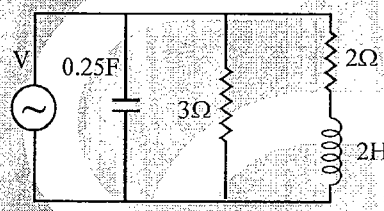
$$I_s = \frac{15-10}{200} = 25\text{mA}$$

$$I_s = I_z + I_{600\Omega}$$

$$25 = I_z + 16.67$$

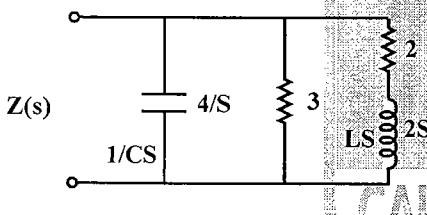
$$I_z = 8.33\text{mA}$$

). Find the resonance frequency (rad/sec) of the circle shown in the figure below **[JEST 2014]**



(a) 1.41 (b) 1.0 (c) 2.0 (d) 1.73

oln.



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$$\Rightarrow \quad z(s) = \frac{(2+2S)\times 3}{3+2+2S} \Rightarrow \left.\frac{6+6S}{5+2S}\right\} (3\Omega) \| (2+2S) \qquad \text{(Here, } S = j\omega\text{)}$$

$$\Rightarrow \quad z(s) = \frac{\frac{4}{S}\times\left(\frac{6+6S}{5+2S}\right)}{\frac{4}{S}+\left(\frac{6+6S}{5+2S}\right)} \Rightarrow \frac{\frac{4}{S}\times\left(\frac{6+6S}{5+2S}\right)}{\frac{4(5+2S)+(6+6S)S}{S(5+2S)}}$$

$$\Rightarrow \quad z(s) = \frac{\frac{4}{S}\times\left(\frac{6+6S}{5+2S}\right)}{\frac{20+8S+6S+6S^2}{S(5+2S)}} \Rightarrow Z(S) = \frac{24+24S}{6S^2+14S+20}$$

$$\Rightarrow \quad z(s) = z(j\omega) = \frac{24+24j\omega}{-6\omega^2+14j\omega+20} \text{ divide by (2)}$$

$$= \frac{12+12j\omega}{-3\omega^2+7j\omega+10}$$

$$\Rightarrow \quad z(j\omega)=\frac{12+12j\omega}{10-3\omega^2+7j\omega}\left\{\frac{(10-3\omega^2)-7j\omega}{10-3\omega^2-7j\omega}\right\}$$

Considering-numerator

$$\left\{12(10-3\omega^2)+12j\omega(10-3\omega^2)-12\times7j\omega+12\times7\omega^2\right\}$$

Since, at resonance frequency both I and V remain same phase.
Therefore, the imginary part of the numerator should be zero.

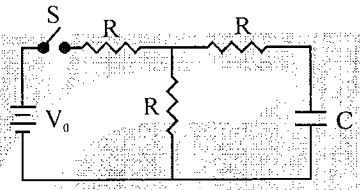
$$\Rightarrow \quad 12j\omega(10-3\omega^2)-12\times7j\omega=0 \Rightarrow 12(10-3\omega^2)=12\times7 \Rightarrow 10-7=3\omega^2 \Rightarrow 3=3\omega^2$$

$$\Rightarrow \quad \omega=\pm1 \text{ rad/sec} \Rightarrow \omega=1 \text{ rad/sec}$$

Discard negative frequency

Correct option is (b)

61. A capacitor C is connected to a battery V_0 through three equal resistors R and a switch S as shown below: **[JEST 2012]**



The capacitor is initially uncharged. At time t = 0, the switch S is closed. The voltage across the capacitor as a function of time 't' for t > 0 is given by

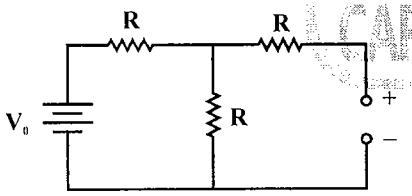
(a) $(V_0/2)(1-e^{-t/2RC})$ (b) $(V_0/3)(1-e^{-t/3RC})$

(c) $(V_0/3)(1-e^{-3t/2RC})$ (d) $(V_0/2)(1-e^{-2t/3RC})$

Soln. At t = 0, capacitor is uncharged

$$\therefore \quad V_c(0)=0$$

At $t\to\infty$ capacitor fully charged, behaving as open circuit.



$$V_c(\infty)=\frac{V_0}{2}$$

$$\boxed{V_c(t)=V_c(\infty)+\left[V_c(0)-V_c(\infty)\right]\cdot e^{-\frac{t}{\tau}}}$$

For $t\geq0$,

$$V_c(0^-)=V_c(0)=V_c(0^+)=0$$

Property of capacitor,

$$\tau=\text{Req}\cdot C \Rightarrow \tau=\frac{3R}{2}\cdot C \quad \because \quad \text{Req}=R\,||\,R+R=\frac{3R}{2}$$

$$V_c(t)=\frac{V_0}{2}+\left[0-\frac{V_0}{2}\right]\cdot e^{-\frac{t}{\frac{3R}{2}C}} \Rightarrow V_c(t)=\frac{V_0}{2}\left[1-e^{\frac{-2t}{3RC}}\right]$$

Correct option is (d)

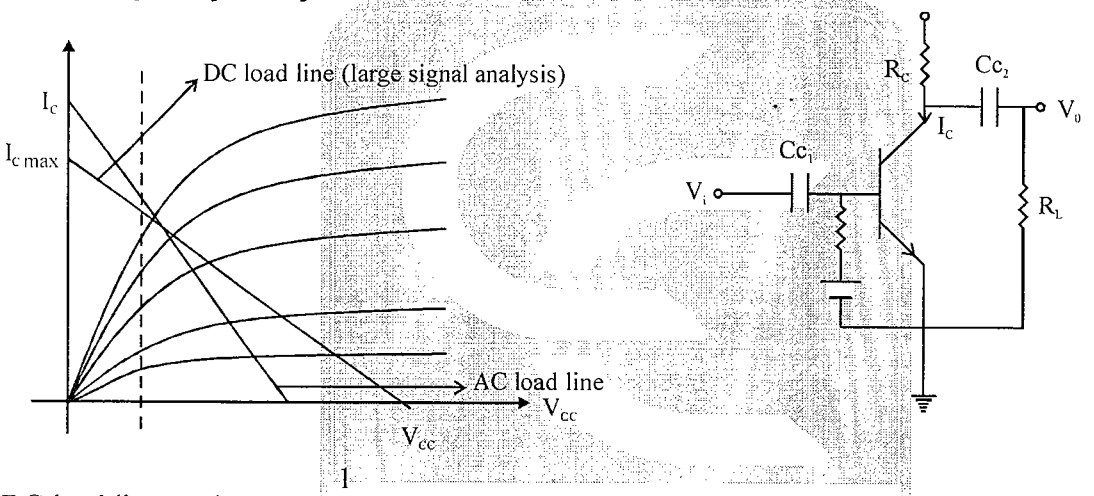
Chapter 5

OP-AMP, SINGLE STAGE AMPLIFIERS & OSCILLATORS

Single stage BJT amplifiers:

There are basically three configurations for implementing single stage BJT amplifiers
(i) The common emitter (ii) The common collector (iii) The common base configuration
All there are studied below utilizing the same basic structure with the same biasing arrangement.

Low Frequency Analysis:



DC load line $\rightarrow$ slope $= -\frac{1}{R_C}$

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For AC analysis (Small signal analysis) : All the DC source are zero voltage source short circuit current source open circuit coupling capacitor and by pass capacitor are VV large.

Cc_1, Cc_2, act on AC short.

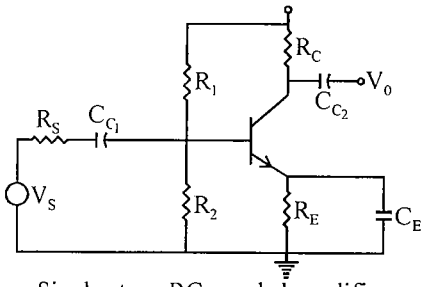
C_E by pass capacitor

$R'_L = R_C \parallel R_L < R_C$

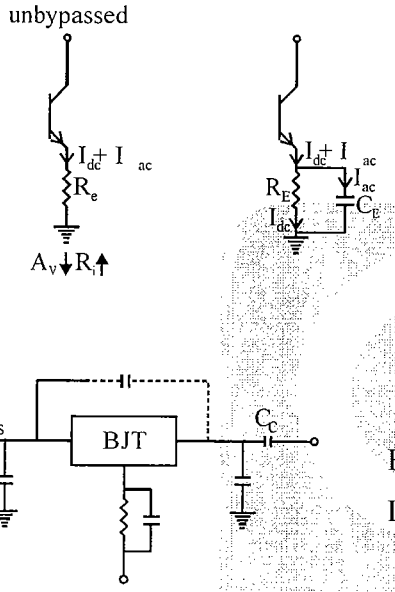
AC load line slope $= -\frac{1}{R'_L} = -\left[\frac{1}{R_C} + \frac{1}{R_L}\right]$

Slope of AC line > slope of DC line.

Single stage RC coupled amplifier: C_{C_1}, C_{C_2} are called coupling capacitor. They practically offer short circuited path to ac but for dc they act as open circuit.



Single stage RC coupled amplifier



C_E v.v.high, Z_{C_E} = v.v. low compared to R_E

C_{C_2} act as S.C. for A.C.

High $C_{C_1} = C_{C_2}, C_E \rightarrow \mu F$

Low $C_D, C_T, C_W \rightarrow PF$

During A.C. Analysis:

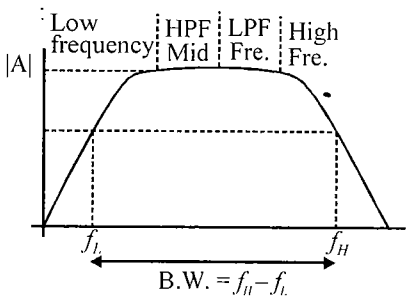
Lower frequency region	*Middle frequency region*	*High frequency region*
$F\downarrow \lvert Z\rvert\uparrow$	$\lvert Z_C\rvert$ *dc* decided by	$f\uparrow \lvert Z_C\rvert\downarrow$
$C \rightarrow O.C.$	value of C	$C \rightarrow S.C$
gain $\downarrow$ due to	$C_{C_1}, C_{C_2}, C_Z \rightarrow S.C.$	gain $\downarrow$ due to C_D, C_T, C_W
C_{C_1}, C_{C_2}, C_E	$C_D, C_T, C_W, C_S \rightarrow O.C$	and C_S

Note: At d.c. it will be out.

Frequency response curve gain versus frequency

At $f = f_H$ or f_L i.e. 70% of maximum

$P_{out} = \frac{P_{max}}{2}$ (Power). Half power band width



RC coupled amplifier:

1. It is audio frequency voltage amplifier
2. Operating frequency range 20Hz to 20 Khz.

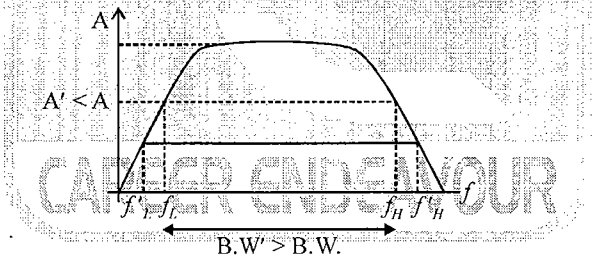
- Single stage RC coupled amplifier introduce a phase shift of 180° and two stages introduces 360° of 0°.
- Coupling capacitor (C_{C_1} and C_{C_2}) is also called d.c. blocking capacitor.
- It is used to couple A.C. signal and simultaneously block d.c. signal and d.c. biasing current.
- By using emitter resistance R_E without a by pass capacitor, there will be a negative feedback across R_E and this reduces the voltage gain of amplifier.
- By pass capacitor (C_Z or C_E) is used to by pass the A.C. signal current through it and for d.c. current or biasing current C_2 is open. By using C_Z negative feedback across R_E {during A.C. analysis only} is dominated therefore voltages increases but input resistance decreases. In an amplifier for better performance always R_E and C_E combination is used.

requency response curve:

- Fall of gain in low frequency region is due to C_{C_1}, C_{C_2} and C_Z
- In mid frequency region all coupling and by pass capacitor will be treated as a.c. short and all in capacitance {C_D and C_T} wiring capacitor C_W, stray capacitance C_S will be treated as A.C. open the gain of amplifier is maximum at mid frequency and it almost independent of frequency. Therefore amplifier analysis is generally done at mid frequency.
- Frequency of gain at high frequency is due to the effect of
  (i) J^n capacitor {C_D and C_T} (ii) C_W (iii) C_S (iv) Early effect
- High frequency fall is mainly due to J^n capacitance.

andwidth:

- It is the band of input signal frequency where the gain of amplifier is almost a constant.
- A larger band width indicate better reproduction of input signal
- In an amplifier gain band width product is constant i.e. one increases and other decreases.

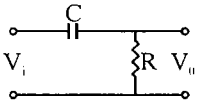


gain decreases due to negative feedback

Disadvantage: Disadvantage of single stage RC coupled amplifier is smaller gain band width product.

Low frequency analysis of RC coupled amplifier:

Amplifier behaves as a HPF



$$\frac{V_0}{V_i} = \frac{R}{R + \frac{1}{SC}} = \frac{1}{1 + \frac{1}{SCR}}$$

$$A = \frac{1}{1 - J/\omega RC} \qquad \boxed{A = \frac{1}{1 - J\omega_L/\omega} = \frac{1}{1 - Jf_L/f}}$$

$\omega_L = 2\pi f_L = \frac{1}{RC}$, $\phi = \tan^{-1} f_L / f$ {leading} total phase shift

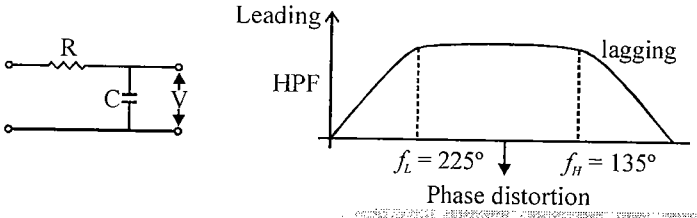
$\phi_T = 180° + \tan^{-1}\frac{f_L}{f}$ at $f = f_L$, $\phi = 180° + 45° = 225°$

$A = \frac{V_0}{V_i} = \frac{1/SC}{R + 1/SC}$, $A = \frac{1}{1+SCR} = \frac{1}{1+J\omega CR}$, $A = \frac{1}{1+J\omega/\omega H} = \frac{1}{1+J_f/f_H}$

$\phi = -\tan^{-1} f/fH$ {lagging} total phase shift

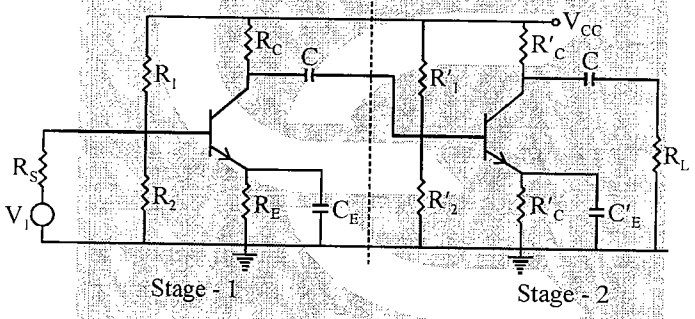
$\phi_T = 180° - \tan^{-1} f/f_H = 180° - 45° = 135°$

High frequency region: Behaves as LPF



Cascaded or multistage amplifier:

1. Amplifier are connected in cascade to get larger gain bandwidth product.



2. When amplifier are connected such that output of one is given to input of another they are said to be cascaded.
3. When amplifier are cascaded proper impedance matching must be provided in between the stages so that
   (i) Maximum power will be transferred from one to another stage
   (ii) Output will not be distorted.
   Note: If mismatch is more in amplifier output will be highly distorted.
4. Three different type of coupling
   (i) RC coupling {for voltage amplifier}
   (ii) Direct coupling {for direct coupled amplifier}
   (iii) Transformer coupling for proper amplifier
5. In multistage amplifier gain bandwidth product is constant.
6. In multistage amplifier bandwidth is reduced.
7. The gain bandwidth product of two stage amplifier is greater than that of single stage amplifier.

Bandwidth of multistage amplifier:

B.W.* = B.W. of multistage amplifier

B.W. = B.W. of single stage amplifier

B.W.* = $f_H^{\pm} - f_L^*$

B.W. = $f_H - f_L$

f_H^* = high 3dB cutoff frequency of multistage amplifier

f_L^* = low 3dB cutoff frequency of multistage amplifier

f_H = high 3dB cutoff frequency of single stage amplifier

f_L = low 3dB cutoff frequency of single stage amplifier

n = identical non-interacting stages of cascade.

"proper impedance matching"

Rise time: $t_r = \frac{0.35}{f_H} = \frac{0.35}{B.W.}$

Disadvantage: $t_r \uparrow$ when $B.W. \uparrow$ and response will sluggish.

$t_r^* = 1.1 \times \sqrt{t_{r_1}^2 + t_{r_2}^2 + ...t_{r_n}^2}$, t_{r_0} = Rise time of signal

$$\boxed{t_r^* = 1.1 \times \sqrt{t_{r_0}^2 + t_{r_1}^2 + t_{r_2}^2 + ... + t_{r_n}^2}}$$

Oscillators:

A transistor with proper positive feed back can act as an oscillator i.e. It can generate oscillators without any external source.

The circuit needs only a quick trigger signal to start the oscillators. Once the oscillations have started, no external signal source is needed.

In order to get continuous undamped output from the circuit, the following condition must be met

$$\boxed{\beta A = 1} \qquad ...(1)$$

where, A = voltage gain of amplifier without feedback, β = feedback fraction

This relation is called Bark hauren criterion

Mathematical explanation: The voltage gain of positive feedback amplifier is given below:

$A_{Vf} = \frac{A}{1 - \beta A}$, if $\beta A = 1$, then $A_{Vf} = \infty$

Oscillator can produces sinusoidal and non-sinusoidal [square] waves.

Different type of transistor oscillators:

1. The difference between these oscillators lies in the method by which energy is supplied to the tank circuit to meet the losses.
2. Tunned collector oscillator
3. Colpitt's oscillator
4. Hartely oscillator
5. Phase shift oscillator

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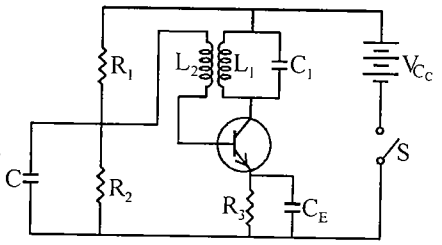
6. Wien bridge oscillator
7. Crystal oscillator

Tunned collector oscillator: The frequency of oscillator depend upon the values of L_1 and C_1 and is given by $f = \frac{1}{2\pi\sqrt{L_1C_1}}$



Tunned collector oscillator

- Figure shows the circuit of tunned collector oscillator. It contains tunned circuit $L_1 - C_1$ in the collector.
- The biasing is provided by potential divider arrangement.
- The capacitor C connected in the base circuit provides low reactance path to the oscillations.

Circuit Operation:

- When switch S is closed, collector current starts increasing and charges the capacitor C_1.
- When this capacitor is fully charged, it discharges through coil L_1, setting up oscillations of frequency $f = \frac{1}{2\pi\sqrt{L_1C_1}}$
- A phase shift of 180° is created between the voltage of L_1 and L_2 due to the transformer action.
- A further phase shift of 180° takes place between base emitter and collector circuit due to transistor property. As a result, the energy feedback to the tank circuit is in phase with the generated oscillations.

Colpitt's Oscillator:

1. The tank circuit is made up C_1, C_2 and L. The frequency of oscillations: $f = \frac{1}{2\pi\sqrt{LC_{eq}}}$

where. $C_{eq} = \frac{C_1C_2}{C_1 + C_2}$

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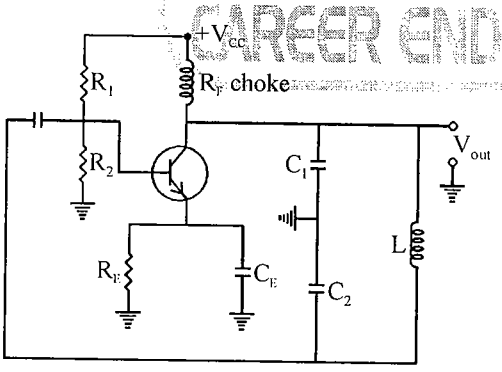
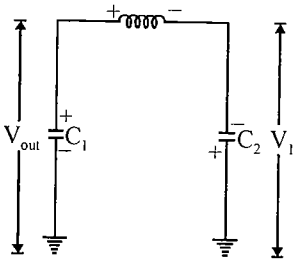


Colpitt's oscillator



Feedback circuit

Circuit operation:

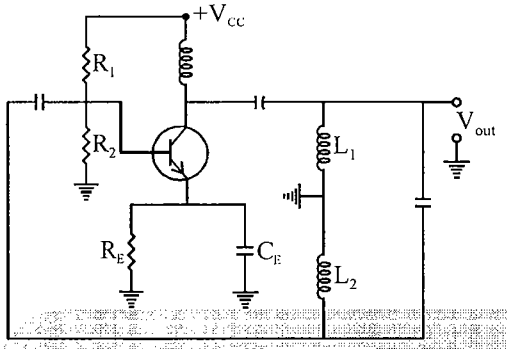
1. When the circuit is turned on, the capacitor C_1 and C_2 are charged. The capacitor discharge through L_1 setting up oscillations of frequency
2. The output voltage of the amplifier appears across C_1 and feedback voltage is developed across C_2.
3. Feedback fraction: $b = \frac{Vf}{V_{out}} = \frac{X_{C_2}}{X_{C_1}} = \frac{C_1}{C_2}$

Hartley oscillator:

The hartley oscillator is similar to colpitt's oscillator only feedback circuit changed.

The frequency of oscillations

$$\boxed{f = \frac{1}{2\pi\sqrt{CL_{eq}}}}$$ where $L_{eq} = L_1 + L_2 + 2M$, M = mutual inductance between L_1 and L_2.



Hartley oscillator

Note: $L_1 - L_2 - C$ is also the feedback network that produces a phase shift of 180°.

Wien bridge oscillator:

The wien bridge oscillator is the most frequency used type of audio oscillator as the output is free from circuit fluctuations and ambient temperature.

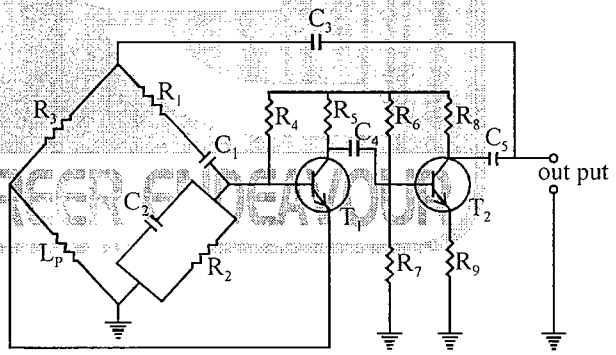
It is the standard oscillator circuit for all frequency in the range of 10Hz to about 1Mhz

The frequency of oscillations

$$f = \frac{1}{2\pi\sqrt{R_1C_1R_2C_2}},$$

if $R_1 = R_2 = R$ and $C_1 = C_2 = C$

then $f = \frac{1}{2\pi RC}$



Wien bridge oscillator

It is essential a two stage amplifier with R-C bridge circuit.

Resistance R_3 and tungsten lamp L_p are used to stabilize the amplitude of the output

Circuit operation:

The transistor T_1 serves as an oscillator and amplifier while the other transistor T_2 serves as an inverter (i.e. to produce a phase shift of 180°).

The circuit uses positive and negative feedbacks.

The positive feedback is through R_1C_1, R_2C_2 to transistor T_1.

The negative feedback is through the voltage divider to the input of transistor T_2.

The two transistors produce a total phase shift of 360° so that proper feedback is ensured.

Advantages: It gives constant output due to using temperature sensitive tungsten lamp L_p.

- The overall gain is high because of two transistors.
- The frequency of oscillators can be easily changed by using a potentiometer
- The circuit works quite easily

Disadvantages:

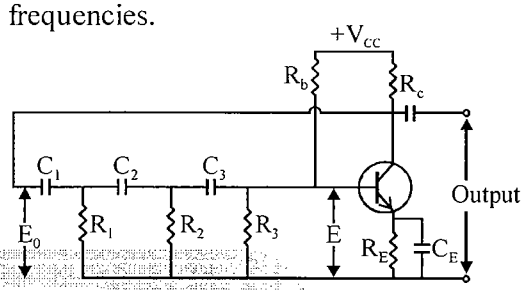
- It can not generate very high frequency.
- The circuit requires two transistors and a number of component

Phase shift oscillator

- Good frequency stability and wave form can be obtained from oscillators employing resistive and capacitive elements such amplifier are called R-C or phase shift oscillators and have the additional advantage that they can be used for very low frequencies.
- The frequency of oscillations is given by

$$f_0 = \frac{1}{2\pi RC\sqrt{6}},$$

where $R_1 = R_2 = R_3 = R$,
$C_1 = C_2 = C_3 = C$



Phase shift oscillator

- A phase shift of 180° is produced by the transistor amplifier. A further phase shift of 180° is produced by the RC network.
- As a result the phase shift around the entire loop is 360°
- Figure shows the three reaction of RC network.
- Each reaction produces a phase shift of 60°, consequently a total phase shift of 180° is produced.

Advantages:

- It can be used to produce very low frequencies
- The circuit provides good frequency stability
- It does not require transformer or inductors

Disadvantages:

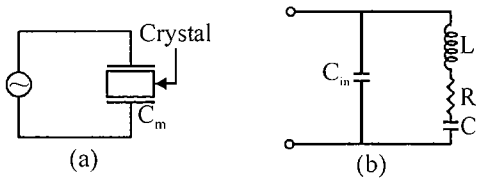
- It is difficult for the circuit to start oscillations as the feedback is generally small.
- The circuit gives small output.

Limitation of L-C and R-C:

- Due to some limitations of LC and RC oscillators, we need crystal oscillators.
- L-C oscillators are not used at low frequencies because inductor become bulky and costly.
- R-C oscillators have not constant frequency and Q-factor because of temperature variations.

Crystal oscillator:

- Electrical oscillators circuit are replaced by mechanical vibration circuit that is called crystal oscillator.
- Certain crystalline material, namely, rohelle salt, quartz and four maline exhibit the piezoelectric effect.
- Of the various piezoelectric crystals, quartz is most commonly used because it is expensive and their great mechanical strength.



- When the circuit is not vibrating, it is equivalent to capacitance C_{in} because it has two metal plates separated by a dielectric.
- When a crystal vibrates, it is equivalent R-L-C series circuit.

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Therefore, the equivalent circuit of a vibrating crystal is R-L-C series circuit shunted by the mounting capacitance C_m as shown in figure (b)

Q factor of crystal $Q = \frac{1}{R}\sqrt{\frac{L}{C}}$

Q factor of crystal is very high

The extremely high Q of a crystal leads to frequency stability.

Frequency response of crystal:

Case-I: When $X_C = X_L$ at series resonant

Impedance of the crystal is very low Z = R

Series resonant frequency $f_S = \frac{1}{2\pi\sqrt{LC}} Hz$

Case-II: At a slightly higher frequency, the net reactance of branch R-L-C inductive and equal to X_{cm}.

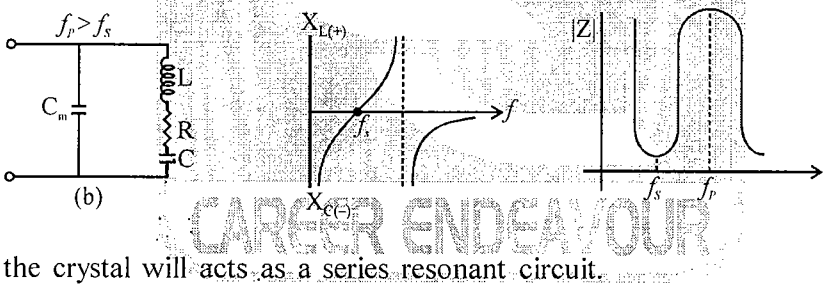
$X_L - X_{cm}$

The crystal now acts as a parallel resonant circuit

Impedance of crystal is very high

Parallel resonant frequency

$f_P = \frac{1}{2\pi\sqrt{LC_{eq}}}$ where, $C_{eq} = \frac{C \times C_m}{C + C_m}, C_{eq} < C$



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Note: At f_S, the crystal will acts as a series resonant circuit.

At f_P, the crystal will acts as a parallel resonant circuit.

Advantages:

It has a high order of frequency stability.

The quality factor (Q) of the crystal is very high.

Disadvantages:

Hardly tunned

It is fragile and consequently can only be used in low power circuit.

Questions Based on Oscillators

1. The Darlington pair is mainly used for

(a) Impedance matching
(b) Wide band voltage amplification
(c) Power amplification
(d) Reducing distortion

Ans. **(a)**

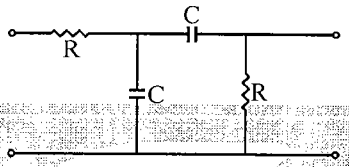
2. Consider the following with the reference of CE transistor amplifier.

(a) The use of negative feedback
(b) The concession of d.c. power to a.c.
(c) High voltage and current gains
(d) The use of stop-up transformer

The power gain is due to

Ans. **(a) and (c)**

3. An FET oscillator uses the given phase shift network as shown below. The minimum gain required for oscillation is



(a) –29 (b) 1 (c) 3 (d) 29

Soln. Apply KVL in 1st loop

$$V_0 = I_1\left(R-\frac{J}{\omega C}\right)+I_2\left(\frac{J}{\omega C}\right) \quad \ldots (1)$$

Apply KVL in second loop

$$O = I_2\left(R-\frac{2J}{\omega C}\right)+I_1\left(\frac{J}{\omega C}\right)$$

$$I_2\left(R-\frac{2J}{\omega C}\right) = -I_1\frac{J}{\omega C} \quad \ldots (2)$$

from (1) and (2) $\frac{V_0}{I_2 R} = \frac{V_0}{V_f} = 3 + J\frac{(1-X^2)}{X}$

where $X = \frac{1}{\omega RC}$

for phase shift to be 180° (for oscillations)

$$\tan^{-1}\left(\frac{1-X^2}{3X}\right) = \pi \Rightarrow \frac{1-X^2}{3X} = 0$$

$x = 1$ i.e. $\omega = \frac{1}{RC}$ $\therefore \frac{V_0}{V_y} = 3$ at $\omega = \frac{1}{RC}$ $\therefore \beta = \frac{V_f}{V_0} = \frac{1}{3}$

loop gain $\beta A \geq 1$ (for oscillations)

$\therefore A_{minimum} = \frac{1}{\beta} = 3$

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Bark hausen condition:

Every oscillator circuit to produce sustained oscillations must satisfy the following two conditions

Loop gain = –AB = 1

Gain Aj = ∞

The total phase shift suffered by the signal in travelling from input to output and back to input must introduce precisely 0° phase shift.

In practical oscillators loop gain is kept slightly greater than 1, this is oversome the circuit internal losses.

For ideal oscillator loop gain is 1.

In a practical oscillator the amplitude of oscillations are limited by on set of non-linearity.

In a practical oscillator, suppose if Bark-Hausen's criterion is failed then the circuit will stop producing the frequency of oscillators.

Sinusoidal oscillators:

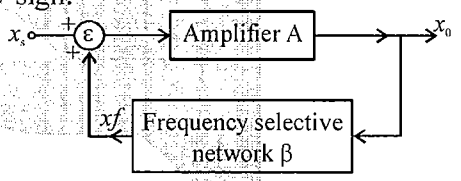
Basic principles of sinusoidal oscillators: In spite of the name linear oscillator, some form of non-linearity has to be employed to provide central of the amplitude of the output sine wave. In fact, all oscillators are essentially non linear circuits. This complicates the task of analysis and design of oscillators, no longer is one able to apply transform (S-plane) methods directly. Nevertheless, technique have been developed by which the design of sinusoidal oscillators can be performed in two steps. The first stop is a linear one and frequency domain method of feedback circuit analysis can be readily employed. Subsequently, a non-linear mechanism for amplitude control can be provided.

The oscillator feedback loop: The basic structure of a sinusoidal oscillator consists of an amplifier and a frequency selective network connected in a positive feedback loop. Diagram is given below:

Here feedback signal *xy* is summed with a positive sign.

Thus the gain with feedback is given by

$$\boxed{A_{f(s)} = \frac{A(s)}{1 - A(s)\beta(s)}}$$



Sinusoidal oscillator with positive feedback

where we note negative sign in the denominator.

For our purpose here it is more convenient to drop the minus sign and define the loop gain L(s) as:

$$L(s) = A(s)\beta(s) \quad \ldots (1)$$

the characteristic equation thus becomes

$$1 - L(S) = 0 \quad \ldots (2)$$

The oscillation criterion: If at a specific frequency f_0 the loop gain Aβ is equal to unity it follows that $L(J\omega_0) = A(J\omega_0)\beta(J\omega_0) = 1$

That is at ω_0 the phase of the loop gain should be zero and the magnitude of the loop gain should be unity. This is known as Bark hausen criteria.

For this loop to produce and sustain an output x_0 with no input applied (x_5 = 0), the feedback signal x_y. $x_y = \beta x_0$

Should be sufficiently large that when multiplied by A it produces x_0, that is $A_{xy} = x_0$

i.e. $A\beta x_0 = x_0$ which result in $\boxed{A\beta = 1}$

An alternative approach to the study of oscillator circuits consists of examining the circuit poles, which are the roots of the characteristic equation

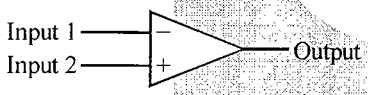
2. For the circuit to produce sustained oscillations at a frequency ω_0 the characteristic equation. For the circuit to produce sustained oscillations at a frequency ω_0 the characteristic equation has to have roots at $S = \pm J\omega_0$. Thus $1 - A(s)\beta(s)$ should have a factor of the form $S^2 + \omega_0^2$.

Non-linear amplitude control: The oscillation condition, the Bark hausen criteria, guarantees sustained oscillations in a mathematical sense. It is well known, however, that the parameter of any physical system cannot be obtained constant for any length of time. In otherwards, suppose we work hard to make $A\beta = 1$ at $\omega = \omega_0$ and then the temperature changes and $A\beta$ becomes slightly less than unity. There are two basic approaches to the implementation of non-linear amplitude stabilization mechanism. The first approach makes use of a limited circuit. Oscillations are allowed to grow until the amplitude reaches the level to which the limiter is set. When the limiter comes into operation, the amplitude remains constant. Obviously, the limiter should be shoft to minimize non-linear distortion such distortion however, is reduced by the filtering action of the frequency selective network in the feedback loop.

The other mechanism for amplitude control utilizes an element whose resistance can be controlled by the amplitude of the output sinusoid. By placing this element in the feedback circuit so that its resistance determines the loop gain. The circuit can be designed to ensure that the loop gain reaches unity at the desired output amplitude. Diodes or JFETs operated in the triode region are commonly employed to implement the controlled resistance element.

Operational Amplifier:

Symbol:



(+) Non-inverting terminal, (–) Inverting terminal

Input impedance : Few mega Ω (Very high), Output impedance : Less than 100 Ω (Very low)

Differential and Common Mode Operation: One of the more important features of a differential circuit connection as provided in an op-amp is the circuit ability to greatly amplify signals that are opposite at the two inputs while only slightly amplifying signals that are common to both inputs. An op-amp provides an output component that is due to the amplification of the difference signals applied to the plus and minus input and a component due to the signals common to both inputs. Since amplification of the opposite input signals is much greater than that of common input signals the circuit provides a common-mode rejection as described by a numerical value called COMMON MODE REJECTION RATIO (CMRR).

Differential Input: When separate inputs are applied to the op-amp, the resulting difference signal is the difference between the two inputs. $\boxed{V_d = V_1 - V_2}$.

Common Input: When both input signals applied to an op-amp is common, signal element due to the two inputs can be defined as the average of the sum of the two signals. $\boxed{V_C = \left(\frac{V_1 + V_2}{2}\right)}$.

Output Voltage: Since any signal applied to op-amp in general have both in phase and out of phase components the resulting output can be expressed as $\boxed{V_0 = A_d V_d + A_c V_c}$.

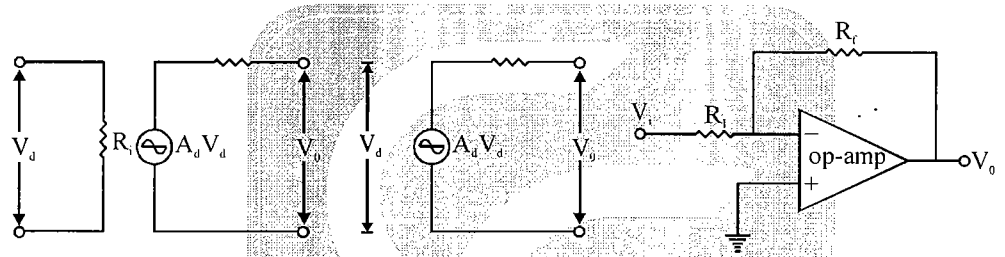
Where V_d = difference voltage, V_C = common voltage,
A_d = difference mode gain of the amplifier, A_c = Common mode gain of the amplifier.

CMRR {Common Mode Rejection Ratio}: $\boxed{CMRR = \frac{A_d}{A_c}}$

The value of CMRR can also be expressed in log term as

$$\boxed{CMRR(\text{in } d_B) = 20 \log_{10} \frac{A_d}{A_c} (dB)}$$

Equivalent Circuit: While an input to the minus (–) input results in on opposite polarity output. The ac equivalent circuit of the op-amp is shown in figure. As shown the input signal applied between input terminals sees as input impedance Ri typically very high. The output voltage is shown to be the amplifier gain times the input signal taken through output impedance R_0, which is typically very low. An ideal op-amp circuit, as shown in figure would have infinite input impedance zero output impedance and infinite voltage gain.



Characteristic of ideal OPAMP

(*i*) Open loop voltage gain in infinite (A = ∞)
(*ii*) Input impedence is infinite is infinite
(*iii*)Output impedence is zero
(*iv*) Bandwidth is infinite
(*v*) Characteristic do not drift with temperature

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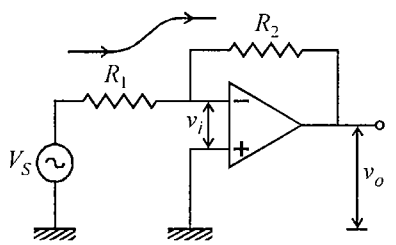
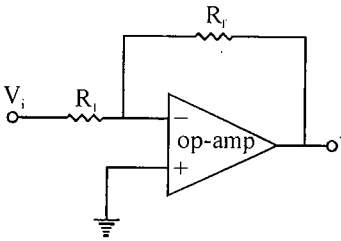
Concept of virtual ground

Figure shows an circuit of inverting opamp which employs negative feedback through the resistor R_2. The negative feedback voltage R_2 tends to cancel the input signal at the point A and tries to keep the point A at ground potential. If v_i is the input voltage then output would be $v_o = -A \cdot v_i$. Since open loop gain $A \to \infty$, for finite v_o we should have $v_i \to 0$. Since, $R_i \to \infty$, no current will enter into the opamp. Thus, there is a 'virtual ground' at point A. The word virtual is used to imply that through the point A is effectively connected to ground.

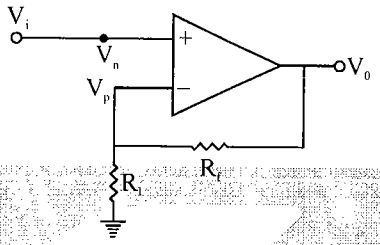


Inverting Amplifier: The most widely used constant gain amplifier circuit is the inverting amplifier.



. We can write $\boxed{V_0 = -\frac{R_f}{R_1} V_1}$

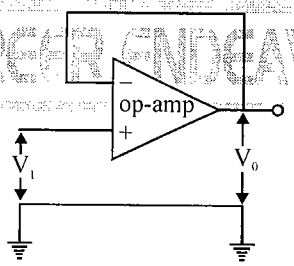
Non-inverting Amplifier: The connection of figures shows an op-amp that works as a non-inverting amplifier or constant gain multiplier. It should be noted that the inverting amplifier connection is more widely used because it has better frequency stability.



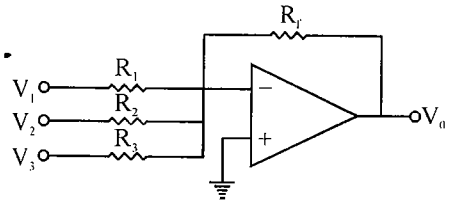
By virtual ground law: $V_n = V_p = V_i$

$$\Rightarrow V_i = \frac{R_1 V_0}{R_1 + R_f} \Rightarrow \frac{V_0}{V_i} = \frac{R_1 + R_f}{R_1} = 1 + \frac{R_f}{R_1} \Rightarrow \boxed{V_0 = \left(1 + \frac{R_f}{R_1}\right) V_i}$$

Voltage Follower or Unity Follower: The unity follower circuit as shown in figure provides a gain of unity (1) with number polarity or phase reversal. From the equivalent circuit, it is clear that $\boxed{V_0 = V_1}$ and that the output is the same polarity and magnitude as the input. The circuit operates like an emitter or source follower circuit except that the gain is exactly unity.



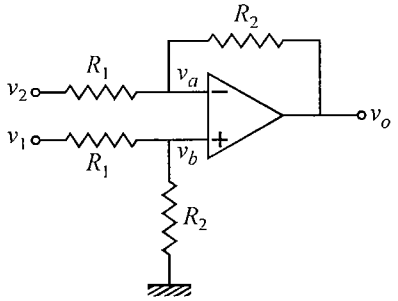
Summing Amplifier: Three input summing amplifier.



$$\boxed{V_0 = -\left(\frac{R_f}{R_1} V_1 + \frac{R_f}{R_2} V_2 + \frac{R_f}{R_3} V_3\right)}$$

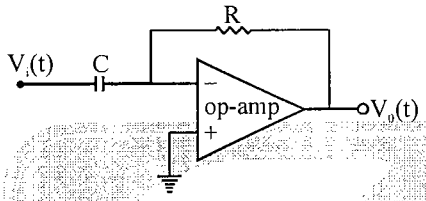
Differential Amplifier

$$v_o = \frac{R_2}{R_1}(v_1 - v_2)$$



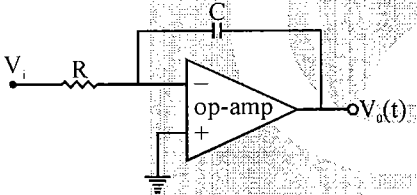
Differentiator : A differentiator circuit is shown in figure while not as useful as the circuit forms covered above the differentiator does provide a useful operation, the resulting for the circuit being

$$V_0(t) = -RC\frac{dV_1(t)}{dt}$$



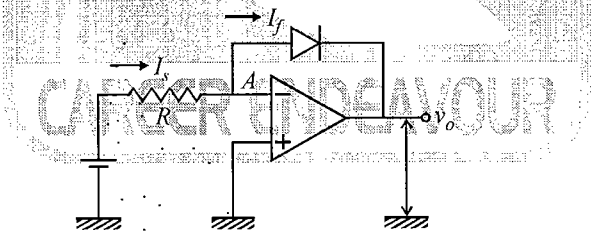
where the scale factor is –RC.

Integrator:



$$V_0(t) = -\frac{1}{RC}\int V_1(t)dt$$

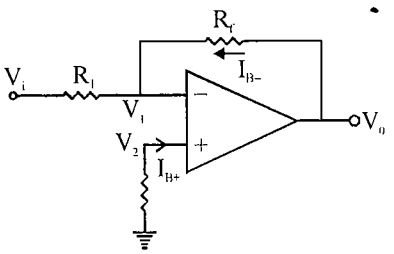
Logarithmic Amplifier



Due to virtual ground at point A, $I_S = I_f$

$$\Rightarrow \quad \frac{V_S - 0}{R} = I_0\left(e^{\frac{v_f}{\eta kT}} - 1\right) \simeq I_0\, e^{\frac{ev_f}{\eta kT}} \quad \text{or} \quad v_o = -v_f = -\frac{\eta kT}{e}\ell n\frac{V_S}{I_0 R}$$

Offset Currents and Voltages {d.c. characteristic of op-amp}:



(1) Input bias current : $\frac{i_B^+ + i_B^-}{2}$

(2) Input offset current: $I_{0s} = |I_B^+| - |I_B^-|$

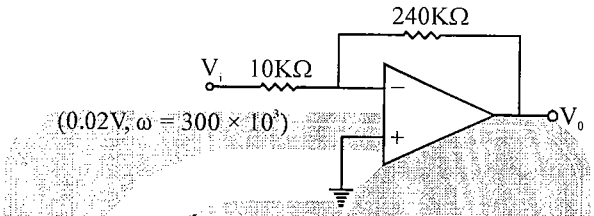
(3) Input offset voltage : $V_{0s} = V_2 - V_1$

Note: Due to mismatching between V_1 and V_2 output voltage may be positive or negative so we apply offset voltage (V_{os}).

Slew Rate: Another parameter reflecting the op-amp's ability to handling varying signal is slew rate, defined as slew rate = maximum rate at which amplifier output can change in volts per micro second.

$\boxed{SR = \frac{\Delta V_0}{\Delta t} V/\mu s}$ with t in μs.

Problem: Calculate the slew rate of given circuit.



Soln. For a gain of magnitude $A_{CL} = \left|\frac{R_F}{R_1}\right| = \frac{240K\Omega}{10K\Omega} = 24$. The output voltage provides.

$K = A_{CL}$, $V_i = 24(0.2V) \Rightarrow 0.48V$

$\omega \le \frac{SR}{K} = \frac{0.5v/\mu s}{0.48} = 1.1 \times 10^6 \text{ rad/sec}$

Voltage Buffer: A voltage buffer circuit provides a means of isolation on input signal from a load by using a stage having unity gain with no phase or polarity inversion.

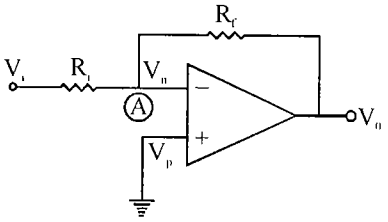
Controlled Sources: Op-amp can be used to form various types of controlled sources. An input voltage can be used to control on output voltage or current or an input current can be used to control on output voltage or current. There type of connections are suitable for use in various instrument system (circuit). It has four types:

(1) Voltage Controlled Voltage Source (2) Voltage Controlled Current Source

(3) Current Controlled Current Source (4) Current Controlled Voltage Source

(1) Voltage Controlled Voltage Source: An ideal form of a voltage source whose output V_0 is controlled by on input voltage V_J is shown in figure. The output voltage is seen to be independent on the input voltage. This type of circuit can be built using an op-amp as shown in figure.

(i) Inverting op-amp:

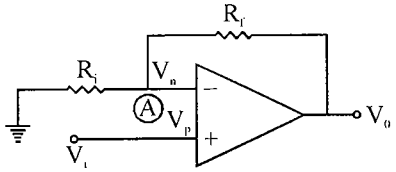


$\boxed{K = -\frac{R_f}{R_i}}$

By virtual ground condition $V_n = V_p = 0$

Now KCL at point A $\frac{V_i-0}{R_i}=\frac{0-V_0}{R_f}\Rightarrow\boxed{\frac{V_0}{V_i}=-\left(\frac{R_f}{R_i}\right)}$

i) **Non-inverting op-amp:**



$\boxed{K=1+\frac{R_f}{R_i}}$

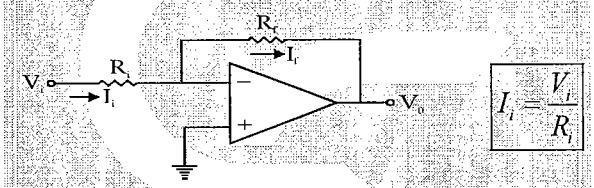
By virtual ground condition $V_p = V_n = V_i$

Now KCL at point A

$$\frac{O-V_i}{R_i}=\frac{V_i-V_o}{R_f}\Rightarrow\frac{V_0}{R_f}=\left(\frac{R_f+R_i}{R_f+R_i}\right)V_i\Rightarrow\boxed{\frac{V_0}{V_i}=1+\frac{R_f}{R_i}}$$

ii) **Voltage Controlled Current Source:** An ideal form of circuit providing an output current controlled by an input voltage is that of figure. The output current is dependent on the input voltage.

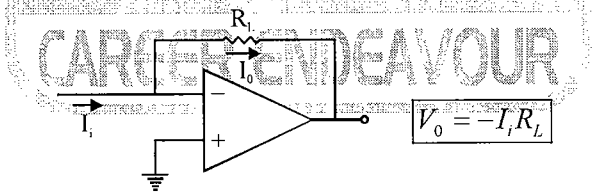
Practical Circuit:



$\boxed{I_i=\frac{V_i}{R_i}}$

iii) **Current Controlled Voltage Source:** An ideal form of a voltage source controlled by a input current is shown in figure. The output voltage is dependent on the input current.

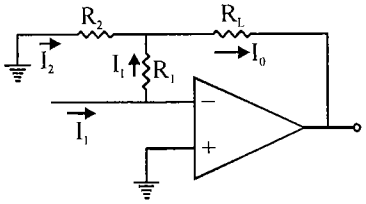
Practical Circuit:



$\boxed{V_0=-I_iR_L}$

iv) **Current Controlled Current Source:** An ideal form of a circuit providing on output current dependent on an input current is shown in figure. In this type of circuit on output current is provided dependent on the input current.

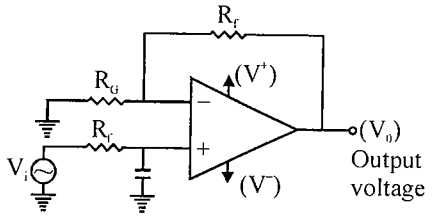
Practical Circuit:

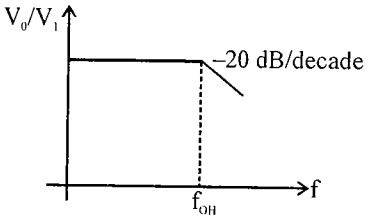


$$I_0=I_1+I_2=I_1+\frac{I_1R_1}{R_2}=\left(1+\frac{R_1}{R_2}\right)I_1=KI_1$$

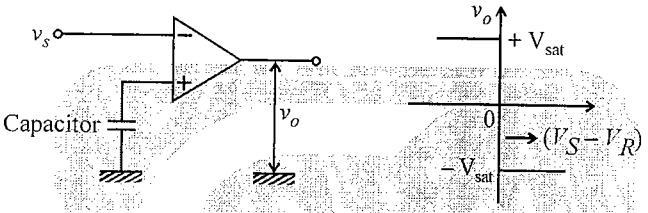
Low Pass Filter: A 1st order, low pass filter using resistor and capacitor as in figure shown has a practical slope of –20 dB per decade as shown in figure (rather them the ideal response of figure). The voltage gain below the cutoff frequency is constant at

$\boxed{A_v = 1 + \frac{R_f}{R_G}}$, at a cut off frequency of $\boxed{f_{OH} = \frac{1}{2\pi R_1 C_1}}$





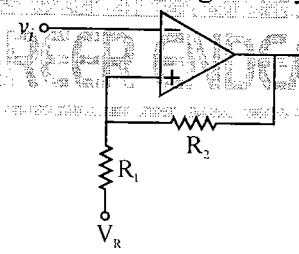
Voltage comparator



Comparator is a circuit which can compare two voltages. In the figure an opamp voltage comparator compare two voltage, input signal V_S with reference voltage V_R. When $V_S < V_R$ the output voltages goes to maximum saturation value $(+V_{sat})$ when $V_S > V_R$ the opamp output switches to the negative saturation value $(-V_{sat})$.

Schmitt Trigger

Its a comparator which uses positive or regenerative feedback and exhibits the phenomenon of hysteresis is called a regenerative comparator or more generally Schmitt trigger.



when $v_i < v_1$ the output v_o is at positive saturation v_{sat}, so we have $v_1 = \frac{R_2 V_R}{R_1 + R_2} + \frac{R_1 V_{sat}}{R_1 + R_2} = v_i$.

Now, until $v_i = v_1$, the output remains at $+v_{sat}$. A slight increase in v_i makes the output to $-v_{sat}$, and it remains their when $v_i > v_1$.

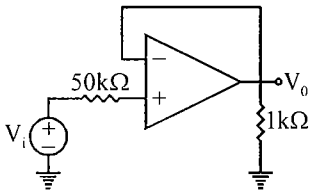
In this time, $v_1 = \frac{R_2 V_R}{R_1 + R_2} - \frac{R_1 v_{sat}}{R_1 + R_2} = v_2$

If voltage v_i now reduced the output stays at $-v_{sat}$, but when v_i just cross v_2, output switches to $+v_{sat}$.

Since, v_2 is smaller as compared to v_1 so a hysteresis will from, where $v_H = v_1 - v_2 = \frac{2R_1}{R_1 + R_2} v_{sat}$.

SOLVED PROBLEMS

. The circuit is as shown in figure



(i) The ideal closed loop voltage gain is

(a) 1 (b) –1 (c) ∞ (d) 50

oln. Voltage follower circuit, $V+=V_i$, $V-=V_i=V_0$, $\frac{V_0}{V_i}=1$. Hence, (a) is correct answer.

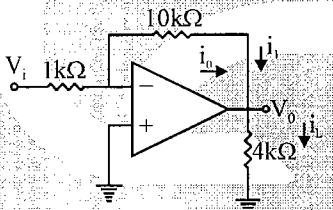
(ii) The open loop gain is A_{od} = 999, then closed loop gain is

(a) –0.999 (b) 0.999 (c) 1.001 (d) –1.001

$V+=V_i$, $V-=V_0$, $A_{od}(V_i-V_0)=V_0$, $A_{od}V_i=(A_{od}+1)V_0$, $A_{od}=999$

$$\frac{V_0}{V_i}=\frac{A_{od}}{1+A_{od}}=\frac{999}{1+999}=0.99$$

. The input to the circuit in figure is $V_i=2\sin\omega t mV$. The current I_0 is



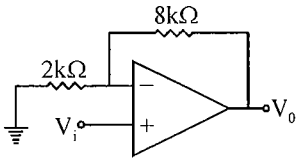
(a) $-2\sin\omega t\,\mu A$ (b) $-7\sin\omega t\,\mu A$ (c) $-5\sin\omega t\,\mu A$ (d) 0

oln. $V_0=\frac{-10}{1}(2\sin\omega t)mV=-\frac{R_2}{R_1}.V_S=-20\sin\omega t\,mV$

$i_L=\frac{V_0}{4K}=-5\sin\omega t\mu A$, $i_1=i_i=\frac{2\sin\omega t}{1K}=2\sin\omega t\mu A$

KCL at V_0, $i_0=i_L-i_1=-5\sin\omega t-2\sin\omega t=-7\sin\omega t$. Hence, (b) is correct answer.

. The op-amp of figure has a very poor open-loop voltage gain of 45 but is otherwise ideal. The closed loop gain of amplifier is



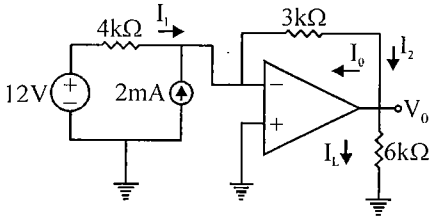
(a) 20 (b) 4.5 (c) 4 (d) 5

oln. A closed loop gain,

$$A_{CL}=\frac{V_0}{V_i}=\frac{A_{od}}{1+\beta A_{od}},\beta=\frac{2K}{8K+2K}=0.2,A_{CL}=\frac{45}{1+(45)(0.2)}=4.5$$

Hence, (b) is correct answer.

4. Find the value of current I_0?



(a) 12mA (b) 8.5mA (c) 6mA (d) 7.5mA

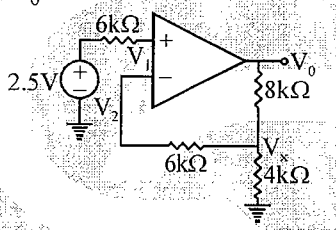
Soln. $V+ = V- = 0,\ i_1 = \dfrac{12}{4K} = 3mA$

$i_2 = 3+2 = 5mA,\ V_0 = -(5)(3) = -15V$

$i_2 = i_0 + i_L,\ 5 = i_0 + \dfrac{-15}{6},\ i_0 = 7.5\text{mA}$

Hence, (d) is correct answer.

5. Find the value of output voltage V_0.



(a) –7.5V (b) 7.5V (c) 8V (d) –8V

Soln. $V+ = 2.5V = V-,\ \dfrac{V_0(4)}{8+4} = 2.5,\ V_0 = 7.5V,\ V_1 = 2.5V$

(since zero current inside op-amp)

$V_1 = V_2 = 2.5V,\ V_X = 2.5\text{V}$

(zero current inside op-amp, so zero voltage drop at 6KΩ)

CAREER ENDEAVOUR

6. A multistage amplifier consists of three stages. The voltage gains of the stages are 30, 50 and 80 respectively. The overall voltage gain in dB will be **[BHU 2013]**

(a) 101.58 (b) 50.79 (c) 33.98 (d) 38.06

Soln. Overall gain = $Av_1 \times Av_2 \times Av_3 = 30\times50\times80 = 120000$

Therefore, the overall voltage gain in dB = 20 log Av_T = 20 log 120000 = 101.58

Correct option is (a)

7. An amplifier with negative feedback has a voltage gain of 100. It is found that with feedback an input signal of 0.6 V is required to produce a given output whereas without feedback the input signal must be only 50 mV for the same output. Then the voltage gain without feedback A and feedback factor β are

(a) $A = 1000,\ \beta = \dfrac{9}{1000}$ (b) $A = 1100,\ \beta = \dfrac{10}{1100}$ **[BHU 2013]**

(c) $A = 1400,\ \beta = \dfrac{13}{1200}$ (d) $A = 1200,\ \beta = \dfrac{11}{1200}$

Soln. We know that gain with negative feedback is given by

$$A_f = \frac{A}{1+AB} = 100$$

The output with negative feedback is given by

$$V_0 = 100 \times 0.6 = 60V$$

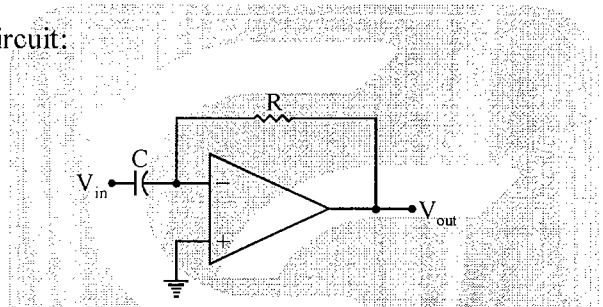
And output without negative feedback is given by

$$V_0 = A \times 50mV = 60V \Rightarrow A = \frac{60}{50} \times 10^3 = 1.2 \times 10^3 = 1200$$

Now, $A_f = 100 = \frac{A}{1+A\beta} \Rightarrow \beta = \left(\frac{1200}{100} - 1\right)/1200 = \frac{11}{1200}$

Correct option is (d)

Consider the following circuit: **[TIFR 2012]**

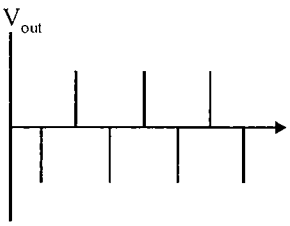


If the waveform given below is fed in at V_{in},

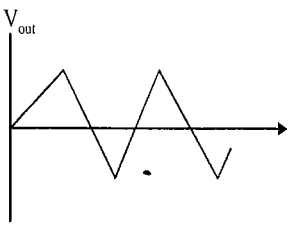
CAREER ENDEAVOUR

then the waveform at the output V_{out} will be

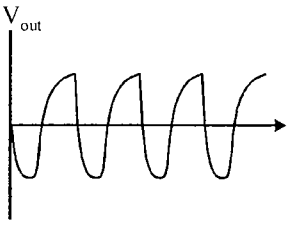
(a)



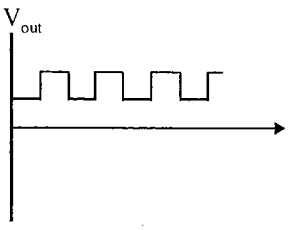
(b)



(c)



(d)



Soln. From the given circuit we may write

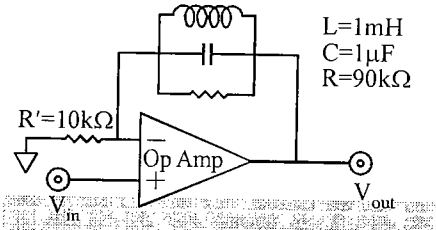
$$\frac{d}{dt}(v_i - 0)C = \frac{0 - V_0}{R}$$

$$\Rightarrow \quad V_0 = -CR\frac{dV_i}{dt}$$

So, it is a differator circuit. So, for square wave input out put will be spike (Impulse)

Correct option is (a)

9. An input of 1.0V DC is given to the ideal Op-Amp circuit depicted below. What will be the output voltage?



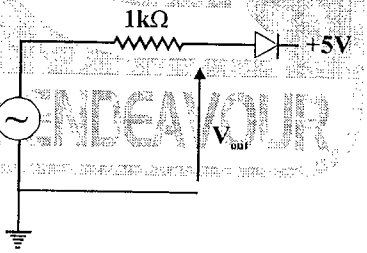
(a) 10.0 V (b) –9.0 V (c) 1.0 V (d) 0 V **[TIFR 2013]**

Soln. For DC current, $R_2 = 0$

$$\therefore \quad V_0 = V_{in}\left(1 + \frac{R_2}{R_1}\right) = 1\left(1 + \frac{0}{10}\right) = 1V$$

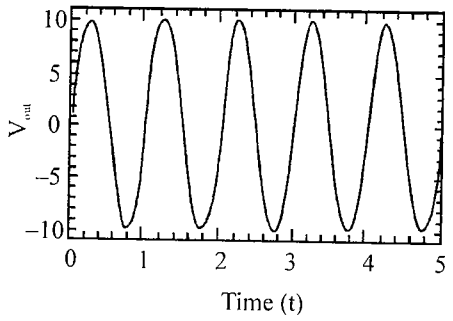
Correct option is (c)

10. Consider the following circuit **[TIFR 2014]**

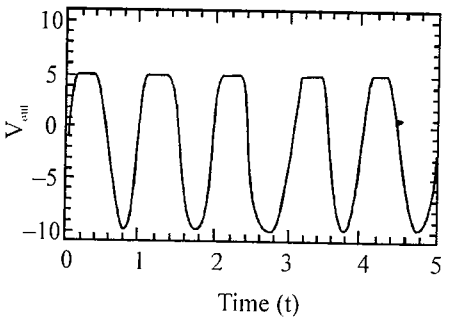


Which of the graphs given below is a correct representation of V_{out}?

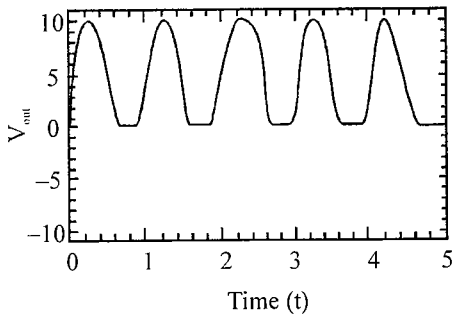
(a)



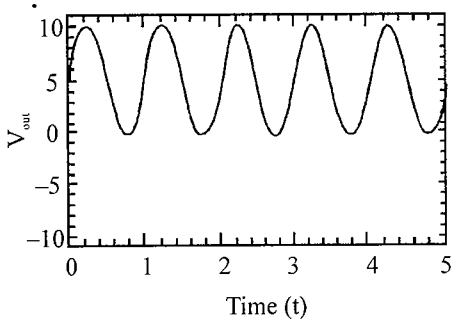
(b)



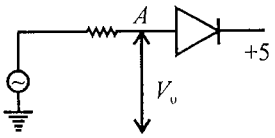
(c)



(d)



Soln.

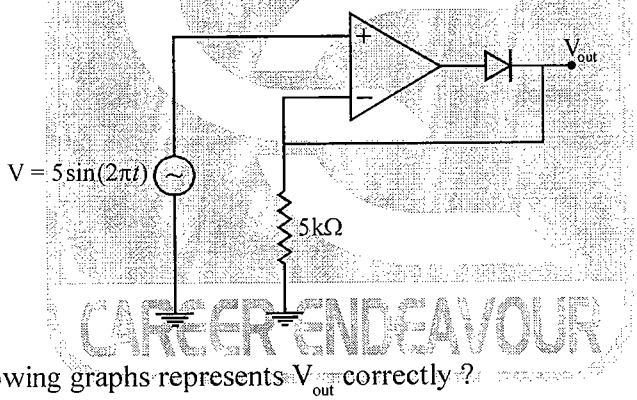


For negative half cycle output will same input as during the negative cycle diode will be in reversed biased.

But for positive half cycle, when voltage at the point A will just greater than the 5 volt diode will work in forwards baised. So, peak voltage will be +5 volt.

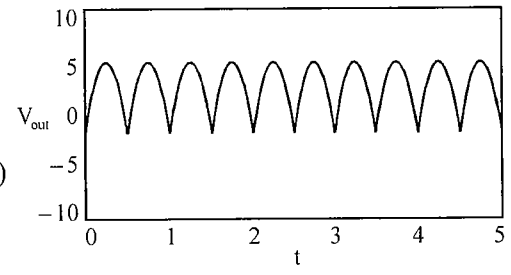
Correct option is (b)

1. In the circuit shown below, the op-amp is powered by a bipolar supply of ±10 V. **[TIFR 2015]**

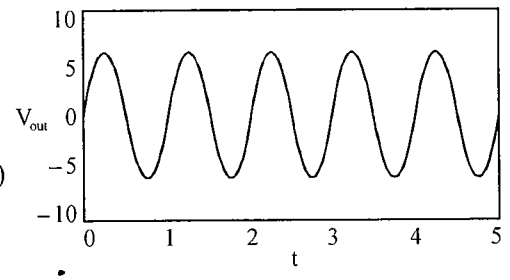


Which one of the following graphs represents V_{out} correctly ?

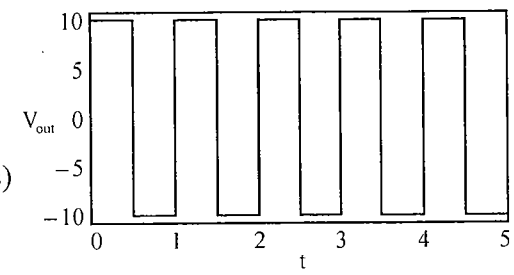
(a)



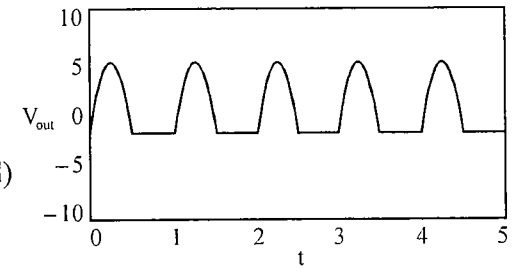
(b)



(c)

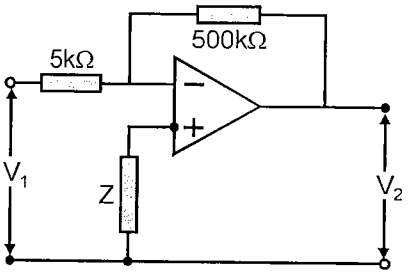


(d)



Soln. It is unit given non-inverting circuit (if don't include diode) during the positive half cycle diode will conducting. So, output will be same as input.
But, during negative half cycle diode with be in reversed baised.
Correct option is (d)

12. In the generalized operational amplifier circuit shown on the right, the amp. has a very high input impedance $(Z > 50\,M\Omega)$ and an open gain of 1000 for the frequency range under consideration. Assuming that the op. amp. draws negligible current, the voltage ratio V_2/V_1 is approximately
[TIFR 2016]



(a) −190 (b) −190 (c) −90 (d) 80

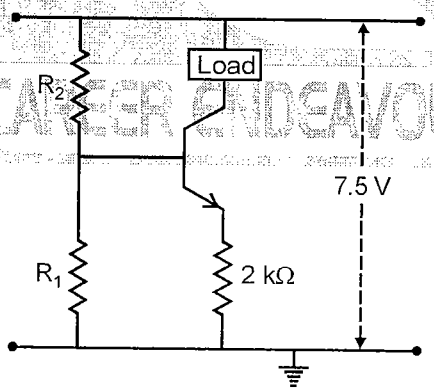
Soln. $$A_{CL} = \frac{-A_{OL} \cdot R_f}{R_f + (1 + A_{OL})R}$$

Put value

$A_{CL} = -90$

Correct option is (c)

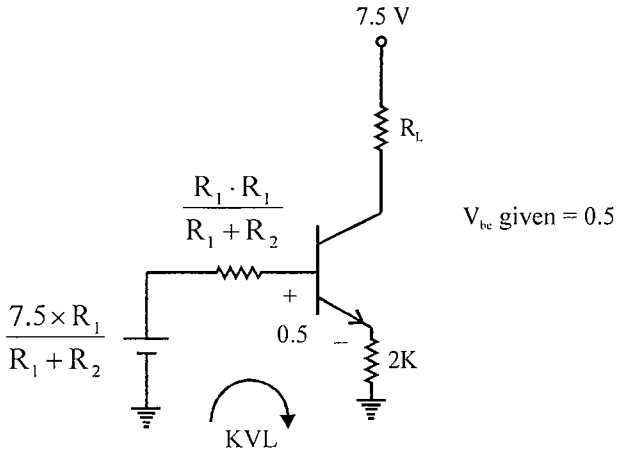
13. In the transistor circuit shown on the right, assume that the voltage drop between the base and the emitter is 0.5V.
[TIFR 2016]



What will be the ratio of the resistances R_2/R_1, in order to make this circuit function as a source of constant current, $I = 1$ mA ?

(a) 4.5 (b) 3.0 (c) 2.5 (d) 2.0

oln.



Thevenin equivalent circuit

$$I_C = I_E = 1mA$$

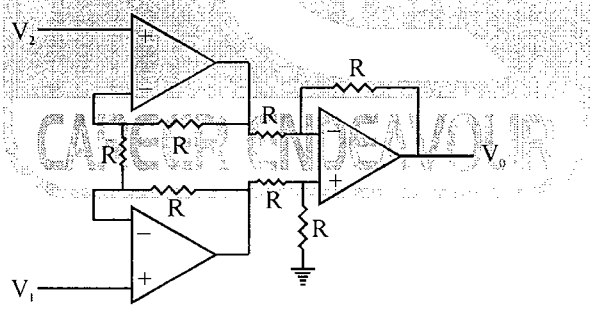
$$\frac{7.5 \times R_1}{R_1 + R_2} = I_B \times \left[\frac{R_1 \cdot R_2}{R_1 + R_2}\right] + 0.5 + 2 \times 1$$

No 'β' neglect I_B given in question

$$\frac{7.5 \times R_1}{R_1 + R_2} = 2.5 \Rightarrow \frac{R_1}{R_1 + R_2} = \frac{1}{3} \Rightarrow \frac{R_1 + R_2}{R_1} = 3 \Rightarrow \frac{R_2}{R_1} = 3 - 1 = 2$$

Correct option is (b)

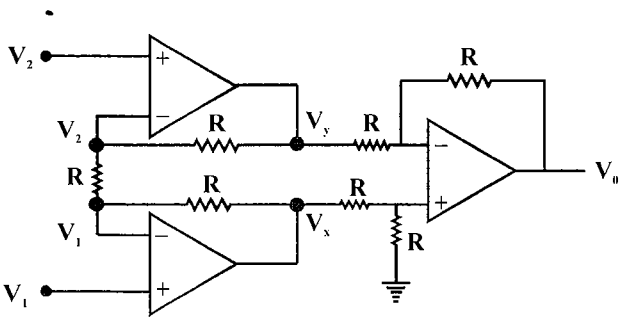
4. The classic three op-amp instrumentation amplifier configuration is shown below:



The op-amp are ideal and all resistors are of equal value R. The gain, defined as the output voltage V_0 divided by the differential input voltage $V_1 - V_2$, is equal to

(a) 2 (b) 3 (c) 4 (d) 6

oln. Its an instrumentation amplifier



KCL at V_2

$$\frac{V_2 - V_1}{R} + \frac{V_2 - V_y}{R} = 0$$

$$2V_2 - V_1 = V_y \qquad \ldots (1)$$

KCL at V_1

$$\frac{V_1 - V_2}{R} + \frac{V_1 - V_x}{R} = 0$$

$$2V_1 - V_2 = V_x \qquad \ldots (2)$$

$$V_0 = \frac{R}{R}\left[V_x - V_y\right]$$

Subtractor circuit, put value of V_x, V_y

$$V_0 = \frac{R}{R}\left[2V_1 - V_2 - 2V_2 + V_1\right]$$

$$\frac{V_0}{V_1 - V_2} = 3$$

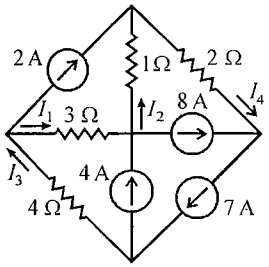
Correct option is (b)

CAREER ENDEAVOUR

PRACTICE SET

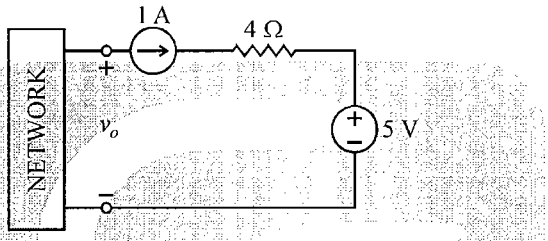
Multiple Choice Questions (MCQ):

In the given circuit of figure current I_2 is



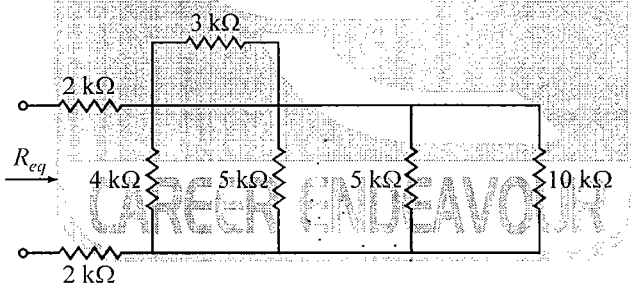
(a) 13 A (b) –3 A (c) 5 A (d) 3 A

The voltage v_0 in figure is always equal to



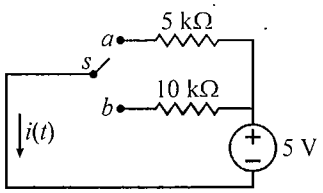
(a) 1 V (b) 5 V (c) 9 V (d) None of these

For the circuit of figure R_{eq} is



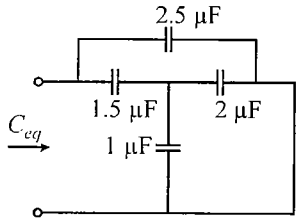
(a) 4 kΩ (b) 6 kΩ (c) 9 kΩ (d) 5.33 K

In the circuit of figure, the ideal switch S alternately stays at position A for 1 ms and at position B for 4 ms. The average value of current $i(t)$ is



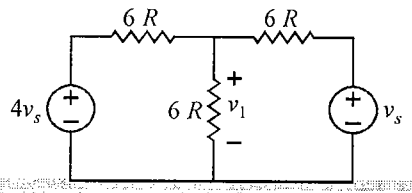
(a) 0.6 mA (b) 1.5 mA (c) 0.75 mA (d) 2.5 mA

5. For the circuit shown in figure the value of C_{eq} is



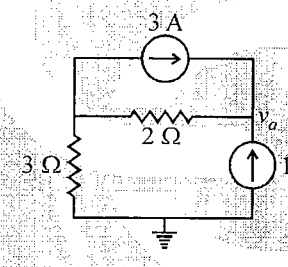
(a) 3.5 μF (b) 7 μF (c) 2.4 μF (d) 2.6 μF

6. In the following circuit the voltage v_1 is



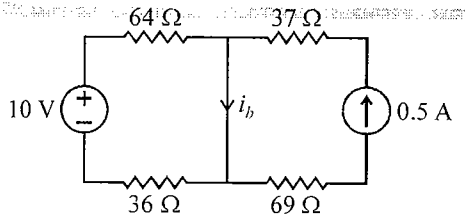
(a) $0.4v_s$ (b) $1.6v_s$ (c) $0.67v_s$ (d) $2.5v_s$

7. In the following circuit the voltage v_o is



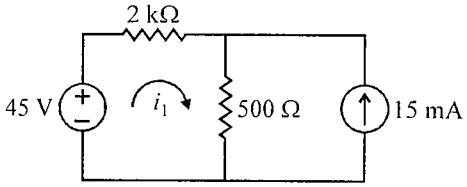
(a) –11 V (b) 11 V (c) 3 V (d) –3 V

8. In the following circuit the current i_b is



(a) 0.6 A (b) 0.5 A (c) 0.4 A (d) 0.3 A

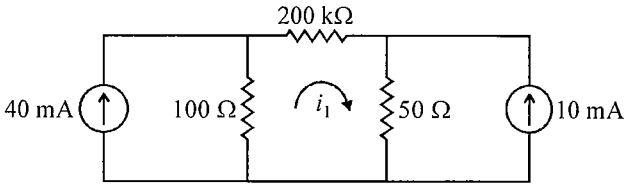
9. In the following circuit the current i_1 is



(a) 20 mA (b) 15 mA (c) 10 mA (d) 5 mA

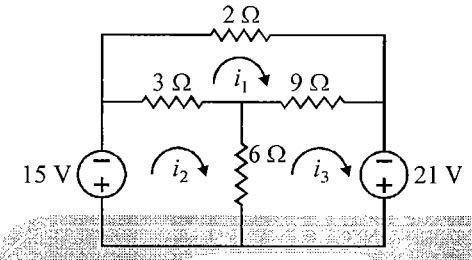
CAREER ENDEAVOUR

0. In the following circuit the current i_1 is



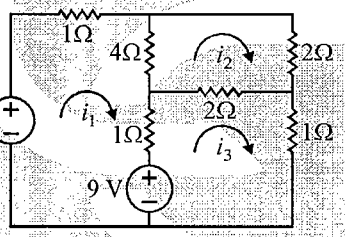
(a) 10 mA (b) –10 mA (c) 0.4 mA (d) –0.4 mA

1. In the following circuit the currents i_1, i_2 and i_3 is



(a) 3A, 2A and 4A (b) 3A, 3A and 8A (c) 1A, 3A and 4A (d) 1A, 2A and 8A

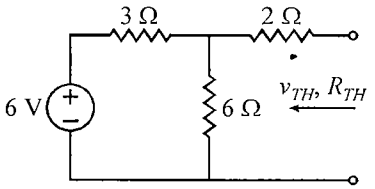
2. The mesh equation for the following circuit are



(a) $\begin{bmatrix} 6 & 4 & -1 \\ 4 & -8 & -2 \\ -1 & -2 & 4 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ 9 \end{bmatrix}$ (b) $\begin{bmatrix} 6 & 4 & -1 \\ 4 & 8 & -2 \\ -1 & -2 & 4 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ 9 \end{bmatrix}$

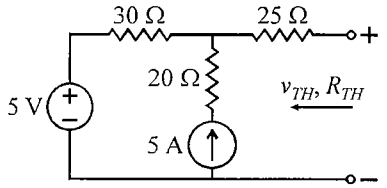
(c) $\begin{bmatrix} 6 & -4 & -1 \\ -4 & 8 & -2 \\ -1 & -2 & 4 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ 9 \end{bmatrix}$ (d) $\begin{bmatrix} -6 & 4 & 1 \\ 4 & -8 & -2 \\ 1 & 2 & -4 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ 9 \end{bmatrix}$

3. In the following circuit the value of v_{TH} and R_{TH} are



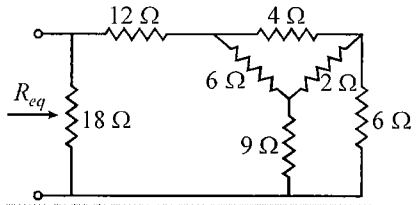
(a) 2V, 4Ω (b) 4V, 4Ω (c) 4V, 5Ω (d) 2V, 5Ω

14. For the following circuit the value of v_{TH} and R_{TH} are



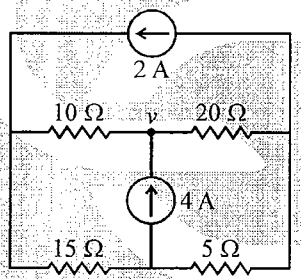
(a) −100 V, 75 Ω (b) 155 V, 55 Ω (c) 155 V, 37 Ω (d) 145 V, 75 Ω

15. In the following circuit the value of R_{eq} is



(a) 18 Ω (b) $\frac{72}{13}\Omega$ (c) $\frac{36}{13}\Omega$ (d) 9 Ω

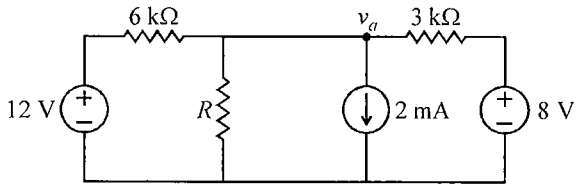
16. In the following circuit the voltage v is



(a) 60 V (b) −60 V (c) 30 V (d) −30 V

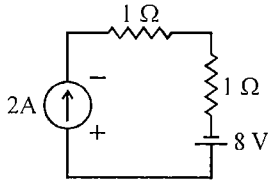
CAREER ENDEAVOUR

17. The node voltage v_a in the following circuit is 4 V. The value of R would be



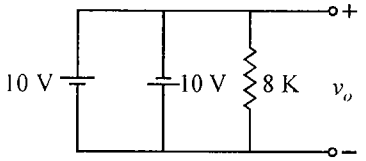
(a) 6 kΩ (b) 12 kΩ (c) 16 kΩ (d) 18 kΩ

18. Find V_{2A} = ?



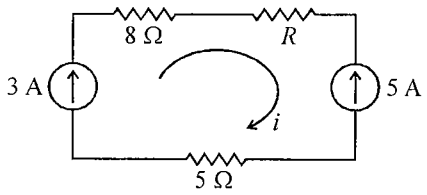
(a) 4V (b) −4V (c) 8V (d) −8V

9. Find v_o ?



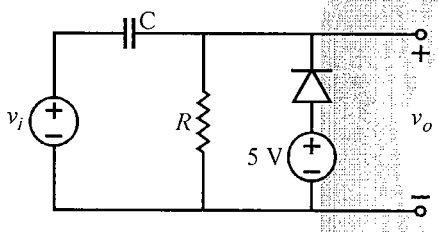
(a) 10 V (b) −10 V (c) 20 V (d) None of these

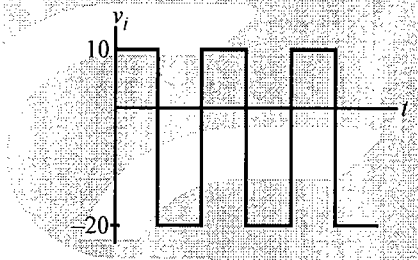
0. Find i ?



(a) −2A (b) 2A (c) Data insufficient (d) None of these

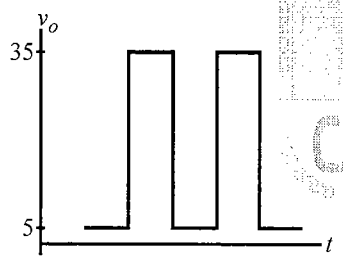
1. For the circuit shown below, the input voltage v_i is as shown in figure.



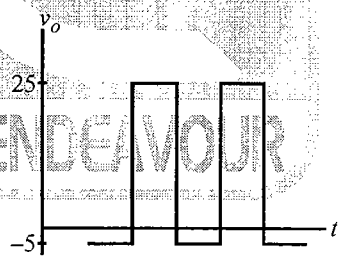


Assume the RC time constant large and cuting voltage $V_\gamma = 0$. The output voltage v_o is

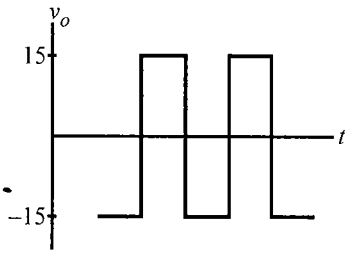
(a)



(b)



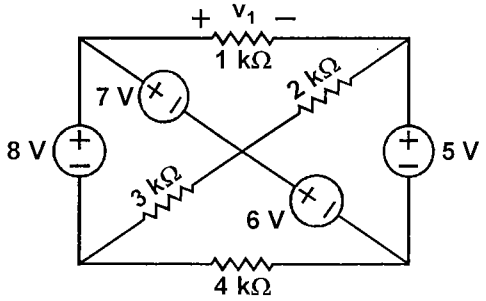
(c)



(d) None of these

CAREER ENDEAVOUR

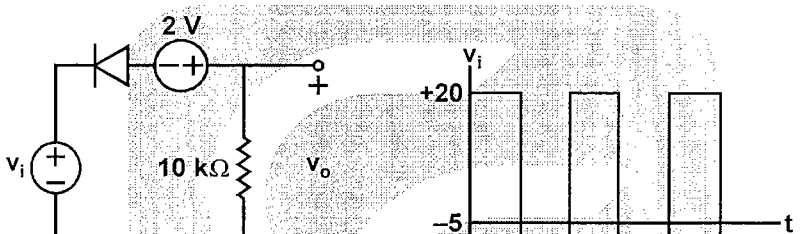
22. In the circuit of figure the voltage $v_1 = ?$



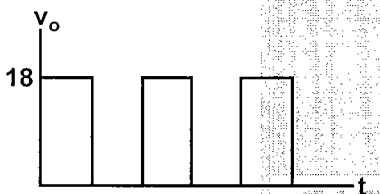
(a) −11 V (b) 5 V (c) 8 V (d) 18 V

23. Consider the given a circuit and a waveform for the input voltage. The diode in circuit has cutin voltage $V_\gamma = 0$.

The waveform of output voltage v_0 is

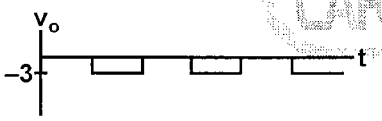


(a)



(b)

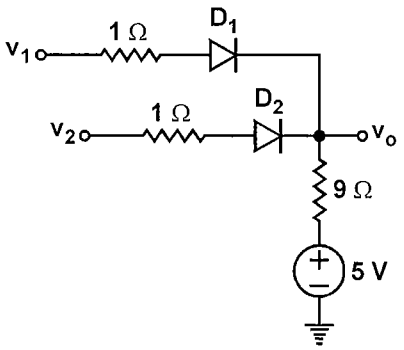
(c)



(d)

CAREER ENDEAVOUR

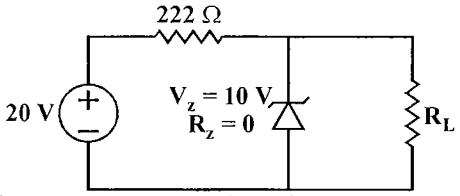
24. Consider the circuit shown below. Assume diodes are ideal.



If $v_1 = v_2 = 10$ V, then output voltage v_o is

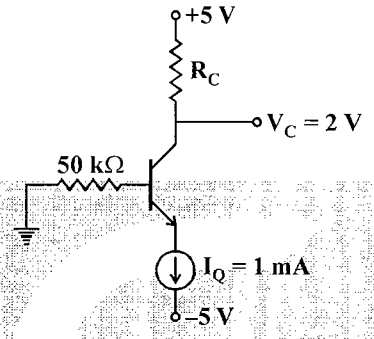
(a) 0 V (b) 9.737 V (c) 9 V (d) 9.5 V

5. In the voltage regulator circuit shown below the power rating of Zener diode is 400 mW. The value of R_L that will establish maximum power in Zener diode is



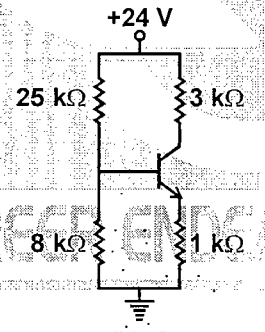
(a) 5 kΩ (b) 2 kΩ (c) 10 kΩ (d) 8 kΩ

6. The common-emitter current gain of the transistor is $\beta = 75$. The voltage V_{BE} in ON state is 0.7 V. The value of I_C and R_C is



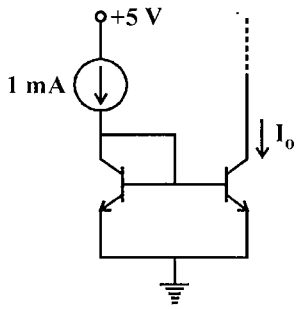
(a) 0.987 mA, 3.04 kΩ (b) 1.013 mA, 2.96 kΩ
(c) 0.946 mA, 4.18 kΩ (d) 1.057 mA, 3.96 kΩ

7. For the circuit shown below, let $\beta = 75$. The Q-point (I_{CQ}, V_{CEQ}) is



(a) (4.68 mA, 16.46 V) (b) (3.12 mA, 1.86 V)
(c) (3.12 mA, 8.46 V) (d) (4.68 mA, 5.22 V)

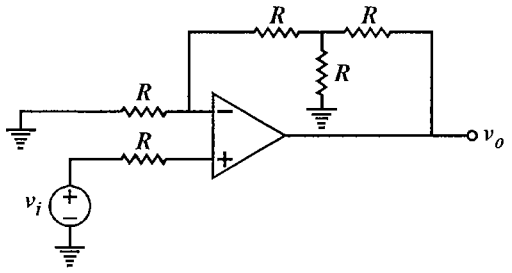
8. In the current mirror circuit shown below the transistor parameters are $V_{BE} = 0.7$ V, $\beta = 50$ and the Early voltage is infinite. Assume transistor are matched.



The output current I_o is

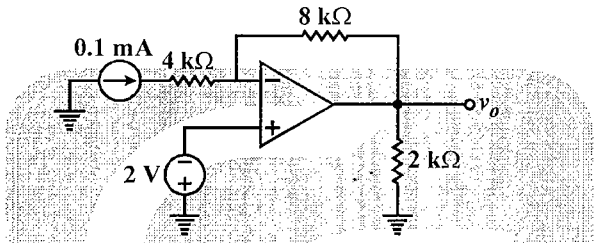
(a) 1.04 mA (b) 1.68 mA (c) 962 μA (d) 432 μA

29. For the circuit shown below the value of $A_v = \frac{v_o}{v_i}$ is



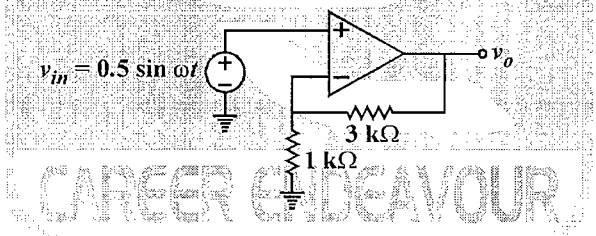
(a) 5 (b) –5 (c) 6 (d) –6

30. For the circuit shown below the value of v_o is



(a) –10 V (b) 18 V (c) –18 V (d) 28 V

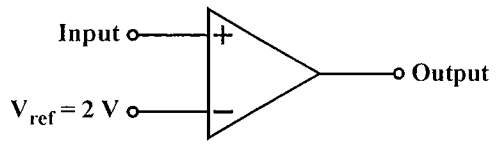
31. The op-amp shown in figure, has a slew rate of 1V/ns, what is the highest input frequency for which no slewing occurs



CAREER ENDEAVOUR

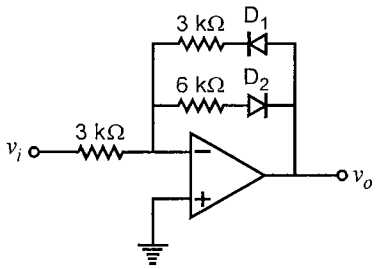
(a) 153.8 MHz (b) 63.7 MHz (c) 79.6 MHz (d) 127.4 MHz

32. If the input to the following ideal comparator is a sinusoidal signal of 8 V (peak to peak) without any DC componen., the output of the comparator has a duty cycle of



(a) 1/2 (b) 1/3 (c) 1/6 (d) 1/12

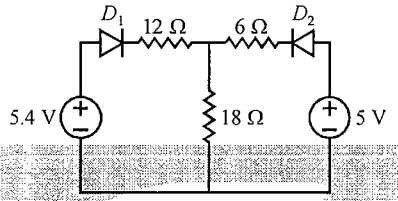
3. Consider the circuit shown below



If $v_i = 2$ V, then output v_o is

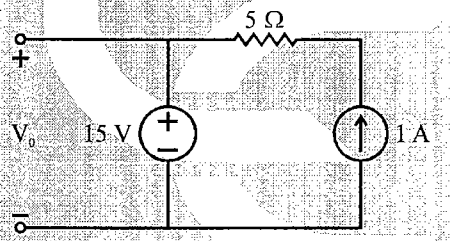
(a) 4 V (b) –4 V (c) 3 V (d) –3 V

4. In the circuit shown below diodes has cutin voltage of 0.6 V. The diode in ON state are



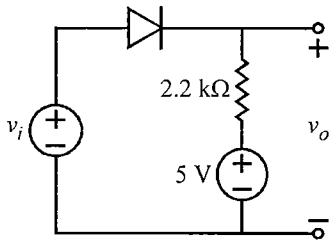
(a) only D_1 (b) only D_2 (c) both D_1 and D_2 (d) None of these

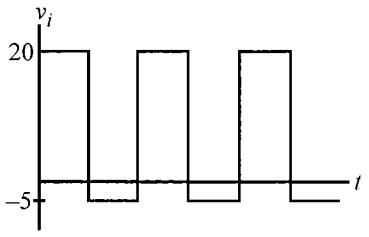
5. In the circuit of figure the value of voltage v_o is



(a) 10 V (b) 15 V (c) 20 V (d) None of these

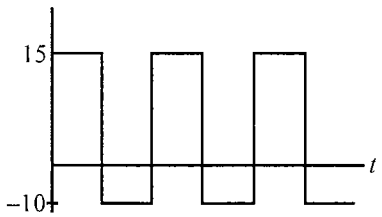
6. Consider the given a circuit and a waveform for the input voltage. The diode in circuit has cutin voltage $V_\gamma = 0$.



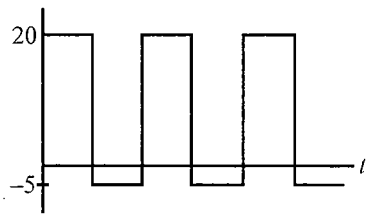


The waveform of output voltage v_o is

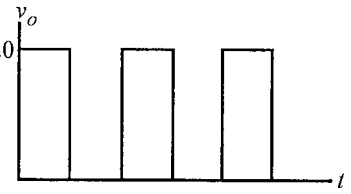
(a)



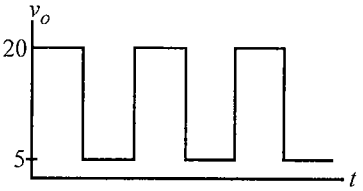
(b)



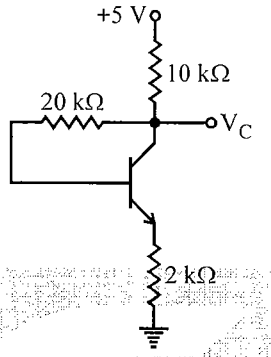
(c)



(d)

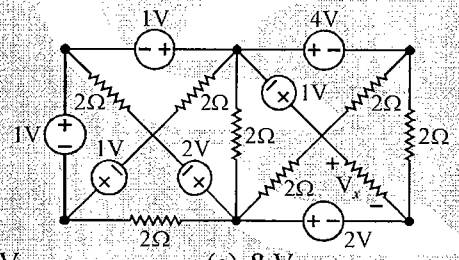


37. The common-emitter current gain of the transistor is $\beta = 75$. The voltage V_{BE} in ON state is 0.7 V. The value of V_C is



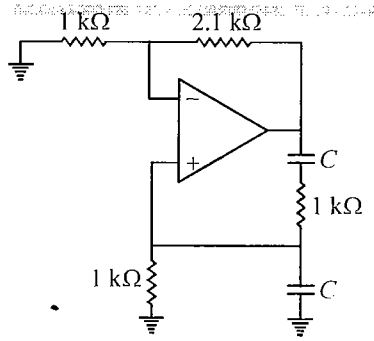
(a) 1.49 V (b) 2.9 V (c) 1.78 V (d) 2.3 V

38. In the circuit of figure the voltage V_x is



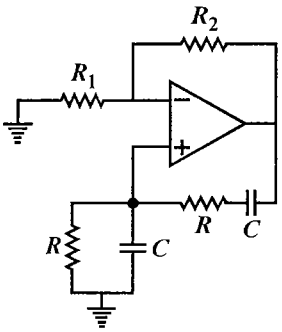
(a) 3 V (b) 2 V (c) 8 V (d) 4 V

39. The value of C required for sinusoidal oscillation of frequency 1 kHz in the following circuit is



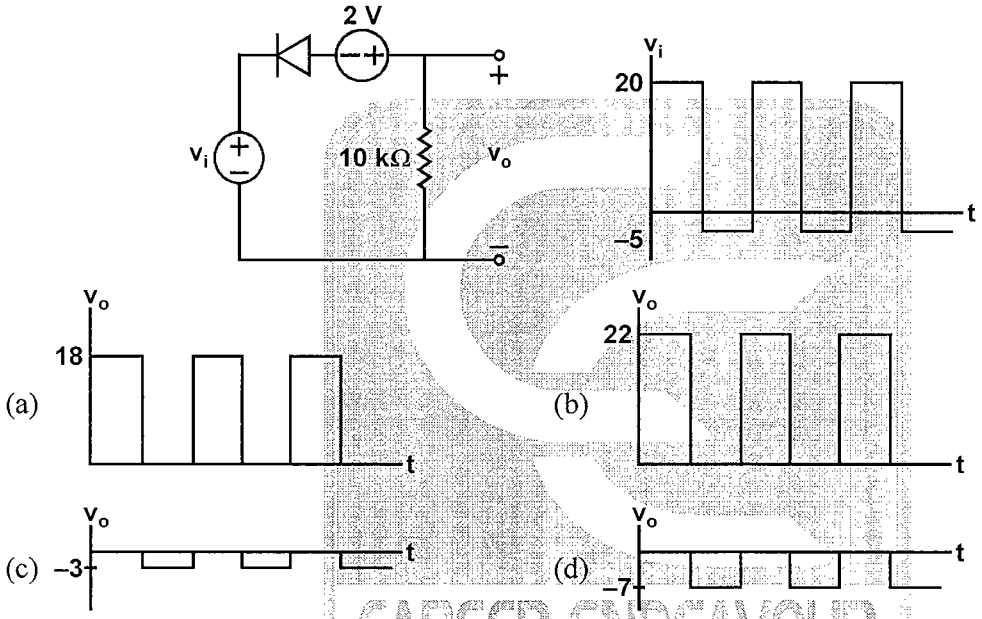
(a) $\frac{1}{2\pi}\mu F$ (b) $2\pi\,\mu F$ (c) $\frac{1}{2\pi\sqrt{6}}\mu F$ (d) $2\pi\sqrt{6}\,\mu F$

0. In the following circuit minimum required value of R_2/R_1 to sustain oscillation is

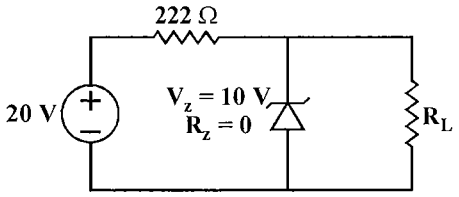


(a) 1.5 (b) 1 (c) 2 (d) 4

1. Consider the given a circuit and a waveform for the input voltage. The diode in circuit has cutin voltage $V_\gamma = 0$. The waveform of output voltage v_0 is

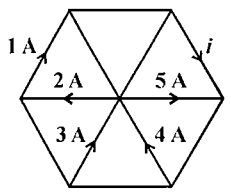


2. In the voltage regulator circuit shown below the power rating of Zener diode is 400 mW. The value of R_L that will establish maximum power in Zener diode is



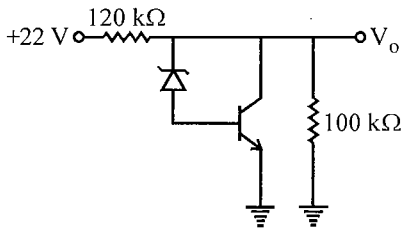
(a) 5 kΩ (b) 2 kΩ (c) 10 kΩ (d) 8 kΩ

3. For the circuit shown in figure the current i is



(a) 1 A (b) 2 A (c) 3 A (d) 4 A

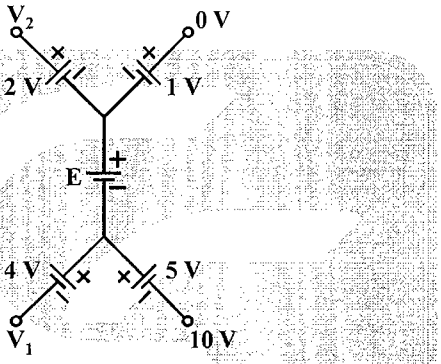
46. In the shunt regulator shown below, the $V_Z = 8.2$ V and $V_{BE} = 0.7$ V. The regulated output voltage V_o is



(a) 11.8 V (b) 7.5 V (c) 12.5 V (d) 8.9 V

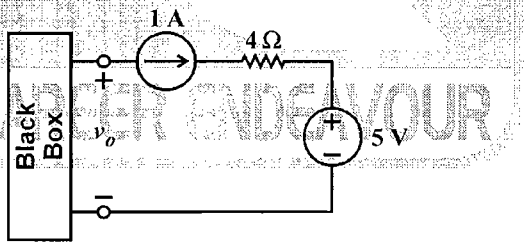
Multiple Select Questions (MSQ):

1. In the circuit of the figure, the value of the voltage source E is



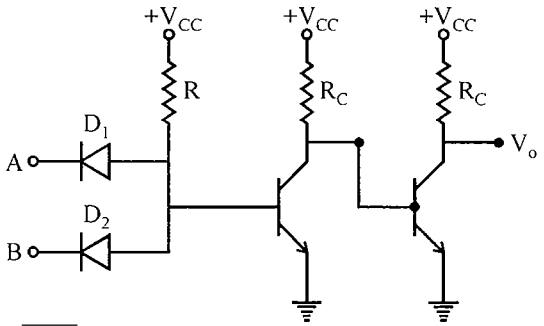
(a) –16 V (b) 4 V (c) –6 V (d) 16 V

2. The voltage v_o in figure is always equal to



(a) 1 V (b) 5 V (c) 9 V (d) None of these

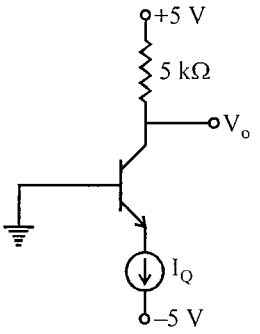
3. Below figure acts as :



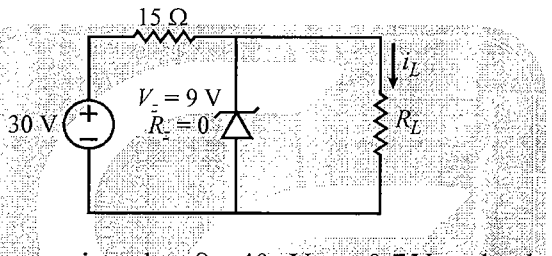
(a) $A \cdot B$ (b) $\overline{\overline{A} \cdot \overline{B}}$ (c) $\overline{\overline{A} + \overline{B}}$ (d) $A[\overline{A} + AB]$

Numerical Answer Type (NAT) :

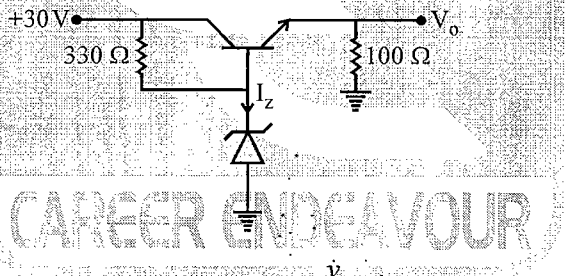
For the circuit shown below, $V_{CB} = 0.5\ V$ and $\beta = 100$. The value of I_Q is _____mA.



In the voltage regulator circuit shown below the maximum load current i_L that can be drawn is _____ mA.

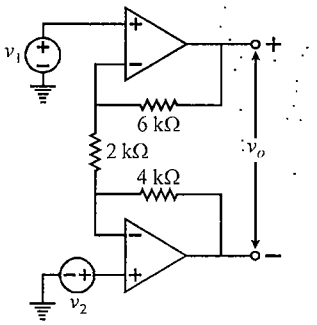


For circuit shown in figure, the transistor has $\beta = 40$, $V_{BE} = 0.7\ V$ and voltage across zener diode is 15 V. The current (in mA) through the zener diode is _____



CAREER ENDEAVOUR

For the circuit shown below the voltage gain $A_{vd} = \dfrac{v_o}{(v_1 - v_2)}$ is _____



Multiple Choice Questions (MCQ)

1. (b)	2. (d)	3. (d)	4. (a)	5. (a)
6. (b)	7. (b)	8. (a)	9. (b)	10. (a)
11. (a)	12. (c)	13. (b)	14. (b)	15. (d)
16. (a)	17. (a)	18. (a)	19. (b)	20. (d)
21. (a)	22. (c)	23. (c)	24. (b)	25. (b)
26. (a)	27. (d)	28. (c)	29. (a)	30. (a)
31. (c)	32. (b)	33. (b)	34. (c)	35. (b)
36. (d)	37. (a)	38. (d)	39. (a)	40. (c)
41. (c)	42. (b)	43. (a)	44. (d)	

Multiple Select Questions (MSQ)

1. (d) 2. (d) 3. (a),(c),(d)

Numerical Answer Type (NAT)

1. (0.85 – 0.95) 2. (1.3 to 1.5) 3. (40 to 43) 4. (6)

CAREER ENDEAVOUR

Chapter 6

DIGITAL ELECTRONICS

Boolean Algebra:

(1) $A+A=A$, (2) $A+1=1$, (3) $A+0=A$, additive identity

(4) $A+\overline{A}=1$, (5) $A.A=A$, (6) $A.1=A$ multiplicative identity

(7) $A.0=0$, (8) $A.\overline{A}=0$, (9) $\overline{\overline{A}}=A$,

(10) $A+A.B=A$, (11) $A(A+B)=A$ (12) $A.(\overline{A}+B)=AB$

(13) $A+\overline{A}B=A+B$, (14) $\overline{A}+AB=\overline{A}+B$, (15) $A(\overline{A}+B)=AB$

Note: $A+BC=(A+B)(A+C)$

Boolean Law:

) **Commutative Law:** $A+B=B+A, AB=B.A$

) **Associative Law:** $A+(B+C)=(A+B)+C$, $A.(B.C)=(A.B).C$

) **Distributive Law:** $A+B.C=(A+B)(A+C)$, $A.(B+C)=AB+AC$

) **Demorgan's Law:** $\overline{A.B}=\overline{A}+\overline{B}$,

$\overline{A+B}=\overline{A}.\overline{B}$

CAREER ENDEAVOUR

SOLVED PROBLEMS

The Boolean expression $(X+Y)(X+\overline{Y})(\overline{X}+Y)$ is equivalent to

(a) $\overline{X}Y$ (b) $X\overline{Y}$ (c) XY (d) $\overline{X}\overline{Y}$

)ln. $(X+Y)(X+\overline{Y})(\overline{X}+Y)=(X+Y)(XY+\overline{X}\overline{Y})=XY+XY=XY$

Given that $AB+\overline{A}C+BC=AB+\overline{A}C$, then $(\overline{A}+C)(B+C)(A+B)$ is equivalent to

(a) $(\overline{A}+B)(A+C)$ (b) $(A+B)(\overline{A}+C)$

(c) $(A+\overline{B})(\overline{A}+C)$ (d) $(A+\overline{B})(\overline{A}+\overline{C})$

)ln. Using duality $(A+B)(\overline{A}+C)(B+C)=(A+B)(\overline{A}+C)$

Expression $A+\overline{A}B+\overline{A}\overline{B}C+\overline{A}\overline{B}\overline{C}D+\overline{A}\overline{B}\overline{C}\overline{D}E$ would be similar to

(a) $A+\overline{A}B+CD+E$ (b) $A+B+CDE$

(c) $A+BC+CD+DE$ (d) $A+B+C+D+E$

Soln. $F = A + \bar{A}B + \bar{A}\bar{B}C + \bar{A}\bar{B}\bar{C}(D + \bar{D}E)$

$= A + \bar{A}B + \bar{A}\bar{B}(C + \bar{C}(D + E)) = A + \bar{A}(B + \bar{B}(C + D + E)) = A + B + C + D + E$

Complement and Dual of Expression:

For calculating dual of a expression multiply is replaced by plus sign, while plus sign is replaced by multiply. After calculating dual primed variable are replaced by unpaired. Variable while unprimed by primed variables.

4. Calculate dual complement of given expression $Y(A,B,C) = A\bar{B}C + \bar{A}B\bar{C}$

$Y_D(A,B,C) = (A + \bar{B} + C)(\bar{A} + B + \bar{C})$, $\bar{Y}(A,B,C) = (\bar{A} + B + \bar{C})(A + \bar{B} + C)$

Minterms:

For n variables we have 2^n minterms. Here each minterm is obtained by from AND operation of n variables of n variables with each variable being primed if corresponding bit of binding number is zero unprimed if corresponding bit is 1. Sum of all minterms is always equal to 1.

Minterms for No. 110 is $AB\bar{C}$.

Maxterms:

It is nothing but simply complement of minterm and each minterm is complement of maxterm.

Maximum term for 011 is Maxterm for number 110 is $\bar{A} + \bar{B} + C$.

SOP $\rightarrow$ SUM OF PRODUCT, POS $\rightarrow$ PRODUCT OF SUM

Examples:

f(A, B, C) = $\sum(0, 3, 5, 6, 7)$ ..... SOP, f(A, B, C) = $\pi(0, 3, 5, 6, 7)$ ..... POS

5. f(A, B, C) = $\sum(0, 5, 6)$ ..... POS

Soln. $m_0 + m_5 + m_6 = \bar{A}\bar{B}\bar{C} + A\bar{B}C + AB\bar{C}$

6. f(A, B, C) = $\pi(0, 2, 4, 7)$ ..... SOP = $M_0M_2M_4M_7$

$= (A + B + C)(A + \bar{B} + C)(\bar{A} + B + C)(\bar{A} + \bar{B} + \bar{C})$

Deriving sum of product (SOP) expression from the truth table

Derive a minterm expression for the following table

Input			Output
C	B	A	Y
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	0

$Y = \bar{A}\bar{B}C + \bar{A}BC + A\bar{B}\bar{C} + AB\bar{C}$

Deriving product of sum (POS) expression from a truth table

Derive a Maxterm expression for the following truth table

Input			Output
C	B	A	Y
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	0

$Y = (A+B+C)(A+\overline{B}+C)(\overline{A}+B+C)(\overline{A}+\overline{B}+\overline{C})$

Binary Number:

A decimal number such as 7392 represents a quantity equal to 7 thousands, plus 3 hundreds, plus 9 tens, plus 2 units. The thousands, hundreds etc. are power of 10 implied by the position of the coefficients. To be more exact, 7392 should be written as

$7 \times 10^3 + 3 \times 10^2 + 9 \times 10 + 2 \times 10^6$

In general, a number with a decimal point is represented by a series of coefficients as follows:

$a_5 a_4 a_3 a_2 a_1 . a_{-1} a_{-2} a_{-3}$

The decimal number system is said to be of base or radix, 10 because it uses ten digits and the coefficients are multiplied by the power of 10. The binary system is a different number system. The coefficients of the binary system have two possible values: 0 and 1 each coefficient multiplied by 2^J.

For example the decimal equivalent of the binary number 11010.11 is 26.75

$1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 + 1 \times 2^{-1} \times 1 \times 2^{-2} = 26.75$

In the hexadecimal (base 16) number system, the first term digits are borrowed from the decimal system. The letters A, B, C, D, E and F are used for digits 10, 11, 12, 13, 14 and 15 respectively. An example of a hexadecimal number is

$(B65F)_{16} = 11 \times 16^3 + 6 \times 16^2 + 5 \times 16 + 15 = (46687)_{10}$

CAREER ENDEAVOUR

Number base conversion:

A binary number can be converted to decimal by forming the sum of the powers of 2 of those coefficients whose value is 1. For example

$(1010.011)_2 = 2^3 + 2^1 + 2^{-2} + 2^{-3} = (10.375)_{10}$

The conversion from decimal to binary or to any other base-*r* system is more convenient if the number is separated into an integer part and fraction part and the conversion of each part done separately. The conversion of an integer from decimal to binary is best explained by example.

Integer quotient	Remainder	Coefficient
$\frac{41}{2} = 20$	$+\frac{1}{2}$	$a_0 = 1$
$\frac{20}{2} = 10$	$+0$	$a_1 = 0$
$\frac{10}{2} = 5$	$+0$	$a_2 = 0$

$\frac{5}{2}=2$ $+\frac{1}{2}$ $a_3=1$

$\frac{2}{2}=1$ $+0$ $a_4=0$

$\frac{1}{2}=0$ $+\frac{1}{2}$ $a_5=1$

Answer $(41)_{10} = (a_5a_4a_3a_2a_1a_0)_2 = (101001)_2$

The arithmetic process can be manipulated more conveniently as follows:

Integer	Remainder
41	
20	1
10	0
5	0
2	1
1	0
0	1

Answer: 101001

The conversion from decimal integers to any base-r system is similar to the example, except that division is done by r instead of 2.

The conversion of a decimal fraction to binary is accomplished by a method similar to that used for integers. However, multiplication is used instead of division and integers are accumulated instead of remainders.

7. Convert $(0.6875)_{10}$ to binary.

	Integer	Fraction	Coefficient
Soln.	$0.6875 \times 2 = 1+$	0.3750	$a_{-1} = 1$
	$0.3750 \times 2 = 0+$	0.7500	$a_{-2} = 0$
	$0.7500 \times 2 = 1+$	0.5000	$a_{-3} = 1$
	$0.5000 \times 2 = 1+$	0.0000	$a_{-4} = 1$

$(0.6875)_{10} = (0.a_{-1}\ a_{-2}\ a_{-3}\ a_{-4}) = (0.1011)_2$

8. Convert $(0.513)_{10}$ to octal

Soln. $0.513 \times 8 = 4.104$

$0.104 \times 8 = 0.832$

$0.832 \times 8 = 6.656$

$0.656 \times 8 = 5.248$

$0.248 \times 8 = 1.984$

$0.984 \times 8 = 7.872$

The answer to seven significant figures is obtained from the integer part of the product.

$(0.513)_{10} = (0.406517....)_8$

The conversion of decimal number with both integer and fraction parts is done by converting the integer and fraction separately and then combining the two answers.

Octal and hexadecimal numbers:

The conversion from the binary to octal and hexadecimal plays an important part in digital computers. Since $2^3 = 8$ and $2^4 = 16$, each octal digit corresponds to three binary digits and each hexagonal digit corresponds to four binary digits. The conversion from binary to octal is easily accomplished by partitioning the binary number into groups of three digits each, starting from the binary point and proceeding to the left and to the right. The corresponding octal digit is then assigned to each group. The following examples illustrates the procedure.

$(\underbrace{10}\ \underbrace{110}\ \underbrace{001}\ \underbrace{101}\ \underbrace{011}\ \underbrace{111}\ \underbrace{100}\ \underbrace{000}\ \underbrace{110})_2 = (26153.7460)_8$

Conversion from binary to hexadecimal is similar, except that the binary number is divided into groups of four digits:

$(\underbrace{10}_{2}\ \underbrace{1100}_{C}\ \underbrace{0110}_{6}\ \underbrace{1011}_{B}\ \underbrace{1111}_{F}\ \underbrace{0010}_{2})_2 = (2C6B.F2)_{16}$

Conversion from octal or hexadecimal to binary is done by a procedure reverse to the above. Each octal digit is converted to its three digit binary equivalent. Similarly each hexadecimal digit is converted to its four-digit binary equivalent. This is illustrated in the following examples:

$(673.124)_8 = (\underbrace{110}_{6}\ \underbrace{111}_{7}\ \underbrace{011}_{3}\ \underbrace{001}_{1}\ \underbrace{010}_{2}\ \underbrace{100}_{4})_2$

$(306.D)_{16} = (\underbrace{0011}_{3}\ \underbrace{0000}_{0}\ \underbrace{0110}_{6}\ \underbrace{1101}_{D})_2$

Logic Gates: Positive Logic: logic 0 = 0V, logic 1 = 5V

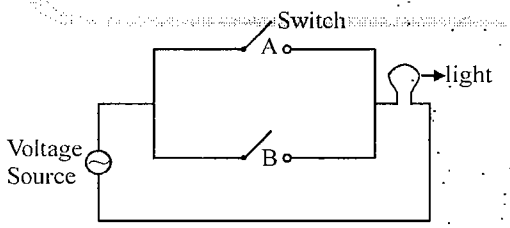
Negative Logic: logic 1 = 0V, logic 0 = 5V.

Logic-Gates: They are the most basic and most important building blocks of any digital system including computers. Three logic gates are AND, OR and NOT gates. These three gates together can be used to construct the logic circuit for any given logic or boolean expression.

OR Gates: $\boxed{Y = A + B}$

It is a logic circuit with 2 or more than two inputs and one output.

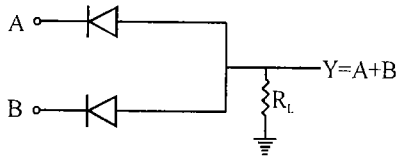
Switching Circuit of OR Gate:



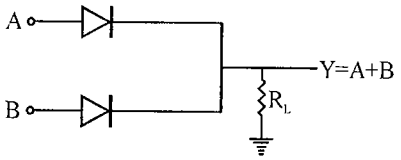
Truth Table for OR gate:

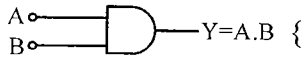
A	B	$Y = A + B$
0	0	0
0	1	1
1	0	1
1	1	1

(1) Negative Logic OR Gate:



(2) Positive Logic OR Gate:

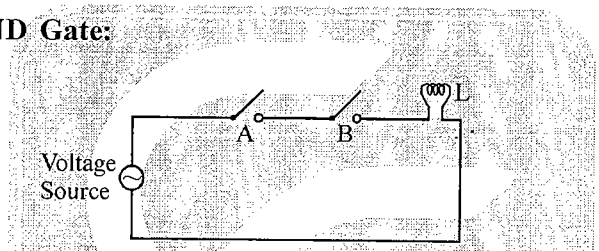


AND Gate: $\boxed{Y = A.B}$ 

{Symbol of AND Gate}

It is a logic circuit having two or more than two input and one output.

Switching Circuit of AND Gate:



Truth Table:

A	B	$Y = A.B$
0	0	0
0	1	0
1	0	0
1	1	1

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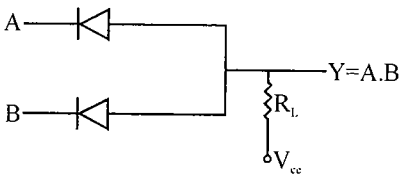
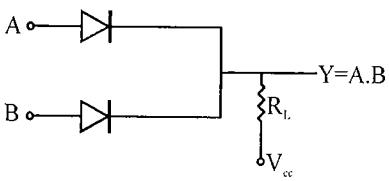
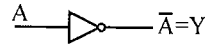
(1) Positive logic AND gate:



(2) Negative logic AND gate:



Note: Negative logic OR gate is same as positive logic AND gate and vice-versa while positive logic OR gate is same as negative logic AND gate and vice-versa.

NOT Gate: $\boxed{Y = \overline{A}}$ 

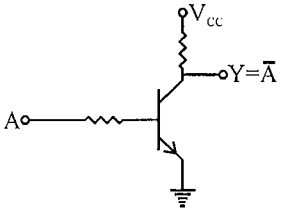
(Symbol of NOT Gate)

It is one input and one output logic gate whose output is always the complement of the Input.

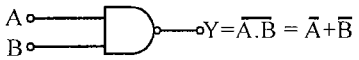
Truth Table:

A	$Y=\bar{A}$
0	1
1	0

Switch Circuit of NOT Gate:



NAND Gate: NAND stands for NOT-AND. An AND gate followed by a NOT circuit makes it a NAND gate. The truth table of a NAND gate is obtained from truth table of an AND gate by complementing the output entries.



Input		Output
A	B	NAND
0	0	1
0	1	1
1	0	1
1	1	0

When both inputs are high in a NAND gate the output is low. NAND gate gives zero output only in high input condition.

Switching Circuit of NAND Gate:

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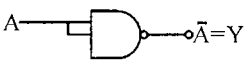
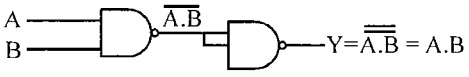
he NAND gate is called an universal gates as all functions OR, AND, NOT can be realized from t

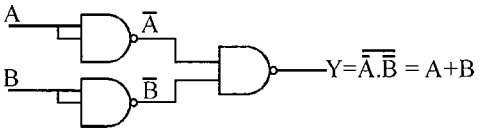
OT:



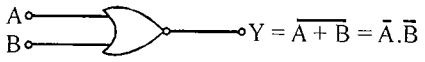
ND:



OR:



NOR Gate: NOR stands for NOT-OR. Thus the NOR Gate output is obtained by complementing the output of OR gate. Thus output of NOR gate logic 1 when all its input are logic 0. For all other cases the output is logic 0. NOR gate also called as universal gate because all the gates can be designed by this gate.



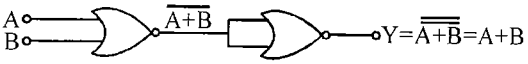
Input		Output
A	B	NOR
0	0	1
0	1	0
1	0	0
1	1	0

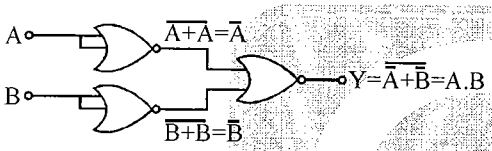
Output of NOR will gate be high when all of its input are low.

NOR gates as universal Gate:

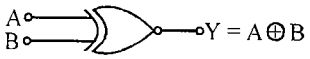
OR, NOT, AND can be derived using only NOR gate.

NOT:

OR: 

AND: 

XOR/EX-OR Gate: The output of 2 input EX-OR gate is logic 1 when the two inputs are unequal and output is logic 0 when two inputs are equal (Odd function)



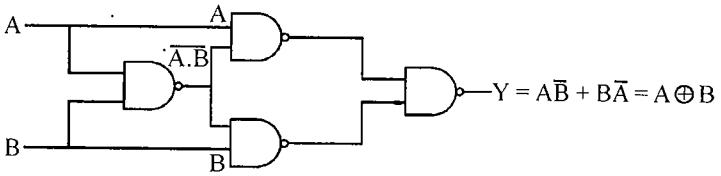
Truth Table of EX-OR Gate:

Input		Output
A	B	XOR
0	0	0
0	1	1
1	0	1
1	1	0

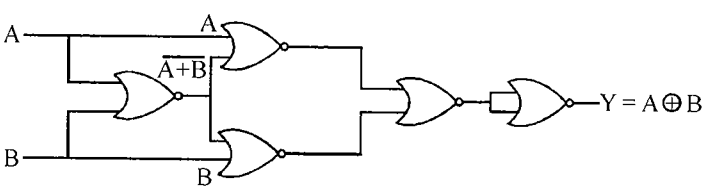
The symbol ⊕ means the terms are XORed together. $Y = A\bar{B} + \bar{A}B$ is boolean expression for EX-OR.

Implementation of EX-OR gate with 2 input NAND and NOR gate:

Using NAND Gate:



Using NOR Gate:



Application of EX-OR/XOR Gate:

1. XOR Gate can be used as an inequality comparator.
2. It can also be used as controlled inverter.
3. It can be used for complementing in adder circuit.
4. XOR is an important logic gate used in arithmetic operations, code conversion, and in error detector and correction circuit.

EX-NOR Gate: The output of the XOR Gate is connected to an inverter and the output taken.

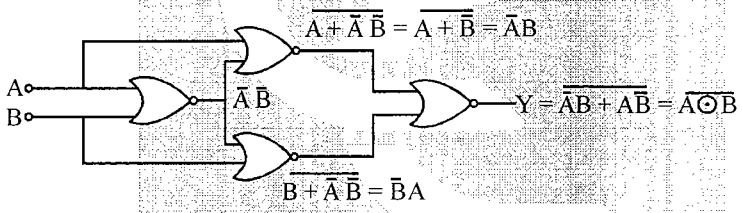
Example:

Truth Table:

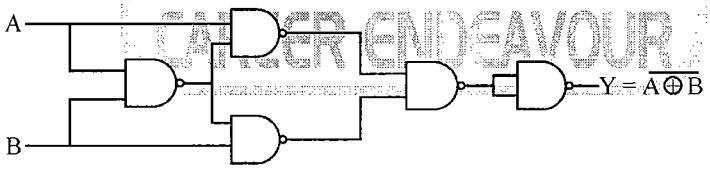
Input		**Output**
A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

So, If XOR is an odd number detector then XNOR is an even number detector.

Implementation of EX-NOR gate with 2 input NOR and NAND gate.


$\overline{A + \bar{A}\bar{B}} = \overline{A + \bar{B}} = \bar{A}B$
A
B
$\bar{A}\bar{B}$
$Y = \overline{\bar{A}B + A\bar{B}} = \overline{A \odot B}$
$\overline{B + \bar{A}\bar{B}} = \bar{B}A$

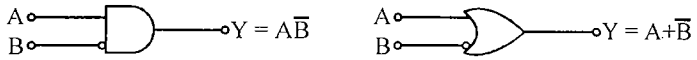
Using NOR Gate



Using NAND Gate

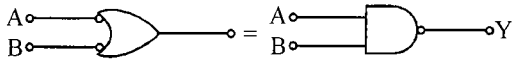
Logic gate	*No. of NAND gate used*	*No. of NOR gate used*
NOT	1	1
AND	2	3
OR	3	2
EX – OR	4	5
EX – NOR	5	4
NOR	4	1
NAND	1	4

Inhibitor: If one of the inputs to an AND or OR gate is inverted it becomes an INHIBITOR. An inhibitor is also known as AND-NOT or OR-NOT gate.

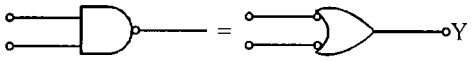


If we add on inverter at the output only the outputs get inverted. How ever, if we invert the input, the gate functions change.

1. If both the inputs to one OR gate are inverted, the gate becomes a NAND gate.



2. If both the inputs to an NAND gate are inverted, the gate becomes a NOR gate.
If we invert both inputs and outputs.



If we invert both inputs and outputs.

Design of a Boolean expression by minimum number of NAND gate.

1st Step: Write boolean expression is form of AND and OR gate.
2nd Step: Place a bubble at the output of each AND gate and place a bubble at each input of OR gate.
3rd Step: NOT followed by NOT means original literal.

Demorgans Theorem:

$\overline{ABC} = \bar{A} + \bar{B} + \bar{C}$, $\overline{A + B + C} = \bar{A}.\bar{B}.\bar{C}$

1. Minimise logic expression

$(A + B)(A + C) \rightarrow A + BC$

Note: A is present in both the bracket

2. $(\bar{X} + Y)(\bar{X} + Z) \Rightarrow \bar{X} + YZ$

Note: $\bar{X}$ is present in both the bracket

3. $(\underline{A + B} + C)(A + \bar{B} + C)(\underline{A + B} + \bar{C})$

$\Rightarrow (\underline{A} + B)(\underline{A} + \bar{B} + C)$

$\Rightarrow A + B(\bar{B} + C)$

$\Rightarrow A + BC$

Concensns Theorem:

$AB + \bar{A}C + (BC) \rightarrow$ redundant term

$\Rightarrow AB + \bar{A}C + BC(A + \bar{A})$

$\Rightarrow AB + \bar{A}C + BCA + BC\bar{A}$

$\Rightarrow AB + \bar{A}C$

Note: (1) Three variable

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(2) Each variable is used twice as pair

(3) Only one variable is complemented

$\bar{A}\bar{B}+\bar{B}\bar{C}+A\bar{C} \Rightarrow \bar{A}\bar{B}+A\bar{C}$

Transposition Theorem:

$(A+B)(\bar{A}+C)$

$\Rightarrow AC+\bar{A}B+BC$

$\Rightarrow AC+\bar{A}B$

SOLVED PROBLEMS

Reduce $AB+ABC+\bar{A}B+A\bar{B}C$ using law's of Boolean algebra **[BHU 2012]**

(a) $B+AC$ (b) $A+BC$ (c) $C+AB$ (d) $A+B+C$

Soln. $AB+ABC+\bar{A}B+A\bar{B}C$

$\Rightarrow \quad (A+\bar{A})B+AC(B+\bar{B})$

$= B+AC \qquad [A+\bar{A}=1]$

Correct option is (a)

10101 binary number corresponds to decimal number **[BHU 2012]**

(a) 31 (b) 21 (c) 11 (d) 3

Soln. 10101

$=1\times2^4+0\times2^3+1\times2^2+0\times2^1+1\times2^0=16+4+1=21$

Correct option is (b)

The unique output for a NAND logic gate is a 0 : **[BHU 2014]**

(a) when all inputs are 0 (b) when all inputs are 1

(c) when any one input is 0 (d) when any one input is 1

Soln. $y=\overline{A\cdot B}$

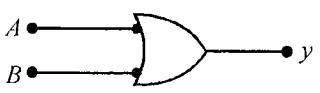
A	B	y
1	1	0
0	1	1
1	0	1
0	0	1

Therefore, correct option is (b)

Bubbled (inputs inverted) OR gate is equivalent to **[BHU 2015]**

(a) NOR gate (b) NAND gate (c) NOT gate (d) XNOR gate

Soln.



$y=\bar{A}+\bar{B}=\overline{AB}$ it is the output of NAND gate

Correct option is (b)

5. The simplified Boolean expression for the given Boolean expression $AB+(A+B)\cdot(B+C)$ is

(a) $B+AC$ (b) $C+AB$ (c) $A+BC$ (d) $AB+AC$ **[BHU 2015]**

Soln. $y=AB+(A+B)\cdot(B+C)$

$=AB+B+AC$

$=(A+1)B+AC$

$=B+AC$

Correct option is (a)

6. The Boolean expression $AB\overline{C}+BC\overline{A}+CA\overline{B}+ABC$ is equivalent to the simplified Boolean expression in POS form **[BHU 2015]**

(a) $(A+B)\cdot(B+C)\cdot(C+A)$ (b) $(A+\overline{B})\cdot(B+\overline{C})\cdot(C+\overline{A})$

(c) $(\overline{A}+\overline{B})\cdot(\overline{B}+\overline{C})\cdot(\overline{C}+\overline{A})$ (d) $(\overline{A}+B)\cdot(\overline{B}+C)\cdot(\overline{C}+A)$

Soln. $AB\overline{C}+BC\overline{A}+CA\overline{B}+ABC$

$=AB\overline{C}+BC\overline{A}+CA\overline{B}+ABC+ABC+ABC$

$=AB(\overline{C}+C)+BC\overline{A}+CA\overline{B}+ABC+ABC$

$=AB+BC\overline{A}+CA\overline{B}+ABC+ABC$

$=AB+BC+CA$

$=AB(C+1)+BC+CA$

$=ABC+AB+BC+CA$

$=ABC+ABC+AB+AB+BC+CA+CA$

$=(A+B)\cdot(B+C)\cdot(C+A)$

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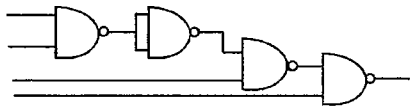
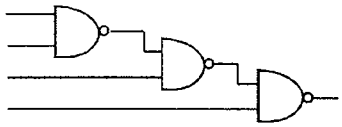
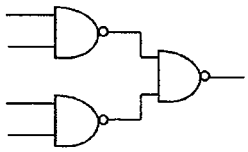
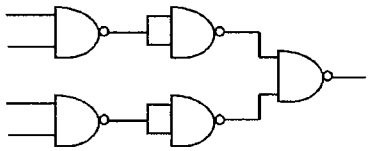
Correct option is (a)

7. Which of the following circuits will act like a 4-input NAND gate? **[JEST]**

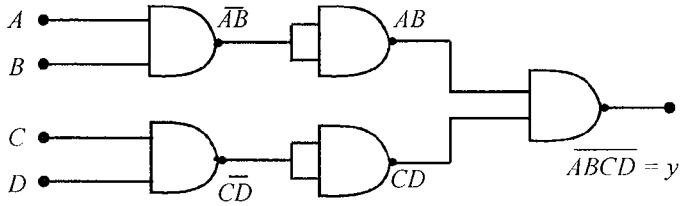
(a) 

(b) 

(c) 

(d) 

Soln.



Correct option is (d)

. For the logic circuit shown in figure 4, the required input condition (A, B, C) to make the output (X) = 1 is,

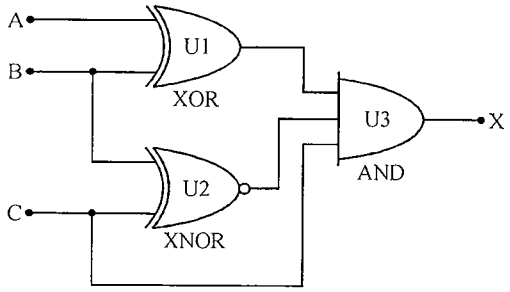
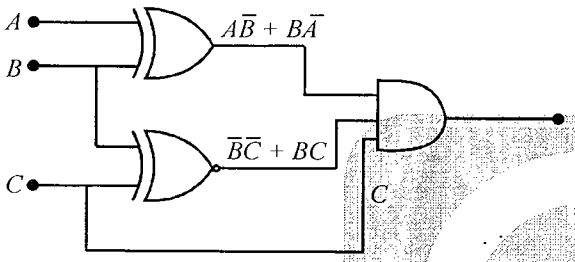


Figure 4

[JEST 2015]

(a) 1, 0, 1 (b) 0, 0, 1 (c) 1, 1, 1 (d) 0, 1, 1

oln.



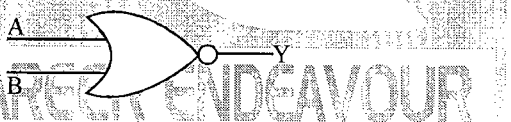
$$X = \left(\bar{A}B + \bar{B}A\right)\cdot\left(\bar{B}\bar{C} + BC\right)\cdot C$$

$$= \left(\bar{A}B + \bar{B}A\right)\left(BC\right) = \bar{A}BC$$

therefore, to get output X = 1 input will be 0, 1, 1

Correct option is (d)

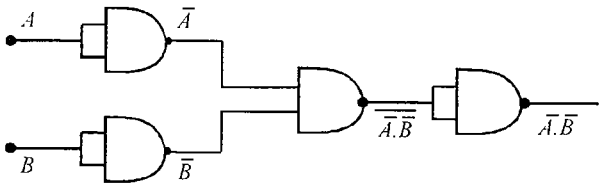
Consider the circuit shown below. **[TIFR 2012]**



The minimum number of NAND gates required to design this circuit is

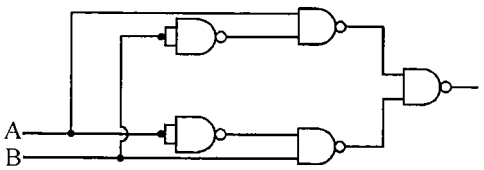
(a) 6 (b) 5 (c) 4 (d) 3

oln. $y = \overline{A + B} = \bar{A}\cdot\bar{B}$



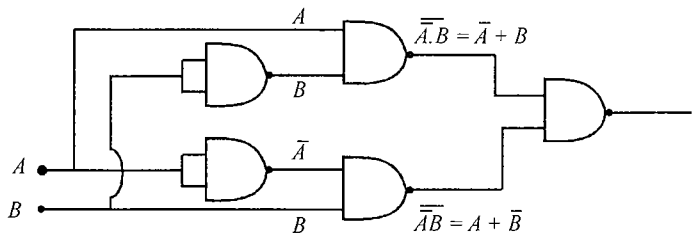
Correct option is (c)

). The circuit shown below uses only NAND gates. Find the final output. **[TIFR 2013]**



(a) A XOR B (b) A OR B (c) A AND B (d) A NOR B

Soln.



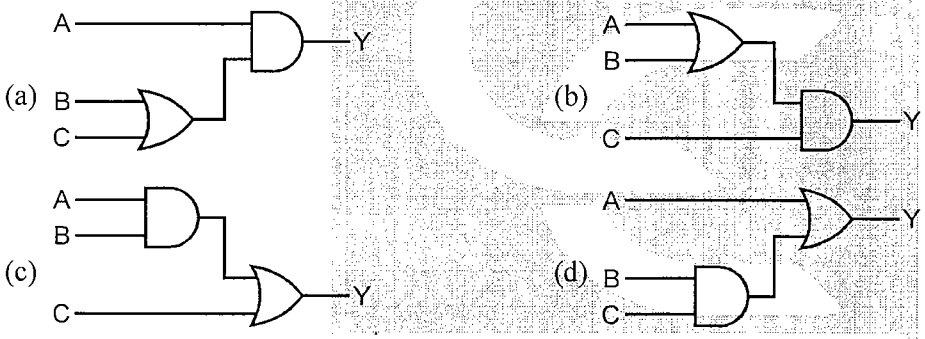
$$\overline{(\overline{A}+B)\cdot(A+\overline{B})} = \overline{\overline{A}+B} + \overline{A+\overline{B}} = A\overline{B} + \overline{A}B = A\ XOR\ B$$

Correct option is (a)

11. In a digital circuit for three input signals (A, B and C) the final output (Y) should be such that for inputs

A	B	C
0	0	0
0	0	1
0	1	0

the output (Y) should be low and for all other cases it should be high. Which of the following digital circuits will give such output ? **[TIFR 2016]**



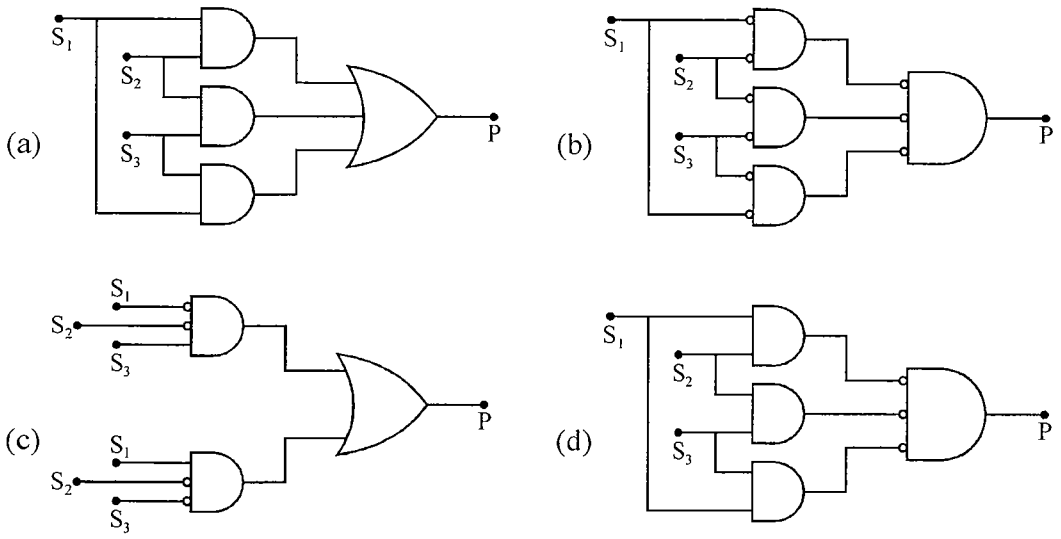
Soln.

A	B	C	y
0	0	0	0
0	0	1	0
0	1	0	0
1	0	0	1

Only option (d) will give such output.

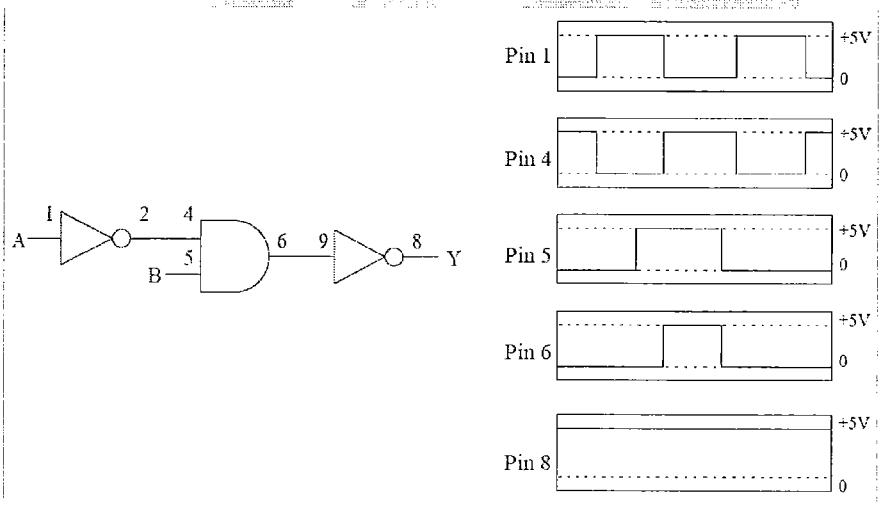
Correct option is (d)

12. A building has three overhead water tanks, each fitted with a sensor (S_1, S_2, S_3) which goes to 0 when the water level in the tank falls below a set value and remains 1 otherwise. A common pump is used to raise water from an underground storage tank to these overhead tanks. Of the following circuits, which one will turn on (P = 1) the pump only when at least two of the tanks have water level below the set value ? **[TIFR 2015]**



oln. According to given condition, the output P will be 1 when any two input will be zero.
Only option (d) satisfy the given condition.

3. The digital electronic circuit shown below (left side) has some problem and is not performing as intended. The voltage at each pin as a function of time is shown in the adjacent figures. **[TIFR 2011]**



The problem in the about circuit may be that

(a) the Pin 6 is shorted to ground
(b) the input inverter is shorted
(c) the Pin 8 is clamped to +5 V
(d) OR gate is used instead of AND gate

oln. $\Rightarrow$ Pin–1 is like input
$\Rightarrow$ Pin–2 = Pin–4
$\Rightarrow$ $\text{Pin} - 2 = \overline{\text{Pin} - 1}$
$\Rightarrow$ Inverter
$\Rightarrow$ here, Pin–4 = Pin–2
$\Rightarrow$ Pin–4 is inverted output of Pin–1, which is correct.
$\Rightarrow$ Pin–6 will be high when both Pin–4 and Pin–5 will be high, which is correct.

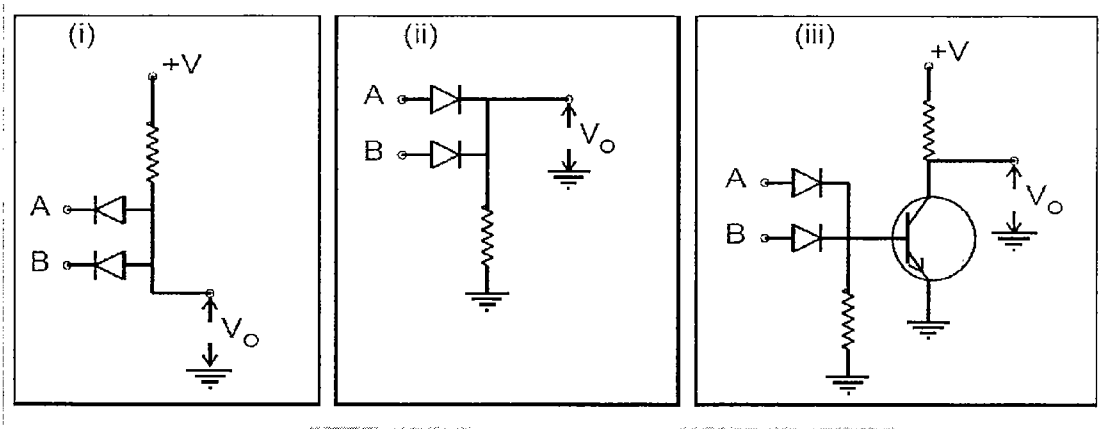
$\text{Pin} - 8 = \overline{\text{Pin} - 9} = \overline{\text{Pin} - 6}$

$\Rightarrow$ Pin–8 is having incorrect output

$\Rightarrow$ Problem is Pin–8 is clamped to +5V/high

Correct option is (c)

14. The three electronic circuits marked (i), (ii) and (iii) in the figure below can all work as logic gates, where the input signals are either 0V or 5 V and the output is V_0. **[TIFR 2011]**



Identify the correct combination of logic gates (i), (ii), (iii), in the options given below.

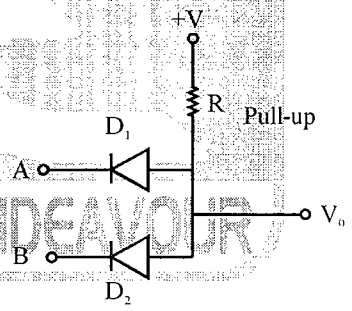
(a) NOR, XOR, AND (b) OR, NAND, NOR

(c) NAND, AND, XOR (d) XOR, AND, NAND

(e) AND, OR, NOR (f) NOR, NAND, OR

Soln.

A	B	D_1	D_2	V_0
0	0	ON	ON	0
0	1	ON	OFF	0
1	0	OFF	ON	0
1	1	OFF	OFF	1



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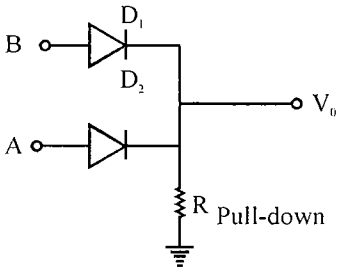
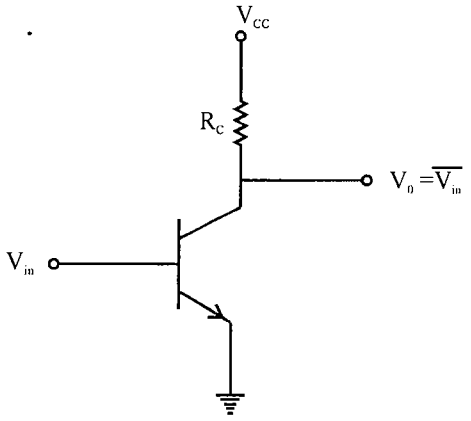
Hence, it is AND gate

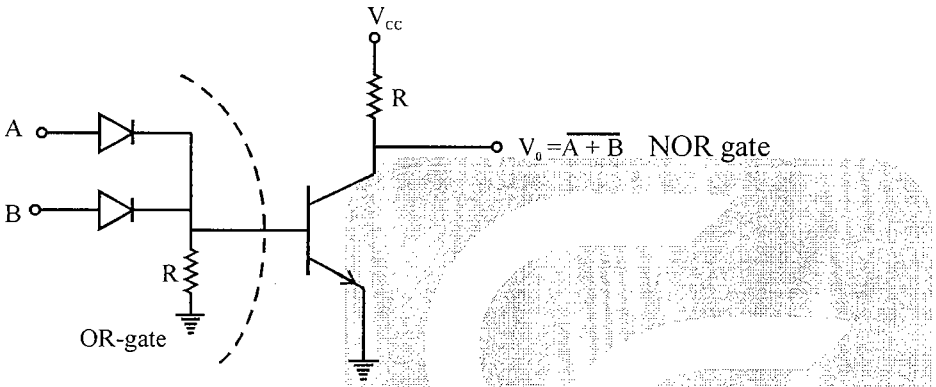
A	B	D_1	D_2	V_0
0	0	OFF	OFF	0
0	1	OFF	ON	1
1	0	ON	OFF	1
1	1	ON	ON	1

hence, OR gate





V_{in}	V_0
0	1
1	0



Correct option is (e)

5. To measure the voltage in the range 0 – 5 V with a precision of 5 mV, the minimum number of bits required in a digital voltmeter is **[TIFR 2015]**

(a) 9 (b) 10 (c) 11 (d) 12

Soln. S = step size = 5 mA

$$S = \frac{V_H - V_L}{2^n}$$

Where, n = number of bits

V_H = high voltage value

V_L = low voltage value

$$5\text{mV} = \frac{5-0}{2^n} \Rightarrow 2^n = 1000 \Rightarrow \boxed{n = 10}$$

Since, n is number of bits (integer)

Correct option is (b)

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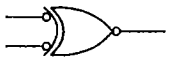
PRACTICE SET

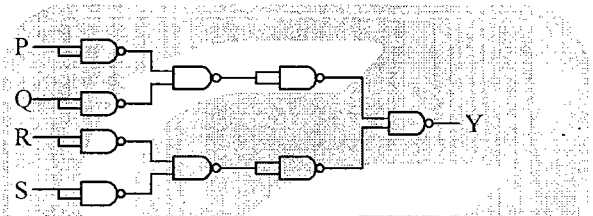
Multiple Choice Questions (MCQ):

1. Match the logic gates in Column A with their equivalents in Column B.

Column - A	Column - B
P.	1. 
Q.	2.
R.	3.
S.	4.

(a) P-2, Q-4, R-1, S-3 (b) P-4, Q-2, R-1, S-3
(c) P-2, Q-4, R-3, S-1 (d) P-4, Q-2, R-3, S-1

2. For the circuit shown in Fig. the boolean expression for the output Y in terms of inputs, P, Q, R and S is

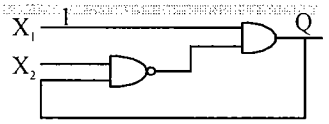


(a) $\overline{P}+\overline{Q}+\overline{R}+\overline{S}$ (b) $P+Q+R+S$ (c) $(\overline{P}+\overline{Q})(\overline{R}+\overline{S})$ (d) $(P+Q)(R+S)$

3. The minimum number of NOR gates required to implement $A(A+\overline{B})(A+\overline{B}+C)$ is equal to
(a) 0 (b) 3 (c) 4 (d) 7

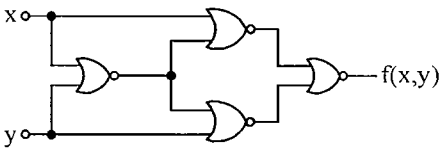
4. If $(211)_x = (152)_8$, then the value of base x is
(a) 6 (b) 5 (c) 7 (d) 9

5. In the figure, as long $X_1 = 1$ and $X_2 = 1$, the output Q remains



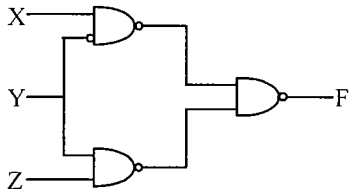
(a) at 1 (b) at 0 (c) at its initial value (d) at unstable

6. Identify the logic function performed by the circuit shown in Fig.



(a) exclusive OR (b) exclusive NOR (c) NAND (d) NOR

. Consider the following circuit.



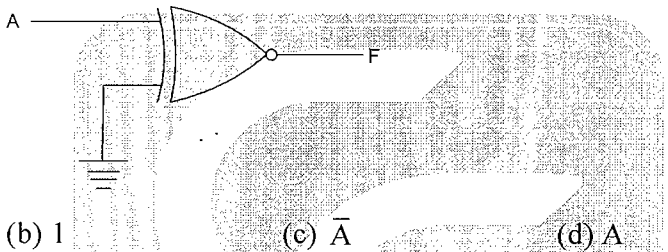
Which one of the following is **TRUE**?

(a) F is independent of X
(b) F is independent of Y
(c) F is independent of Z
(d) None of X, Y, Z is redundant

. Boolean expression for the ouput of XNOR (Equivalent) logic gate with inputs A and B

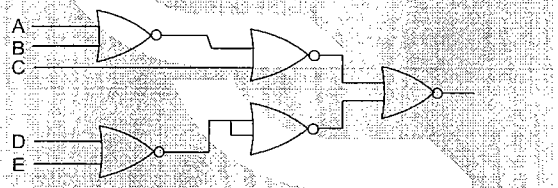
(a) $A\bar{B} + \bar{A}B$ (b) $\overline{AB} + AB$ (c) $(\bar{A} + B)(A + \bar{B})$ (d) $(\bar{A} + B)(A + B)$

. The ouput of the logic gate in fig. is



(a) 0 (b) 1 (c) $\bar{A}$ (d) A

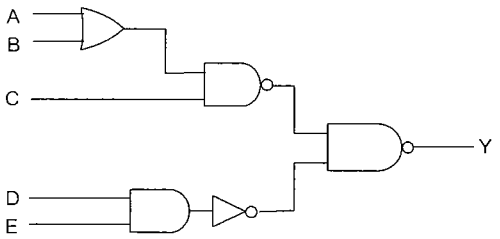
0. The circuit shown in figure realizes the function.



(a) $A + B + C\ (D\bar{E})$ (b) $[\bar{A} \cdot \bar{B} + C]\,\bar{D} \cdot \bar{E}$

(c) $(A+B+C)(\bar{D}\bar{E})$ (d) $(\bar{A}+\bar{B}+\bar{C})(\bar{D}\bar{E})$

1. The ouput Y_0 the circuit shown in the figure is



(a) (A + B) C + DE (b) AB + C (D + E)

(c) (A + B) C + D + E, (d) (AB + C) DE

2. The output of a logic gate is '1' when all its inputs are at logic 'O' Then the gate is either

(a) a NAND or an Ex-OR gate
(b) a NOR or an EX-NOR gate
(c) an OR or an EX-NOR gate
(d) an AND or an EX-OR gate

13. Match List - I (Boolean statement/ Statements is true) with List-II (Conditions under which) and select the corrent answer using the codes given below the lists:

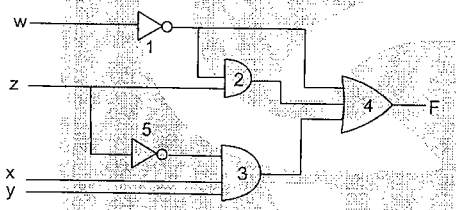
List - I	List - II
A. $A \oplus B = 0$	1. $A \neq B$
B. $\overline{A+B} = 0$	2. $A = B$
C. $\overline{A} \cdot B = 0$	3. $A = 0$ and $B = 1$
D. $A \oplus B = 1$	4. $A = 1$ and $B = 0$

(a) A - 2, B - 3, C - 4, D - 1 (b) A - 1, B - 3, C - 4, D - 2
(c) A - 2, B - 4, C - 3, D - 1 (d) A - 1, B - 4, C - 3, D - 2

14. The minimized form of the logical expression
$(\overline{A}\overline{B}\overline{C} + \overline{A}B\overline{C} + \overline{A}BC + AB\overline{C})$ is
(a) $\overline{A}\overline{C} + B\overline{C} + \overline{A}B$ (b) $A\overline{C} + \overline{B}C + \overline{A}B$ (c) $\overline{A}C + \overline{B}C + \overline{A}B$ (d) $A\overline{C} + \overline{B}C + A\overline{B}$

15. For the identity, $AB + \overline{A}C + BC = AB + \overline{A}C$. the dual form is
(a) $(A + B)(\overline{A}+C)(B+C) = (A+B)(\overline{A}+C)$ (b) $(\overline{A} + B)(A+\overline{C})(\overline{B}+\overline{C}) = (\overline{A}+\overline{B})(A+\overline{C})$
(c) $(A + B)(\overline{A}+C)(B+C) = (\overline{A}+\overline{B})(A+\overline{C})$ (d) $\overline{A}\overline{B} + A\overline{C} + \overline{B}\overline{C} = \overline{A}\overline{B} + A\overline{C}$

16. Consider the following gate network.

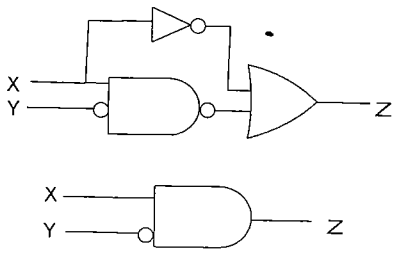


Which one of the following gates is redundant?
(a) Gate No. 1 (b) Gate No. 2 (c) Gate No. 3 (d) Gate No. 4

17. How is inversion achieved using EX-OR gate?
(a) Giving input signal to the two input lines of the gate tied together
(b) Giving input to one input line and logic zero to the other line
(c) Giving input to one input line and logic one to the other line
(d) Inversion cannot be achieved using EX-OR gate

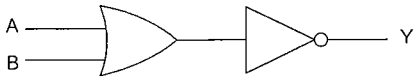
18. The logic funciton $f = \overline{(x.\overline{y}) + (\overline{x} + y)}$ is the same as
(a) $f = 0$ (b) $f = \overline{(\overline{x}.\overline{y}) + (x.y)}$ (c) $f = \overline{(x.y)}\,(\overline{x}.\overline{y})$ (d) $f = \overline{x}.\overline{y} + xy$

19. The logic operation of two combinational circuits shown in the figures below are



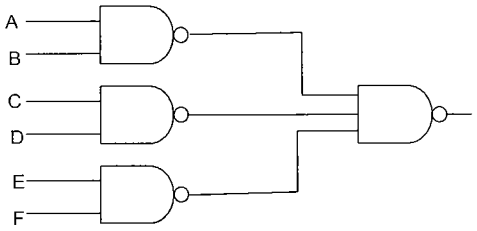
(a) Complementary (b) identical (c) Entirely different (d) None of these

0. For the circuit shown, Y is given by



(a) $\overline{A} + \overline{B}$ (b) $\overline{A}\overline{B}$ (c) $\overline{AB}$ (d) none of these

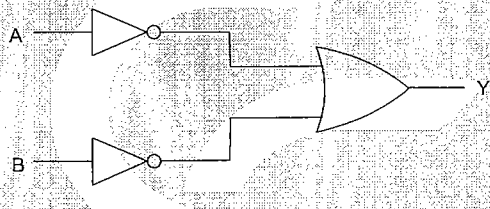
1. The output of the gated network shown in the given figure is



(a) $(\overline{AB})\,(\overline{CD})\,(\overline{EF})$ (b) $\overline{AB} + \overline{CD} + \overline{EF}$

(c) AB + CD + EF (d) (A + B (C + D) (E + F)

2. In the circuit shown, the input word is AB and ouput is Y. The value of input for Y = 0 is.



(a) 00 (b) 01 (c) 10 (d) 11

3. For the circuit shown below, output F is

(a) F = 1 (b) F = 0 (c) F = X (d) $F = \overline{X}$

4. The addition of two binary variables A and B results into a SUM and a CARRY ouput. Consider the following expressions for the SUM and CARRY ouput:

1. $SUM = A.B + \overline{A}\overline{B}$
2. $SUM = A.\overline{B} + \overline{A}.B$
3. CARRY = A.B
4. CARRY = A + B

Which of these expressions are correct?

(a) 1 and 3 (b) 2 and 3 (c) 2 and 4 (d) 1 and 4

5 Which one of the following is equivalent to AND - OR realization?

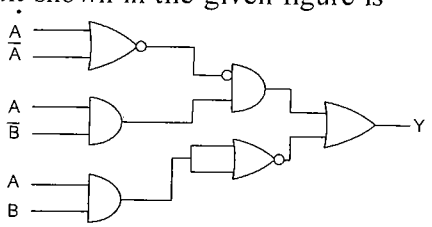
(a) NAND-NOR realizaiton

(b) NOR-NOR realizaiton

(c) NOR-NAND realization

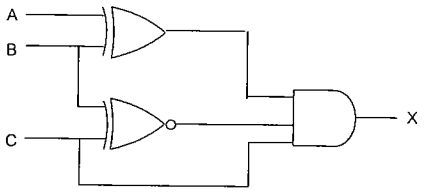
(d) NAND-NAND realization

26. The output Y for the logic circuit shown in the given figure is



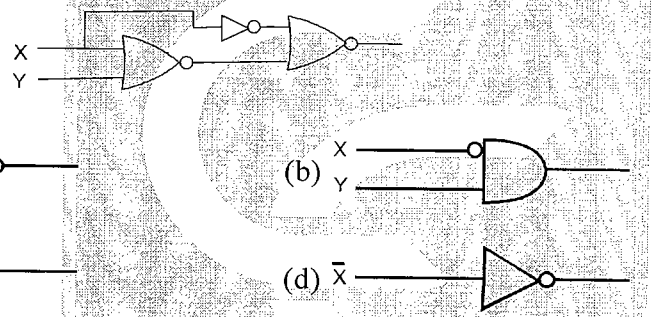
(a) $A\bar{B}$ (b) $\bar{A}+\bar{B}$ (c) $\bar{A}\bar{B}$ (d) $A+\bar{B}$

27. For the logic circuit shown in fig. the required input condition (A,B,C) to make the ouput (X) = 1 is



(a) 1, 0, 1 (b) 0, 0, 1 (c) 1, 1, 1 (d) 0, 1, 1

28. The logic circuit shown in the given figure can be minimized to

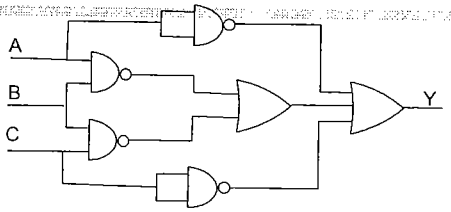


(a) X (b) X Y

(c) X Y (d) $\bar{X}$

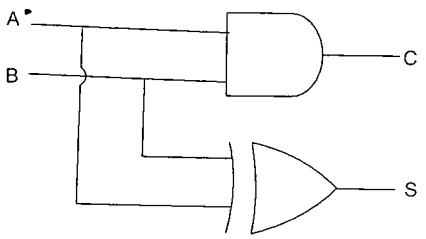
29. Which one of following sets of gates are best suited for parity checking and parity generation?
(a) AND, OR, NOT gates (b) X-OR X-NOR gate
(c) NAND gates (d) NOR gates

30. For the logic circuit shown in the figure, the ouput Y is equal to



(a) $\overline{A\bar{B}C}$ (b) $\bar{A}+\bar{B}+\bar{C}$ (c) $\bar{A}\bar{B}+\overline{BC}$ (d) $\bar{A}\bar{B}+\bar{B}$

31. For the circuit shown, for AB = 00, AB = 01 C, S values respectively are



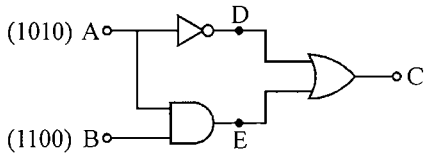
(a) 0,0, and 0.1 (b) 0,0 and 1.0 (c) 0.1 and 0.0 (d) 1.0 and 0.0

Multiple Select Questions (MSQ) :

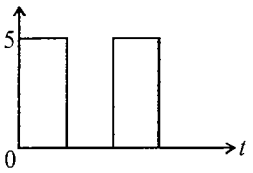
Find out the correct equivalence of the expression $A \oplus B$.

(a) $\bar{A}B + A\bar{B}$ (b) $AB + \bar{A}\bar{B}$ (c) $\overline{\overline{A+B} + \overline{\bar{A}+\bar{B}}}$ (d) $A\bar{B} + \bar{A}B$

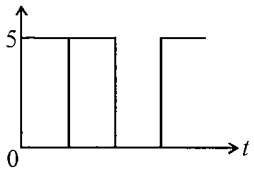
In the circuit shown below find out the correct pulse pattern. Assume 1 as 5 volt and 0 at 0 volt.



(a) At point A,



(b) At point C,

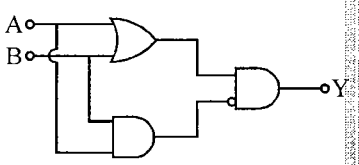


(c) At point D,

(d) At point B,

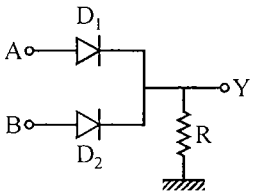
In the circuit shown below, when the inputs A = B = 0, what are the possible logic states of C and D ?

(a) C = 0, D = 1 (b) C = 1, D = 1 (c) C = 1, D = 0 (d) C = 0, D = 0



(a) The output $Y = A\bar{B} + B$ (b) The output $Y = (A + B)\overline{AB}$

(c) Representing XOR gate (d) Representing AND gate



(a) This circuit will act as an OR gate (b) when A = 0, B = 1, output Y = 1

(c) when B = 1, A = 1, output Y = 1 (d) This is an AND gate

An EX-OR gate is used

(a) as an equality detector (b) in arithmetic senction of computer

(c) as universal gate (d) as an inequality detector

A NAND can be buid up using

(a) five NOR gate (b) three NOR gate

(c) two NOT gate and a OR gate (d) One XOR and one NOT gate

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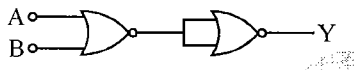
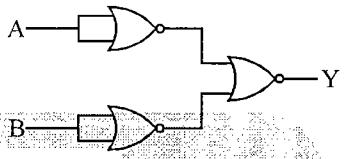
8. Out of the following statements find out the correct one, which represents De Morgan's theorem.
(a) The product of variables is equal to the complements of the sum of the variables
(b) The complement of a sum of variables is equal to the product of the complements of the individual variables
(c) The complement of a product of variables is equal to the sum of the complements of the individual variables
(d) The sum of the complements of individual variable is equal to the product of the complement of the individual variables.

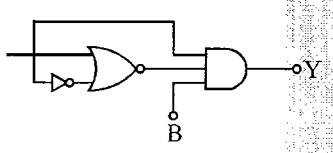
9. Simplified form of the expression, $ABC + \bar{A}BC + A\bar{B}C + \bar{A}B\bar{C} + \bar{A}\bar{B}C + \bar{A}\bar{B}\bar{C}$

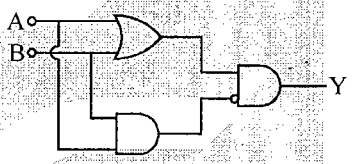
(a) $\bar{C} + AC$ (b) $\bar{A} + C$ (c) $1 + \bar{B}$ (d) $C + \bar{A}\bar{C}$

10. Which of the following is representing a AND gate ?

(a) 

(b) 

(c) 

(d) 

11. Duality theorem converts / simplify
(a) Minimum term to maximum term (b) SOP to POS
(c) Positive logic to negative logic (d) Remove redundancy

Numerical Answer Type (NAT) :

1. The AND function can be realized by using only n number of NOR gates. The n equal to _____

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2. Minimum number of two input NAND gate to design function $f = x\bar{y}z$ _____

3. $(21)_3 = \sqrt{(100)_x}$, the value of x = _____

Multiple Choice Questions (MCQ)

1. (d)	2. (b)	3. (a)	4. (c)	5. (d)
6. (b)	7. (d)	8. (c)	9. (c)	10. (b)
11. (a)	12. (b)	13. (a)	14. (a)	15. (a)
16. (b)	17. (c)	18. (a)	19. (a)	20. (b)
21. (d)	22. (b)	23. (b)	24. (d)	25. (b)
26. (d)	27. (d)	28. (b)	29. (b)	30. (a)
31. (d)				

Multiple Select Questions (MSQ)

1. (a), (c)	2. (a), (b)	3. (a), (c)	4. (b), (c)
5. (a), (b), (c)	6. (b), (d)	7. (a), (c)	8. (b), (c)
9. (b), (d)	10. (b), (c)	11. (a), (b), (c)	

Numerical Answer Type (NAT)

1. (3) 2. (5) 3. (7)

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